

# Endogenous Structural Transformation in Economic Development

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**Abstract:**

This paper proposes a framework to model how a country develops its economy by endogenous structural transformation and efficient resource allocation in a market mechanism. To achieve this goal, the paper first summarizes three attributes of economic structures from the literature, namely, structurality, durationality, and transformality, and discuss their implications for methods of economic modeling. Then, with the common knowledge assumption, the paper studies a Ramsey growth model with endogenous structural transformation in which the social planner chooses the optimal industrial structure, resource allocation with the chosen structure, and consumption to maximize the representative household's total utility subject to the resource constraint. The paper next establishes the mathematical underpinning of the static, dynamic, and structural equilibria. The Ramsey growth model and its equilibria are then extended to economies with complicated economic structures consisting of hierarchical production, composite consumption, technology adoption and innovation, infrastructure, and economic and political institutions. The paper concludes with a brief discussion of applications of the proposed methodology to economic development problems in other scenarios.

**Keywords:** Endogenous structural transformation, structural equilibrium, Hamilton-Jacobi-Bellman equations with variational inequalities, viscosity solutions, structurality, durationality, transformality

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# Contents

<b>1</b>	<b>Introduction</b>	<b>1</b>
<b>2</b>	<b>Structurality, durationality, and transformality</b>	<b>4</b>
2.1	Illustration and discussion . . . . .	5
2.2	Implication for the methodology of economic modeling . . . . .	6
<b>3</b>	<b>A Ramsey growth model with endogenous structural transformation</b>	<b>8</b>
3.1	Representation of economic activities and their structures . . . . .	8
3.2	Knowledge on economic structures . . . . .	11
3.3	Decisions on structural transformation and resource allocation . . . . .	11
3.4	Social planner's objective . . . . .	12
<b>4</b>	<b>Competitive equilibrium</b>	<b>13</b>
4.1	Solution of the extended Ramsey model . . . . .	13
4.2	Static, dynamic, and structural equilibria . . . . .	14
4.3	Transformation region and comparative structural advantage . . . . .	17
4.4	Stationary form of the economy . . . . .	18
4.5	Examples of upgrading industrial structures . . . . .	19
<b>5</b>	<b>Complex economic structures and stagewise development</b>	<b>21</b>
5.1	Intermediate goods and hierarchical production structures . . . . .	22
5.2	Structures of consumer preference . . . . .	25
5.3	Structures of technological progress . . . . .	28
5.4	Infrastructure and economic institutions . . . . .	34
5.5	Political regimes and institutions . . . . .	36
<b>6</b>	<b>Concluding remarks</b>	<b>38</b>
	<b>References</b>	<b>40</b>

# 1 Introduction

The goal of this paper is to develop a framework of modeling endogenous structural transformation and efficient resource allocation in a market economy during different development stages. Following North (1981), we define “structure” as the characteristics of an economy, which are the basic determinants of economic activities with three attributes, namely, structurality, durationality, and transformality, on which we will elaborate in the next section. <sup>1</sup>“Structural transformation” refers to changes in the basic determinants of the composition and organization of economic activities during the process of economic development. “Endogenous structural transformation” means that, for the purpose of economic development, the social planner in the economy makes use of knowledge on economic structures and chooses optimal economic structures to transform during economic development. Finally, “from the early, catching-up stage to the advanced, sustained stage” means the process of how a less developed economy moves from inside the world production possibilities frontier toward its frontier.

To illustrate the idea and present our framework, we begin with the study of structural transformation and economic growth in the neoclassical sense. Structural transformation, or structural change, has been referred to as the process of a country’s productive resources relocating from low-productivity to high-productivity economic activities during the past decades. Lewis (1954) argues the importance of structural transformation from a dual economy to an industrialized market economy in the first stages of economic development, and Kuznets (1966, 1973) explains the economic growth as a sustained increase in per capita income accompanied by “sweeping structural changes.” <sup>2</sup> Kuznets (1966, 1973) further doc-

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<sup>1</sup>The literature has not yet provided a widely accepted definition of the economic structure of an economy. In this paper, economic structure refers to a collection of basic determinants of the composition and organization of production, consumption, distribution, exchange, and other economic activities in an economy. This mainly includes production structures, technological structures, market structures, consumer preference or consumption structures, population structures, financial system structures, trade structures, and even institutional and cultural structures in an economy.

In addition to a country’s economic characteristics, economists consider noneconomic characteristics as determinants of economic activities. For instance, Kuznets (1966, p.437) assumes that “the economic and many noneconomic characteristics of social structure are interrelated as both causes and effects,” and illustrates “some of the noneconomic characteristics associated with differences in economic development and structure between underdeveloped and developed countries bear upon: (1) demographic patterns, (2) political structure, and (3) cultural aspects.”

In his book explaining the stability or change of economic structures in economic history, North (1981, p.1) explains “structure” as follows. “By ‘structure’ I mean those characteristics of a society which we believe to be the basic determinants of performance. Here I include the political and economic institutions, technology, demography, and ideology of a society.”

<sup>2</sup>Kuznets (1966, p.1) explains the economic growth as follows: “We identify the economic growth of nations as a sustained increase in per capita or per worker product, most often accompanied by an increase in population and usually by sweeping structural changes. In modern times these were changes in the industrial structure within which product was turned out and resources employed—away from agriculture toward nonagricultural activities, the process of industrialization; in the distribution of population between the countryside and the cities, the process of urbanization; in the relative economic position of groups within

uments the long-run transformation in several economies at the sector level, that is, the reallocation of economic resources from agriculture to the manufacturing and services sectors<sup>3</sup> and lists it as one of the six major stylized factors of long-term growth.<sup>4</sup> To explain these sectoral reallocation processes, multisector models are developed to explore the linkage between sectoral structural transformation and balanced or non-balanced growth and explain sectoral structural transformation as the result of changes in income and/or relative prices of goods.<sup>5</sup> These multisector models generate results that are consistent with the “stylized facts” of sectoral structural transformation and provide insights on several interesting economic issues, such as economic development, regional income convergence, and aggregate productivity trends. However, they do not feature Kuznets’ (1961) sweeping structural change, due to the following two reasons.

First, Kuznets’ sweeping structural change involves changes in basic determinants of the composition and organization of economic activities, such as agrarian production and other institutions related to modern industrialized production and related institutions. By contrast, sectoral structural transformation in the multisector models deals only with variation in resource allocation among three given sectors under a specific economic structure.<sup>6</sup> Zooming in on structural transformation from the sector level to the industry level, changes in resource allocation among the three given sectors—agriculture, industry, and services—result from the transformation of industrial structures, and such transformation consists of the birth and decay of various industries during economic development, which the multisector growth models cannot describe.<sup>7</sup> Second, Kuznets’ sweeping structural change is referred to as structural transformation involving changes in the composition of industries;

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the nation distinguished by employment status, attachment to various industries, level of per capita income, and the like; in the distribution of product by use—among household consumption, capital formation, and the government consumption, and within each of these major categories by further subdivisions; in the allocation of product by its origin within the nation’s boundaries and elsewhere; and so on.”

<sup>3</sup>Kuznets (1973, p. 248) concludes that “major aspects of structural change include the shift away from agriculture to nonagricultural pursuits and, recently, away from industry to services.”

<sup>4</sup>Stylized facts on sectoral structural transformation have been documented in the literature, which is too large to be entirely listed here. Sectoral structural transformation is usually demonstrated via sectoral shares of employment, value added, and final consumption expenditure; see a recent summary on this in Herrendorf, Rogerson, and Valentinyi (2014, section 2).

<sup>5</sup>See Acemoglu and Guerrieri (2008), Herrendorf, Rogerson, and Valentinyi (2014), and the references therein.

<sup>6</sup>Acemoglu (2009, pp. 693-696) refers to “structural change” as changes in the composition of production and employment and uses the term “structural transformation” to describe changes in the organization and efficiency of production accompanying the process of development. Acemoglu explains that, “one might expect Kuznets’ structural change to be accompanied by a process that involves the organization of production becoming more efficient and the economy moving from the interior of the aggregate production possibilities set toward its frontier.” Therefore, “we would like to develop models that can account for both the structural changes and transformations at the early stages of development and the behavior approximated by balanced growth at the later stages.”

<sup>7</sup>Ju, Lin and Wang (2015) provide an empirical study on this and further explain the phenomenon by an infinite-industry growth model that assumes geometrically distributed production functions and exogenous goods prices.

rural-urban population ratios; hard infrastructure; and social, economic, legal, and political institutions and so forth. Multisector models only deal with sectoral structural transformation in the process of economic growth and hence fail to explain different kinds of structural transformation in the economic history of many countries in Europe, the Americas, Asia, and Africa.<sup>8</sup> Realizing the limits of the multisector growth models, some theoretical economists highlight the importance of characterizing “sweeping structural changes” and advocate for a unified theoretical framework to characterize the process of structural transformation and resource allocation during economic development and growth.<sup>9</sup> However, such a framework has never been developed.

To bridge this gap, this paper starts with a summary of three attributes of economic structures and proceeds step by step to develop a theoretical framework that characterizes an economy’s development and growth via *endogenous structural transformation* (EST) and effective resource allocation. Section 2 argues that all structures possess three important attributes: structurality, durationality, and transformality. In addition to illustrating and discussing these three attributes, the section discusses the implications of the attributes on methods of modeling structural transformation.

Section 3 presents an extended Ramsey model with EST, by assuming that knowledge on the world’s industrial structures can be freely obtained and a unique final good can be produced by different aggregate production functions. The economic structure of the economy is featured with a specific aggregate production function, and the social planner in the economy must optimally and dynamically choose an economic structure, resource allocation within the chosen structure, and a consumption level over time to maximize the representative household’s total utility subject to the endowment constraint.<sup>10</sup>

By using a decoupled representation of economic activities and their structures, the social planner’s objective can be written as an infinite-horizon optimal control problems for hybrid dynamical systems, which evolve on continuous-variable state spaces and subject to continuous controls and discrete transitions. Section 4 solves the social planner’s maximization problem in the extended Ramsey model, shows that the value function satisfies *Hamilton-Jacobi-Bellman equations and quasi-variational inequalities* (HJBQVI), and establishes the associated competitive equilibrium theory. We demonstrate that, in addition to the

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<sup>8</sup>Early efforts characterizing economic development in underdeveloped countries include Hirschman’s (1958) emphasis on unbalanced growth, Nurkse’s balanced growth, and Rosenstein-Rodan’s big push model. Dual economic models were developed to describe the process of structural changes that occurred in the early stages of economic development; see Lewis (1954), Ranis and Fei (1961), and others.

<sup>9</sup>Acemoglu (2009, pp. 693-696) argues as follows: “A useful theoretical perspective might therefore be to consider the early stages of economic development taking place in the midst of—or even via—structural changes and transformations. We may then expect these changes to ultimately bring the economy to the neighborhood of balanced growth, where our focus has so far been. If this perspective is indeed useful, then we would like to develop models that can account for both the structural changes and transformations at the early stages of development and the behavior approximated by balanced growth at the later stages.”

<sup>10</sup>Note that the maximization problem here is distinct from the representative household’s maximization problem in the neoclassical economic model, which only deals with resource allocation (or more specifically, optimal consumption) and can be solved mathematically by the theory of infinite-horizon optimal control.

static and dynamic equilibria, the competitive equilibrium in the extended Ramsey model contains a third type of equilibrium, which we refer to as the *structural equilibrium*. While the former two equilibria determine efficient resource allocation and imply optimal paths of economic activities in a given economic structure, the structural equilibrium characterizes optimal economic structures at each time period and hence their path. In the special case that the economy has only a single economic structure to choose, the structural equilibrium degenerates so that the extended Ramsey model reduces to the neoclassical growth model. From this perspective, the extended Ramsey model extends the neoclassical economy from a single structure to multiple structures with possible transformation among them. Section 4 further discusses the impacts of the factor endowments on the optimal structures and transformation regions of the factor endowments and provides some examples and their implications for economic development.

The extended Ramsey model in section 3 and its associated competitive equilibrium in section 4 deal with transformation of simple structures (or aggregate production functions of the final goods), but the idea of the model can be further extended and applied to economies that have complex structures at different development stages. Section 5 illustrates this by describing EST in different types of structures and discussing their patterns of stagewise development and growth. The first type contains a class of hierarchical production structures with which intermediate and final goods are produced with exogenous technological progress. The second type involves a combined production and consumption structure with which consumption and investment goods are produced under exogenous technological progress and a composite of consumption goods is consumed. The third type deals with structures of endogenous technological progress achieved by adoption and/or *research and development* (R&D) and their transformation. The fourth type deals with transformation of economic institutions involving infrastructures and economic policies. The fifth type handles structural transformation of political institutions.

Section 6 provides a brief discussion and concluding remarks on insights from the EST and proposes areas for future research. Given that the mechanism of EST in the current framework is based solely on the market, the section discusses the significance and necessity of a framework that integrates the market mechanism and government intervention.

## 2 Structurality, durationality, and transformality

In the literature, an economy's structures usually consist of economic and noneconomic characteristics of activities. The former include a set of specifications on the composition and organization of production and other activities, and the latter consist of a collection of rules on economic and political institutions and even the norms, values, and ideologies of the society. Since these characteristics determine the ways in which economic activities are organized and managed, they are different from numerical economic variables that measure the levels of input and output of economic activities. In particular, three specific attributes are found in an economy's structures—structurality, durationality, and transformality.

## 2.1 Illustration and discussion

The first attribute is *structurality*, which refers to the organic relationships of all the economic and noneconomic structures, each with specific characteristics, in the overall structure of an economy or society. An economy's overall structure is a collection of all the structures, which are organized and interconnected according to certain rules. Different schools of thought have proposed different theories to explain the fundamental determinant of an economy's overall structure and the change, say, from an agrarian economy to a modern industrialized economy. For example, according to Weber (1904, 1905), it is value; according to Marx and Engels (1848), it is productive force; and according to North (1981), it is the land-to-labor ratio that determines the overall structure of a society.<sup>11</sup> In neoclassical economics, the competitive equilibrium suggests that, among all the endogenous variables, capital intensity (or generally, the factor endowments) determines efficient resource allocation, which includes production, consumption, and endogenous technological changes. In institutional economics, the role of institutions in shaping economic behavior is highlighted, and institutional structure seems to be more fundamental than the factor endowment. In new structural economics the endowment structure determines the comparative advantages, that is, the industrial structure and appropriate hard infrastructure and soft institutions of an economy in the process of development. In the literature, the word "structure" is sometimes used to refer to the overall structure or alternatively the totality of all (sub)structures in a society; at other times, it is used to refer to a specific (sub)structure in the overall structure. In the latter case, an adjective is often added, for example, the endowment structure, the industrial structure, the preference structure, the financial structure, the legal structure, the institutional structure, the political structure, and so forth.

The second attribute is *durationality*, which means that the overall structure and each of its substructures in an economy will not change instantaneously and will have different levels of stability. Compared with numerical economic variables that measure the input and output levels of economic activities, economic structures describe how economic activities are organized and managed and/or how economic variables are generated and, hence, do not change as fast as numerical economic variables. For instance, an economy's industrial structure does not change instantaneously with variation in firms' output and technology levels. Similarly, a firm's structure, including its equipment and organization, does not change instantaneously with its output level. Furthermore, different (sub)structures show different extents of durationality. One example of this is that quality improvement in Schumpeterian growth occurs during the lifetime of each variety of machine; hence, the characteristics (or structure) of improvement in the quality of each type of machine are less durable than the structure of production that determines whether a particular type of machine should be produced. Another example is that structures of political institutions in an economy are

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<sup>11</sup>In Marx's historical materialism, the superstructure, or institutions, is determined by and impacts the economic base, which consists of the productive forces and the production relations determined by the productive forces. The productive forces are determined by the prevailing technology and industries in an economy.

more durable than other kinds of economic (sub)structures, such as technological progress structures, industrial structures, and even structures of economic institution. In addition, the overall structure of an agrarian society and its related substructures, including economic, social, and political structures, may have existed for millennia in a society, and change of the overall structure to an industrialized society has taken place only in modern times.

The third attribute is *transformality*, which means that a substructure or even the overall structure is not constant forever; it can transform into another structure under certain conditions during the process of economic development and growth. Transformation of economic (sub)structures can be easily found in countries' development and growth processes and has been widely discussed by economists over the past centuries. Since the literature documenting structural transformation is too large to be entirely reviewed here, we emphasize the difference between the transformality of economic structures and the variability of numerical economic variables. In contrast to numerical economic variables, which change almost instantaneously, economic structures change on a much longer time scales due to their durationality. Furthermore, economic (sub)structures have different degrees of transformality in economic development. For instance, it is typically much easier for a country to transform the structure of industry from labor intensive to capital intensive and technological progress from technological borrowing to indigenous innovation than to transform the institutional structures of the economy.

## 2.2 Implication for the methodology of economic modeling

The three attributes distinguish economic structures from economic variables of resource allocation and also from themselves, and such distinction indicates that economic structures should be characterized by a method different from that of modeling economic variables. In the following, we briefly discuss several issues related to modeling economic structures.

### Decoupling economic structures and economic variables

Since a country's economic activities are measured by their input and output levels and determined by their composition and organization, their characterization can be decoupled into two types of variables. The first type is a set of functional variables that describe economic (sub)structures or determinants of economic activities, and the second type is a set of numerical variables that represent the inputs and outputs of economic activities. As an example of this, consider an economy that has a unique final good produced by aggregate production functions. Capital stock, labor, and output are the numerical variables that serve as inputs and outputs of the production functions, and the aggregate production functions are functional variables that determine the production process and hence represent the production structure of the economy. If exogenous or endogenous technological progress is considered in the economy, the structure of technological progress in production is represented by functionals that determine the technological progress.

The decoupling idea separates economic structures from measurements in economic activities and allows us to deal with functional and numerical variables differently in economic modeling. Specifically, given a functional variable (or an economic sub-structure) that determines specific economic activities, the dynamics of numerical economic variables can be analyzed by existing economic methods. When an economic (sub)structure transforms from one to another, it may be considered as a change of the functional variables. Economic (sub)structures with different extents of durationality may be represented as functional variables on different time scales.

### **Structures of economic (sub)structures**

By representing each economic substructure as a functional variable, a country's economic structure becomes a collection of functional variables that determine various economic activities and are organized by certain economic rules. For instance, consider an economy that has intermediate and final goods. The production structure of the economy is represented as a functional that aggregates the production functions of the intermediate and final goods, and the structure of consumers' preference is described by a functional of intermediate (and/or final) goods. These two substructures are aggregated as part of the economic structure of the economy under the resource constraint.

However, the economic rule of organizing economic (sub)structures may not be unique, and sometimes the cause and effect of economic (sub)structures is controversial among economists. As exploration of such rules goes beyond the scope of this paper, we postulate that the cause and effect of economic (sub)structures are known by economists; hence, the structure of economic (sub)structures can be well specified by economic researchers. Once the cause and effect of economic (sub)structures are clear, the pivot or central structure of all the economic (sub)structures may be identified.

### **knowledge on economic structures**

The transformation of economic structures indicates that more than one economic structure appears in a country's development and growth process. To describe the transformation process, we briefly discuss how knowledge on economic structures is generated for a country. In principle, knowledge on economic structures is created or summarized by countries' political, economic, and intellectual elites. Take the structure of technological progress as an example which refers to the organization of economic activities to improve an economy's technology level. For countries inside the world production possibilities frontier, their knowledge on structures of technological progress consists of not only ways to carry out R&D by themselves, but also the paths of other countries' technological progress for adoption.<sup>12</sup> However, this is not the case for countries on the world production possibilities frontier, since they

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<sup>12</sup>A potentially confusing issue here is the difference between the technological structure and the structure of technological progress. As the former usually refers to the composition and level of technology in production, we refer to the latter as how the level of technology is determined in economic activities.

may not need to know paths of technological progress in countries inside the world production possibilities frontier. Thus, their knowledge on structures of technological progress only consists of R&D and/or learning by doing during their development process.

Similar argument can be applied to knowledge on other kinds of structures. But for structures involving noneconomic characteristics, the knowledge on them may be obtained in different ways. Take the knowledge on structures of legal institutions as an example. Since each country's development, culture, and social environment is unique, his/her knowledge on structures of legal institutions is also unique and may not be useful for other countries.

### **Role of the social planner**

Once the knowledge on economic structures is obtained, the next concern is about who will make use of the knowledge and make decisions on the transformation of economic structures. It may be assumed that such decisions are made by interactions and joint efforts of the countries' political, economic, and intellectual elites at different levels. For example, when a new structure of technological progress for a machine type needs to be adopted in countries inside the global production possibilities frontier, an association of entrepreneurs may make the decision; if such decision needs to be supported by workers with different skill sets trained in the schools or through subsidies provided by the government, the government will decide whether a new technology structure should be adopted and provide education and subsidies accordingly. Decision makers for other types of economic substructures usually depend on countries' economic, political, and cultural institutions and should be carefully studied. To model the idea, we do not discuss the details of this here and assume simply that the social planner can make decisions on the transformation of economic structures.

## **3 A Ramsey growth model with endogenous structural transformation**

This section considers an extended Ramsey growth model in which the social planner makes decisions on resource allocation for production activities within an industrial structure to meet the households' consumption demands and transformation of industrial structure within the given overall structure of the economy.

### **3.1 Representation of economic activities and their structures**

#### **Households, firms, production, and technology**

Let  $\mathcal{H}$  be the set of households in the economy and suppose that the economy admits a representative household. That is, the demand side of the economy can be represented as if there were a single household making the aggregate consumption and saving de-

cisions subject to an aggregate budget constraint. Denote this characteristic by  $\mathcal{H} = \{\text{households in } \mathcal{H} \text{ are representative}\}$ . The behavior of households in the economy is characterized by the pair  $(\mathcal{H}, \mathcal{H})$ .

Similarly, let  $\mathcal{F}$  be the set of firms in the economy and suppose that all firms are representative. That is, all firms in the economy access the same aggregate production function for the final good. Denote this characteristic by  $\mathcal{F} = \{\text{firms in } \mathcal{F} \text{ are representative}\}$ . The behavior of firms in the economy is characterized by the pair  $(\mathcal{F}, \mathcal{F})$ .

Suppose that firms use the aggregate production function  $Y(t) = F(K(t), L(t), A(t))$  to produce the final good, where  $K(t)$ ,  $L(t)$ , and  $A(t)$  are, respectively, the capital stock, employment, and technology used in production at time  $t$ . Let  $\mathcal{Y} = \{Y(t) \in \mathbb{R}^+\}$  represent the output of production and  $\mathcal{Y} = \{F\}$  represent the composition and organization of production. Then,  $(\mathcal{Y}, \mathcal{Y})$  characterizes production and its organizational structure.

The level of the technology  $A$  is exogenously determined by a function of technological progress,  $A(t)$ .  $\mathcal{A} = \{A(t) \in \mathbb{R}^+\}$  denotes the level of technology at time  $t$  and  $\mathcal{A} = \{A : \mathbb{R}^+ \rightarrow \mathbb{R}^+ \text{ be exogenously given.}\}$  represents how the technology is exogenously generated. Then, technology and its structure of production can be characterized by  $(\mathcal{A}, \mathcal{A})$ .  $A(t)$  may depend on the production function.

## Institution, labor, resource constraint, and price

To specify a way of allocating resources, we assume that all goods and factor markets are competitive and complete. Let  $\mathcal{M} = \{\text{the demand and supply of labor, capital, and the goods}\}$  and  $\mathcal{M} = \{\text{households and firms are price-takers and pursue their own goals and prices clear markets and all markets are complete.}\}$ . Then the composition and institutional structure of the market can be expressed as  $(\mathcal{M}, \mathcal{M})$ .

Given  $(\mathcal{M}, \mathcal{M})$ , we explore the firms' demand for labor and capital in the aggregate production  $(\mathcal{Y}, \mathcal{Y}_i)$  and their structures. Assume the population  $\bar{L}$  grows exponentially at rate  $\pi$  ( $\pi > 0$ ), that is,

$$\bar{L}(t) := \bar{L}_\pi(t) = \bar{L}(0) \exp(\pi t), \quad \pi > 0. \quad (3.1)$$

Hence, the population structure is expressed as  $(\bar{\mathcal{L}}, \bar{\mathcal{L}}) = (\{\bar{L}(t)\}, \{\bar{L}_\pi\})$ , where  $\bar{L}_\pi$  represents the functional (3.1). To describe the labor market, note that  $L(t) \in \mathbb{R}^+$  is the amount of demand for labor and  $L(\cdot)$  is a functional characterizing such demand; hence, the labor market and its structure are represented by  $(\mathcal{L}, \mathcal{L}) = (\{L(t)\}, \{L(\cdot)\})$ . When the market is competitive, the labor market-clearing condition is  $L(t) = \bar{L}(t)$ , suggesting that the labor market structure coincides with the population structure, that is,  $(\mathcal{L}, \mathcal{L}) \cong (\bar{\mathcal{L}}, \bar{\mathcal{L}})$ .<sup>13</sup>

The households own the capital stock of the economy and rent it to firms. Under the capital market-clearing condition, the demand for capital by firms equals the supply of

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<sup>13</sup>This further implies that, if there exist two growth rates  $\pi_1 > 0, \pi_2 > 0, \pi_1 \neq \pi_2$ , then  $(\{L(t)\}, \{L_{\pi_1}\})$  and  $(\{L(t)\}, \{L_{\pi_2}\})$  are two different representation of the labor market in the economy.

capital by households, which is denoted  $K(t)$ . The aggregate resource constraint, which is equivalent to the budget constraint of the representative household, requires that

$$\dot{K}(t) = Y(t) - \delta K(t) - C(t), \quad (3.2)$$

where  $C(t) := c(t)L(t)$  is the total consumption, and investment consists of new capital  $\dot{K}(t)$ , and replenishment of depreciated capital  $\delta K(t)$ . The constraint (3.2) suggests that the capital market and its structure can be characterized by  $(\mathcal{K}, \mathcal{K})$ , in which  $\mathcal{K} = \{K(t) \in \mathbb{R}^+\}$ , and  $\mathcal{K} = \{K(\cdot) | K(\cdot) \text{ satisfies (3.2)}\}$ .

Under appropriate assumptions (or specifically, (A1) and (A2) in section 3.2), the factor prices can be obtained by solving the profit maximization problem of the representative firm. Denote by  $R(t)$  and  $w(t)$  the rental price of capital and wage at time  $t$ , respectively. Let  $P(t) := 1$  be the normalized price of the final good. Factor and goods prices and their structures can be expressed as  $(\mathcal{P}, \mathcal{P})$ , in which  $\mathcal{P} = \{R(t), w(t), P(t)\}$  and  $\mathcal{P} = \{\text{factor prices are determined by firms to maximize their profit, and } P(t) = 1\}$ . Furthermore, the rental price and wage under  $(\mathcal{P}, \mathcal{P})$  can be expressed as  $R(t) = \frac{\partial F}{\partial K}(K, L, A)$  and  $w(t) = \frac{\partial F}{\partial L}(K, L, A)$ , respectively.

## Consumption and utility function

We now consider consumption in the economy and its characteristic. Suppose that households only consume the unique final good. Then total consumption is given by  $C(t) = c(t)L(t)$  and the characteristic of the consumption in this economy is  $\mathcal{C} = \{\text{households consume the final good}\}$ . Therefore, consumption and its characteristic are expressed as  $(\mathcal{C}, \mathcal{C})$ . Given the consumption per capita  $c(t)$ , we assume that the representative household has an instantaneous utility function  $u(c)$ , which represents the preference of the household's consumption. Then, the preference of consumption and its structure can be expressed as  $(\mathcal{U}, \mathcal{U}) := (\{u(c)\}, \{u(\cdot)\})$ .

## Economic structure

The above argument describes the following agents' behavior and economic activity components in the economy and their characteristics,  $(\mathcal{H}, \mathcal{H})$ ,  $(\mathcal{F}, \mathcal{F})$ ,  $(\mathcal{Y}, \mathcal{Y})$ ,  $(\mathcal{M}, \mathcal{M})$ ,  $(\mathcal{L}, \mathcal{L})$ ,  $(\mathcal{K}, \mathcal{K})$ ,  $(\mathcal{A}, \mathcal{A})$ ,  $(\mathcal{P}, \mathcal{P})$ ,  $(\mathcal{C}, \mathcal{C})$ , and  $(\mathcal{U}, \mathcal{U})$ . To represent these in a more concise way, we let  $\mathcal{E} := (\mathcal{H}, \mathcal{F}, \mathcal{M}, \mathcal{Y}, \mathcal{L}, \mathcal{K}, \mathcal{A}, \mathcal{P}, \mathcal{C}, \mathcal{U})$  represent agents' behavior and economic activities in the economy, and  $\mathcal{E} := (\mathcal{H}, \mathcal{F}, \mathcal{M}, \mathcal{Y}, \mathcal{L}, \mathcal{K}, \mathcal{A}, \mathcal{P}, \mathcal{C}, \mathcal{U})$  represent the economic and noneconomic characteristics of  $\mathcal{E}$ . Then the overall structure and activities of the economy can be characterized by  $(\mathcal{E}, \mathcal{E})$ .

### 3.2 Knowledge on economic structures

Suppose that the final good in the economy can be produced by  $I$  aggregate production functions or industrial structures in the world:

$$Y(t) = F_i[K(t), L(t), A(t)], \quad i \in \mathbb{I} := \{1, \dots, I\}, \quad (3.3)$$

and knowledge on these industrial structures can be freely obtained by economies in the world. The knowledge on the world's industrial structures for the social planner is then given by  $\mathcal{I}_{\mathcal{Y}} = \{\mathcal{Y}_i | \mathcal{Y}_i = \{F_i\}, i \in \mathbb{I}\}$ . When the social planner of the economy chooses  $F_i$  to produce the final good, the industrial structure of the economy is described by the pair  $(\mathcal{Y}, \mathcal{Y}_i) = (\{Y(t)\}, \{F_i\})$ , and correspondingly, the economy with industrial structure  $\mathcal{Y}_i$  in its overall structure is characterized by  $(\mathcal{E}, \mathcal{E}_i)$ . The knowledge on the industrial structures for the social planner can now be expressed as  $\mathcal{I}_{\mathcal{E}} := \{\mathcal{E}_i | i \in \mathbb{I}\}$ .

Provided the economy and its economic structure  $(\mathcal{E}, \mathcal{E}_i)$ , equations (3.3) and (3.2) imply the following aggregate resource constraint in the economy

$$\dot{K}(t) = F_i[K(t), L(t), A(t)] - \delta K(t) - C(t). \quad (3.4)$$

Suppose that the production function  $F_i[K, L, A]$  exhibits constant returns to scale in  $K$  and  $L$ . The output per capita is given by

$$y(t) \equiv Y(t)/L(t) \equiv f_i(t, k(t)), \quad f_i(t, k(t)) = F_i[k(t), 1, A(t)], \quad (3.5)$$

where  $k(t) \equiv K(t)/L(t)$ . Then, the accumulated capital per capita is given by the equation

$$\dot{k}(t) = f_i(t, k(t)) - (\delta + \pi)k(t) - c(t). \quad (3.6)$$

We assume that  $f_i$  ( $i \in \mathbb{I}$ ) satisfy the following conditions for later discussion:

(A1) For each  $i \in I$ , the production function  $f_i(t, k)$  is twice differentiable, strictly increasing, and concave in  $k$ .

(A2) For each  $i$ ,  $f_i$  satisfies the Inada conditions:  $\lim_{k \rightarrow 0} \partial f_i(t, k) / \partial k = +\infty$  and  $\lim_{k \rightarrow +\infty} \partial f_i(t, k) / \partial k = 0$ . Moreover,  $f_i(t, 0) = 0$  for all  $t$ .

Furthermore, we assume that the utility function  $u(c)$  satisfies the following condition:

(A3) The utility function  $u : \mathbb{R}^+ \rightarrow \mathbb{R}$  is strictly increasing, concave, and twice differentiable, with derivatives  $u'(c) > 0$  and  $u''(c) < 0$  for all  $c$  in the interior of its domain.

### 3.3 Decisions on structural transformation and resource allocation

At each time  $t$ , the social planner determines an economic structure (or more precisely, an industrial structure) for the economy and allocate resources under the given overall structure.

Let  $\theta(t)$  be the economic structure (or with a slight abuse of notation, industrial structure) chosen at time  $t$  by the social planner from the information set of economic structures  $\mathcal{I}_{\mathcal{E}} = \{\mathcal{E}_i | i \in \mathbb{I}\}$ , and denote by  $e(t) \in \mathcal{E}$  households' and firms' behaviors and economic activities at time  $t$ . Then, given the industrial structure prior to time  $t$ ,  $\theta(t-)$ , the social planner decides whether the industrial structure  $\theta(t)$  should be the same as the previous one. If yes, then  $\theta(t) = \theta(t-)$ ; otherwise, the social planner chooses an industrial structure  $\theta(t)$  ( $\neq \theta(t-)$ ) from  $\mathcal{I}_{\mathcal{E}}$  and transforms the economic structure from  $\theta(t-)$  to  $\theta(t)$ . After choosing the industrial structure, the social planner will allocate resources and decide the level of  $e(t)$ . Since all the economic structures in  $\mathcal{I}_{\mathcal{E}}$  are different only in industrial structures  $\mathcal{I}_{\mathcal{Y}}$ ,  $e(t)$  can be expressed as a vector of numerical economic variables  $(k(t), R(t), A(t), w(t), c(t))$ .

The above argument indicates that the path of industrial structures  $\{\theta(t)\}$  is piecewise constant and satisfies the durationality and transformality of structures discussed in section 2. Then, to present the idea, we represent  $\{\theta(t)\}$  in another way. Let  $\tau_n$  denote the time of the  $n$ th structural transformation and  $\kappa_n \in \mathcal{I}_{\mathcal{E}}$  denote the transformed structure at time  $\tau_n$ , for  $n \in \{1, 2, \dots\}$ . Assume that this decision problem starts at time  $t_0$ , and let  $\tau_0 = t_0$  and  $\kappa_0 = \mathcal{E}_i \in \mathcal{I}_{\mathcal{E}}$  (or  $i \in \mathbb{I}$ ). The series of  $\tau_n$  and  $\kappa_n$  satisfy

$$(A4) \quad t_0 = \tau_0 \leq \tau_1 < \tau_2 < \dots < \tau_n < \dots, \text{ and } \lim_{n \rightarrow +\infty} \tau_n = +\infty. \quad \kappa_n \in \mathcal{I}_{\mathcal{E}} \text{ for all } n \geq 0.$$

In (A4),  $\tau_1 \geq \tau_0$  means that the social planner may decide to transform the industrial structure at the starting time  $t_0$ . Let  $\xi = \{(\tau_n, \kappa_n)_{n \geq 0}\}$  be the double series of transformation time and transformed structures, and denote by  $\mathfrak{A}$  the set of all series  $\xi$  satisfying (A4). Then given an  $\xi \in \mathfrak{A}$ , the industrial structure of the economy at time  $t$  is expressed as

$$\theta(t) = \sum_{n \geq 0} \kappa_n 1_{[\tau_n, \tau_{n+1})}(t), \quad \text{for } t \geq \tau_0 = t_0,$$

where  $1_{[\tau_n, \tau_{n+1})}(t)$  is an indicator function of  $t$ , taking value 1 if  $t \in [\tau_n, \tau_{n+1})$  and 0 otherwise. By definition,  $\theta(t)$  are right continuous and have left limits at each  $\tau_n$ , and are piecewise constant over time  $t$ .

### 3.4 Social planner's objective

Suppose there is no market failure during the structural transformation, so that the social planner does not need to intervene the transformation. Then, given an  $\xi = \{(\tau_n, \kappa_n)_{n \geq 0}\} \in \mathfrak{A}$ , the capital stock  $K(\cdot)$  is continuous at transformation times  $\tau_n$ , that is,  $K(\tau_n-) = K(\tau_n)$ . Combining this with (3.4) yields the capital accumulation process

$$\begin{cases} \dot{K}(t) = F_{\kappa_n}[K(t), L(t), A(t)] - \delta K(t) - C(t), & \tau_n \leq t < \tau_{n+1}, \\ K(\tau_n-) = K(\tau_n), & t = \tau_n, n = 1, 2, \dots, \end{cases} \quad (3.7)$$

Accordingly, the capital accumulation process per capita is expressed as

$$\begin{cases} \dot{k}(t) = f_{\kappa_n}(t, k(t)) - (\delta + \pi)k(t) - c(t), & \tau_n \leq t < \tau_{n+1}, \\ k(\tau_n-) = k(\tau_n), & t = \tau_{n+1}, n = 1, 2, \dots \end{cases} \quad (3.8)$$

Thus, provided the initial overall structure  $\theta(t_0) = \mathcal{E}_i$ , the initial level of capital intensity  $k(t_0) = k$ , a path of structural transformation  $\xi = \{(\tau_n, \kappa_n)_{n \geq 0}\}$ , and a path of consumption  $\{c(t)\}_{t \geq 0}$ , the total utility for the representative household starting at time  $t_0$  is expressed as

$$J_i(t_0, k; \{c(t), \xi\}) = \int_{t_0}^{\infty} e^{-(\rho-\pi)t} u(c(t)) dt. \quad (3.9)$$

The social planner's objective is to solve the maximization problem

$$\begin{aligned} V_i(t_0, k) &= \max_{\{c(t), \xi\}} J_i(t_0, k; \{c(t), \xi\}) \\ &\text{subject to (3.8) and } k(t_0) = k \in \mathbb{R}^+, \theta(t_0) = \mathcal{E}_i \in \mathcal{I}_{\mathcal{E}}. \end{aligned} \quad (3.10)$$

Since markets are complete and competitive, given an initial industrial structure  $\theta(t_0) = i$  and an initial capital intensity  $k(t_0) = k$ , the competitive equilibrium is defined as the paths of structures and the amount of consumption and savings  $\{\theta(t), c(t), \dot{k}(t)\}_{t \geq t_0}$  that maximize the household's total utility (3.9) subject to the constraint (3.8).

## 4 Competitive equilibrium

We analyze the competitive equilibrium in the extended Ramsey growth model in this section.

### 4.1 Solution of the extended Ramsey model

Note that the competitive equilibrium in the extended Ramsey model consists of paths of consumption, capital stock, wage rates, rental rates of capital, and industrial structures, such that (i) the social planner maximizes the representative household's utility given the initial economic structure and initial capital intensity, wage rates and rental rates, and (ii) the paths of wage rates and rental rates are such that given the paths of capital stock and labor, all markets clear. Using the method in Xing (2021), we can show the following.

**Proposition 4.1.** *For each  $i \in \mathbb{I}$ ,  $V_i(t, k)$  defined by (3.10) is a viscosity solution to*

$$\begin{aligned} \max \left\{ \sup_{c \in \mathcal{U}} \left[ \frac{\partial V_i}{\partial t}(t, k) + [f_i(t, k) - (\delta + \pi)k - c] \frac{\partial V_i}{\partial k}(t, k) + e^{-(\rho-\pi)t} u(c) \right], \right. \\ \left. \max_{j \neq i} V_j(t, k) - V_i(t, k) \right\} = 0, \end{aligned} \quad (4.1)$$

and such solutions are unique on  $[0, \infty) \times \mathbb{R}^+$ .

The above HJB-QVI system contains two components and their economic interpretation is clear. The first component determines optimal consumption and characterizes the transitional dynamics of the capital intensity and optimal consumption, given that the current economic structure  $\mathcal{E}_i$  (or the current production structure  $\mathcal{P}_i$ ) is optimal. The second

component compares the value functions associated with each of the economic structures and chooses the optimal one. In particular, given economic structure  $\mathcal{E}_i$  prior to time  $t$  and another economic structure  $\mathcal{E}_j$  ( $j \neq i$ ), the social planner compares the total utility associated with  $i$  and the total utility after transforming instantaneously from  $\mathcal{E}_i$  to  $\mathcal{E}_j$ . If the former is larger for any  $j \in \mathbb{I}$  and  $j \neq i$ , the economy should stay with economic structure  $\mathcal{E}_i$  at time  $t$  and then use the HJB component to determine the resource allocation within the given industrial structure and optimal consumption. Otherwise, the social planner should transform the industrial structure and overall economic structure from  $\mathcal{E}_i$  to  $\mathcal{E}_j$  at time  $t$ .

Proposition 4.1 has also the following economic implication for some special cases of  $\mathcal{I}_{\mathcal{E}} = \{\mathcal{E}_1, \dots, \mathcal{E}_I\}$ . When  $I = 1$  or  $\mathcal{I}_{\mathcal{E}} = \{\mathcal{E}_1\}$ , that is, there is only one economic structure available to the social planner, hence  $i \equiv 1$  and the second component of HJB-QVI equation (4.1) is gone. Then (4.1) degenerates to the following HJB equation:

$$\sup_{c \in \mathcal{U}} \left[ \frac{\partial V_1}{\partial t}(t, k) + [f_1(t, k) - (\delta + \pi)k - c] \frac{\partial V_1}{\partial k}(t, k) + e^{-(\rho - \pi)t} u(c) \right] = 0. \quad (4.2)$$

Equation (4.2) is the same as the standard HJB equation in the neoclassical growth model. Hence, the competitive equilibrium associated with HJB-QVI system (4.1) with  $I = 1$  is the same as the one associated with the neoclassical growth model. When  $\mathbb{I} = \{1, 2\}$  or  $\mathcal{I}_{\mathcal{E}} = \{\mathcal{E}_1, \mathcal{E}_2\}$ , HJB-QVI equations (4.1) hold for  $i, j = 1, 2, i \neq j$ , and the social planner of the economy only needs to decide when to transform from the current industrial structure and, thus, the economic structure, to the other one. When  $\mathbb{I} = \{1, \dots, I\}$  and  $I \geq 3$ , the social planner needs to choose not only when but also which industrial structure to transform. The latter choice involves comparing two industrial structures other than the current one.

## 4.2 Static, dynamic, and structural equilibria

Proposition 4.1 implies that there exist three types of equilibria in the EST model—static, dynamic, and structural. We now discuss these equilibria and their implications.

### Static and dynamic equilibria and the Euler equation of consumption

The first two types of equilibria are the static and dynamic equilibria when the economic structure  $\mathcal{E}_i$  prior to time  $t$  is still optimal at time  $t$ . The static and dynamic equilibria are characterized by

$$\begin{cases} \sup_{c \in \mathcal{U}} \left[ \frac{\partial V_i}{\partial t}(t, k) + [f_i(t, k) - (\delta + \pi)k - c] \frac{\partial V_i}{\partial k}(t, k) + e^{-(\rho - \pi)t} u(c) \right] = 0, \\ \max_{j \neq i} V_j(t, k) - V_i(t, k) < 0. \end{cases} \quad (4.3)$$

The second component,  $\max_{j \neq i} V_j(t, k) - V_i(t, k) < 0$ , in (4.3) indicates that there does not exist an economic structure that is better than the current economic structure  $\mathcal{E}_i$ . Consequently, economic structure  $\mathcal{E}_i$  (or production structure  $\mathcal{Y}_i$ ) prior to time  $t$  is still optimal at

time  $t$ . Then,  $\theta(t-) = \theta(t) = \mathcal{E}_i$  and the equation

$$\sup_{c \in \mathcal{U}} \left[ \frac{\partial V_i}{\partial t}(t, k) + [f_i(t, k) - (\delta + \pi)k - c] \frac{\partial V_i}{\partial k}(t, k) + e^{-(\rho - \pi)t} u(c) \right] = 0 \quad (4.4)$$

characterizes the static and dynamic equilibria that are associated with the current economic structure  $\mathcal{E}_i$  and, hence, optimal consumption can be determined. The first-order condition of (4.4) implies that optimal consumption  $c_i^*(t, k)$  solves the equation

$$e^{-(\rho - \pi)t} u(c_i^*(t, k)) = \frac{\partial V_i}{\partial k}(t, k). \quad (4.5)$$

Since  $c_i^*(t, k)$  is the optimal consumption associated with economic structure  $\mathcal{E}_i$ , optimal consumption  $c_i^*(t, k)$  and  $c_j^*(t, k)$  under two different economic structures  $\mathcal{E}_i$  and  $\mathcal{E}_j$  ( $i \neq j$ ) are usually different.

To study the dynamics of optimal consumption, we first take the derivative of equation (4.5) with respect to  $k$  and compare the result with (4.5). Then we have

$$\dot{k} \cdot \frac{\partial^2 V_i}{\partial k^2}(t, k) = -\epsilon_u(c_i^*) \cdot \frac{1}{c_i^*} \frac{\partial c_i^*}{\partial t}(t, k) \cdot \frac{\partial V_i}{\partial k}(t, k). \quad (4.6)$$

where  $\epsilon_u(c) = -u''(c) \cdot c/u'(c)$  is the elasticity of the marginal utility  $u'(c)$ . Then taking the derivative of equation

$$\frac{\partial V_i}{\partial t}(t, k) + [f_i(t, k) - (\delta + \pi)k - c_i^*(t, k)] \frac{\partial V_i}{\partial k}(t, k) + e^{-(\rho - \pi)t} u(c_i^*(t, k)) = 0 \quad (4.7)$$

with respect to  $k$  and simplifying the result by (4.6), we obtain the *Euler equation of consumption* associated with economic structure  $\mathcal{E}_i$ :

$$\frac{1}{c_i^*} \frac{\partial c_i^*}{\partial t}(t, k) = \frac{1}{\epsilon_u(c_i^*)} \left[ \frac{\partial f_i}{\partial k}(t, k) - \delta - \pi - \frac{\partial^2 V_i}{\partial t \partial k} / \frac{\partial V_i}{\partial k} \right]. \quad (4.8)$$

This can be viewed as a nonstationary version of the Euler equation in the neoclassical growth models; see Acemoglu (2009, section 8.2.2). Since assumption (A3) suggests that the utility function is monotonically increasing with  $c$ , the procedure of maximizing the total utility by choosing economic structures can be considered a procedure of comparing (4.8) for different economic structures. We will elaborate on this in section 4.4.

Competitive factor markets imply that, when the economic structure at time  $t$  is  $\mathcal{E}_i$ , the rental rate of capital  $R_i(t)$  and the wage rate  $w_i(t)$  are given by

$$R_i(t, k) = \frac{\partial F_i}{\partial K}[K(t), L(t), A(t)] = \frac{\partial F_i}{\partial k}[k(t), 1, A(t)] = \frac{\partial f_i}{\partial k}(t, k), \quad (4.9)$$

and

$$w_i(t, k) = \frac{\partial F_i}{\partial L}[K(t), L(t), A(t)] = f_i(t, k(t)) - k(t) \frac{\partial f_i}{\partial k}(t, k). \quad (4.10)$$

Given economic structure  $\mathcal{E}_i$  and optimal consumption (4.5), the dynamic equilibrium of the capital-labor ratio and optimal consumption is characterized by the equation

$$\dot{k}(t) = f_i(t, k(t)) - (\delta + \pi)k(t) - c_i^*(t, k(t)). \quad (4.11)$$

The static and dynamic equilibria in the EST model are not exactly the same as those in the neoclassical growth models. In the neoclassical growth models, the economic structure is fixed with  $\mathcal{E}_i$ , and hence, static and dynamic equilibria are defined whether the current economic structure  $\mathcal{E}_i$  is optimal or not. By contrast, in the EST model, the static and dynamic equilibria at time  $t$  depend on the optimal economic structure  $\mathcal{E}_i$  at that time. That is, if an economic structure  $\mathcal{E}_i$  is not optimal at time  $t$  and hence is not chosen by the social planner of the economy, static and dynamic equilibria associated with economic structure  $\mathcal{E}_i$  at time  $t$  do not exist.

### Structural equilibria and optimal industrial structures

HJB-QVI system (4.1) also characterizes a third type of equilibrium, which we refer to as *the structural equilibrium*. Under such an equilibrium,

$$\begin{cases} \sup_{c \in \mathcal{U}} \left[ \frac{\partial V_i}{\partial t}(t, k) + [f_i(t, k) - (\delta + \pi)k - c] \frac{\partial V_i}{\partial k}(t, k) + e^{-(\rho - \pi)t} u(c) \right] < 0, \\ \max_{j \neq i} V_j(t, k) - V_i(t, k) = 0. \end{cases} \quad (4.12)$$

The HJB inequality in (4.12) implies that, if the current economic structure were  $\mathcal{E}_i$ , the associated economy would not attain the static and dynamic equilibria no matter how consumption was chosen. Mathematically, the optimality principle fails when the economy is associated with the industrial structure embodied in economic structure  $\mathcal{E}_i$ . In contrast to this, the equality in (4.12) shows that there exists another industrial structure  $\mathcal{E}_j$  (or equivalently, an economic structure  $\mathcal{E}_j \neq \mathcal{E}_i$ ) such that its associated value function (i.e., the maximized total utility) is greater than that associated with economic structure  $\mathcal{E}_i$ . Denoting the new optimal economic structure by  $\mathcal{E}_{j^*(t, k)}$ ,  $j^*$  satisfies the following condition:

$$j^*(t, k) = \arg \max_{j \in \mathbb{I}, j \neq i} V_j(t, k), \quad V_{j^*}(t, k) = V_i(t, k) \quad (4.13)$$

This suggests that the optimal economic structure at time  $t$  is expressed as

$$\theta(t) = \begin{cases} \mathcal{E}_i & \text{if equation(4.3) holds,} \\ \mathcal{E}_{j^*(t, k)} & \text{if equation(4.12) holds,} \end{cases} \quad (4.14)$$

in which  $j^*(t, k)$  is given by condition (4.13).

The discussion above assumes positive switching costs that satisfy assumption (B3). In the degenerate case of vanishing switching costs  $\eta_{ij}(t) \equiv 0$ , the discussion in Section 4.5 implies that the value function  $V_1(t, k) = \dots = V_I(t, k) = V(t, k)$  is a solution of

$$\frac{\partial V}{\partial t}(t, k) + \max_{i \in \mathbb{I}} \left\{ \sup_{c \in \mathcal{U}} \left[ (f_i(t, k) - (\delta + \pi)k - c) \frac{\partial V}{\partial k}(t, k) + e^{-(\rho - \pi)t} u(c) \right] \right\} = 0. \quad (4.15)$$

This suggests that the optimal economic structure at time  $t$  is given by  $\theta(t) = \mathcal{E}_{j^*(t,k)}$ , in which  $j^*(t, k)$  satisfies the following two conditions:

$$j^*(t, k) = \arg \max_{i \in \mathbb{I}} \left\{ \frac{\partial V}{\partial t}(t, k) + \sup_{c \in \mathcal{U}} \left[ (f_i(t, k) - (\delta + \pi)k - c) \frac{\partial V}{\partial k}(t, k) + e^{-(\rho - \pi)t} u(c) \right] \right\}, \quad (4.16)$$

and

$$\frac{\partial V}{\partial t}(t, k) + \sup_{c \in \mathcal{U}} \left[ (f_{j^*}(t, k) - (\delta + \pi)k - c) \frac{\partial V}{\partial k}(t, k) + e^{-(\rho - \pi)t} u(c) \right] = 0. \quad (4.17)$$

The discussion above implies the following property:

**Proposition 4.2.** *The optimal industrial structure in an overall structure at any given time  $t$  and with any given capital intensity  $k$  is determined by structural equilibrium (4.1) at  $(t, k)$ . Moreover, the optimal industrial structure is a function of  $t$  and  $k(t)$ , and hence endogenous to the capital intensity (or more generally, the factor endowments of the economy).*

Furthermore, since  $\mathcal{E}_{j^*(t,k)}$  is the optimal industrial structure for the given  $(t, k)$ , the social planner should transform the economic structure of the economy from  $\mathcal{E}_i$  to  $\mathcal{E}_{j^*(t,k)}$  at  $t$ . Consequently, the optimal consumption, rental rate of capital, and wage rate will shift from  $c_i^*(t, k)$ ,  $R_i(t, k)$ , and  $w_i(t, k)$  to  $c_{j^*}^*(t, k)$ ,  $R_{j^*}(t, k)$ , and  $w_{j^*}(t, k)$ , respectively.

### 4.3 Transformation region and comparative structural advantage

The structural equilibrium tells us how an optimal economic structure is determined for the given  $(t, k)$ . We now consider the issue of finding the set of capital intensities to support structural transformation. Specifically, let  $\mathcal{E}_i \in \mathcal{I}_{\mathcal{E}}$  be the economic structure prior to time  $t$ . We define the transformation region of capital intensities from  $\mathcal{E}_i$  as follows.

$$\mathcal{S}_i(t) = \left\{ k \in (0, \infty) : \max_{j \neq i} V_j(t, k) - V_i(t, k) = 0 \right\}. \quad (4.18)$$

$\mathcal{S}_i(t)$  is a closed subset of  $(0, \infty)$  and represents a set of capital intensities with which the social planner should transform the economic structure away from  $\mathcal{E}_i$ . Consider an economic structure  $\mathcal{E}_j$ , which is different from  $\mathcal{E}_i$ , and define

$$\mathcal{S}_{i,j}(t) = \left\{ k \in \mathcal{S}_i(t) : V_j(t, k) = V_i(t, k) \right\}. \quad (4.19)$$

It is easy to see that

$$\mathcal{S}_i(t) = \bigcup_{j \in \mathbb{I}, j \neq i} \mathcal{S}_{i,j}(t). \quad (4.20)$$

We also define  $\mathcal{N}_i(t)$  as the complement set of  $\mathcal{S}_i(t)$  in  $(0, \infty)$ , which is the so-called *continuation region* or the *no-transformation region* associated with economic structure  $\mathcal{E}_i$ .

$$\mathcal{N}_i(t) = \left\{ k \in (0, \infty) : \sup_{j \neq i} V_j(t, k) < V_i(t, k) \right\}. \quad (4.21)$$

$\mathcal{N}_i(t)$  is an open set and represents a collection of capital intensities with which economic structure  $\mathcal{E}_i$  is optimal. In this open domain, the value function  $V_i(t, k)$  is continuous differentiable and satisfies equation (4.4). Then, by definition,

$$\mathcal{S}_i(t) \cup \mathcal{N}_i(t) = \left[ \bigcup_{j \in \mathbb{I}, j \neq i} \mathcal{S}_{i,j}(t) \right] \cup \mathcal{N}_i(t) = (0, \infty). \quad (4.22)$$

The definition of transformation regions suggests a measure for comparing the comparative advantage of an economic structure over another. Assume that the current economic structure is  $\mathcal{E}_i$  and consider economic structures  $\mathcal{E}_j$  and  $\mathcal{E}_l$  ( $j \neq l, j \neq i$ ). Let

$$\mathcal{H}_{j,l}(t, k) := V_j(t, k) - V_l(t, k), \quad l \neq j, j \neq i. \quad (4.23)$$

$\mathcal{H}_{j,l}(t, k)$  measures the comparative structural advantage of  $\mathcal{E}_j$  over  $\mathcal{E}_l$ . Therefore, we say that economic structure  $\mathcal{E}_j$  dominates economic structure  $\mathcal{E}_l$  if  $\mathcal{H}_i^{j,l}(t, k) > 0$ .

#### 4.4 Stationary form of the economy

The discussion so far has focused on the general or nonstationary form of the maximization problem. We now consider a stationary form of the problem by assuming that  $f_i(t, k) = f_i(k)$  for all  $i \in \mathbb{I}$ . In such case, Proposition 4.1 becomes the following:

**Proposition 4.3.** *For each  $i \in \mathbb{I}$ ,  $v_i(k)$  is a viscosity solution to*

$$\max \left\{ \sup_{c \in \mathbb{U}} \left[ -(\rho - \pi)v_i(k) + [f_i(k) - (\delta + \pi)k - c]v_i'(k) + u(c) \right], \right. \\ \left. \max_{j \neq i} v_j(k) - v_i(k) \right\} = 0, \quad (4.24)$$

and such a solution is unique on  $\mathbb{R}^+$ .

In this economy, the competitive equilibrium consists of time paths of industrial structures, consumption, capital stock, wage rates, and rental rates of capital, such that the social planner maximizes the representative household's total utility, and the time paths of wage rates and rental rates of capital are taken so as to make all markets clear. As an analog of section 5.2, Proposition 4.3 implies that the competitive equilibrium of the economy can be characterized by its three components, the static, dynamic and structural equilibria. Before we characterize the competitive equilibrium, we first note the following results obtained by arguments analogous to those in sections 4.1 and 4.2:

- (i) When  $\mathcal{I}_{\mathcal{E}} = \{\mathcal{E}_1\}$ , HJB-QVI equation (4.24) degenerates to

$$\sup_{c \in \mathbb{U}} \left[ -(\rho - \pi)v_i(k) + [f_i(k) - (\delta + \pi)k - c]v_i'(k) + u(c) \right] = 0,$$

which is the same as the stationary form of the HJB equation for the neoclassical growth model.

(ii) The static and dynamic equilibria of the economy are characterized by the first component in equation (4.24), and the structural equilibrium is characterized by the second component when switching costs satisfy assumption (B3)'.

(iii) When  $\tilde{\eta}_{ij} \equiv 0$ , the discussion in Section 4.5 implies that the value function  $v_i(k)$  satisfies

$$\max_{i \in \mathbb{I}} \left\{ \sup_{c \in \mathbb{U}} \left[ -(\rho - \pi)v_i(k) + [f_i(k) - (\delta + \pi)k - c]v'_i(k) + u(c) \right] \right\} = 0. \quad (4.25)$$

(iv) Since the value function  $v_i(k)$  has a stationary form, the optimal industrial structure is completely determined by capital intensity  $k$ , or generally, by the economy's factor endowments.

(v) Since the value function has a stationary form, regions of transformation and no-transformation can be defined as  $\mathcal{S}_{i,j} = \{k \in (0, \infty) : v_j(k) = v_i(k)\}$ ,  $\mathcal{S}_i = \cup_{j \neq i} \mathcal{S}_{i,j}$  and  $\mathcal{N}_i = \{k \in (0, \infty) : \sup_{j \neq i} v_j(k) < v_i(k)\}$ , respectively. For the vanishing case  $\tilde{\eta}_{ij} \equiv 0$ , regions of transformation and no-transformation can be similarly defined by making use of (4.25).

### Euler consumption equation

Since the stationary form of the value function has the form  $V_i(t, k) = e^{-(\rho - \pi)t}v_i(k)$ , the Euler equation of consumption (4.8) can be simplified as

$$\frac{c_i^*}{c_i^*} = \frac{1}{\epsilon_u(c_i^*)} [f'_i(k) - \delta - \rho]. \quad (4.26)$$

Choosing a *constant relative risk aversion* utility function, that is,  $u(c) = \frac{c^{1-\gamma}}{1-\gamma}$  for  $\gamma \neq 1$ ,  $\gamma \geq 0$  and  $u(c) = \log c$  for  $\gamma = 1$ , the elasticity of the marginal utility of consumption is given by the constant  $\gamma$ , that is,  $\epsilon_u(c_i^*) \equiv \gamma$ . In such case, the increase in consumption depends on the increase in production  $f'_i(k)$ , and hence choosing an optimal economic structure is equivalent to choosing an optimal industrial structure, or more specifically, an optimal production function.

## 4.5 Examples of upgrading industrial structures

To explain this idea, we study an economy of  $I$  industrial structures and its competitive equilibrium and structural transformation regions. Assume that  $L(t) \equiv 1$ , so that  $\pi \equiv 0$ . Let the aggregate production functions, that is, the industrial structures, have the following form:  $f_i(k) = A_i(k - x_i)_+$  for  $i \in \mathbb{I}$ , where  $A_i$  is the technology level,  $(k - x_i)_+ = \max(k - x_i, 0)$ , and  $x_i$  is a threshold for the  $i$ th production function. Assume that  $0 < A_1 < A_2 < \dots < A_I$  and  $0 = x_1 < x_2 < \dots < x_I$ .

Given industrial structures  $\mathcal{Y}_i = \{f_i\}$  at time  $t$ , the accumulation process of the capital per capita is

$$\begin{cases} \dot{k}(t) = (A_i - \delta)(k(t) - x_i)_+ - c(t), & \tau_n \leq t < \tau_{n+1}, \\ k(\tau_n) = k(\tau_n^-), & n = 1, 2, \dots \end{cases} \quad (4.27)$$

Assume that the representative household has a power utility function  $u(c) = c^{1-\gamma}/(1-\gamma)$ ,  $\gamma > 1$ . Then, given the initial condition  $k(0) = k$  and  $\theta(0^-) = i$  ( $i \in I$ ), the representative household's total utility is given by (3.9). The social planner must maximize the total utility subject to the resource constraint (4.27). By Proposition 4.3, we have that, for  $i \in I$ , the value function of the problem is a unique viscosity solution to

$$\begin{aligned} \max \left\{ \sup_c \left\{ [(A_i - \delta)(k - x_i) - c]v'_i(k) - \rho v_i(k) + \frac{c^{1-\gamma}}{1-\gamma} \right\} 1_{\{k \geq x_i\}}, \right. \\ \left. \sup_{j \neq i} v_j(k) - v_i(k) \right\} = 0. \end{aligned} \quad (4.28)$$

Note that the argument in Section 4.5 suggests that  $v(k) := v_1(k) = \dots = v_I(k)$  for all  $k$ .

We first consider the static and dynamic equilibria when the economy is associated with overall structure  $\mathcal{E}_i$  or industrial structure  $f_i(k)$ . Denote the value function in this case by  $\tilde{v}_i(k)$ . The static equilibrium is implied by the first supremum in equation (4.28), which suggests the optimal consumption when the economy has industrial structure  $\mathcal{Y}_i = \{f_i\}$ , then we have  $c_i^* = [v'_i(k)]^{-1/\gamma}$ . Plugging this into the first supremum of equation (4.28), we obtain a partial differential equation for the dynamic equilibrium:

$$-\rho v_i(k) + (A_i - \delta)(k - x_i)v'_i(k) + \frac{\gamma}{1-\gamma} [v'_i(k)]^{\frac{\gamma-1}{\gamma}} = 0, \quad k \geq x_i. \quad (4.29)$$

Solving equation (4.29) yields the value function when the economy stays with industrial structure  $\{f_i\}$ :

$$\tilde{v}_i(k) = Q_i(k - x_i)_+^{1-\gamma}, \quad Q_i := \frac{\gamma^\gamma}{1-\gamma} [\rho + (A_i - \delta)(\gamma - 1)]^{-\gamma}. \quad (4.30)$$

Accordingly, the optimal consumption associated with industrial structure  $\{f_i\}$  is

$$c_i^*(k) = \frac{\rho + (A_i - \delta)(\gamma - 1)}{\gamma} (k - x_i)_+.$$

Plugging  $c^*$  into (4.29) yields the dynamics of the capital intensity

$$\dot{k}(t) = \frac{A_i - \rho - \delta}{\gamma} (k(t) - x_i)_+,$$

and hence

$$(k(t) - x_i)_+ = (k(t_0) - x_i)_+ e^{(A_i - \rho - \delta)(t - t_0)/\gamma}, \quad t \geq t_0.$$

We next discuss the structural transformation regions of the capital intensities. For convenience, we denote for  $1 \leq i < j \leq I$ ,

$$\begin{aligned} a_{ij} &:= \left( \frac{Q_j}{Q_i} \right)^{\frac{1}{1-\gamma}} = \left( 1 + \frac{(A_j - A_i)(\gamma - 1)}{\rho + (A_i - \delta)(\gamma - 1)} \right)^{\frac{\gamma}{\gamma-1}} > 1, \\ k_{ij} &:= x_j + \frac{x_j - x_i}{a_{ij} - 1} > x_j. \end{aligned} \tag{4.31}$$

Obviously,  $a_{ij} = a_{ji}^{-1}$  and  $k_{ij} = k_{ji}$ .

Consider an economy in which only one of  $I = 2$  production functions can be chosen by the social planner. Since  $f_1 \prec f_2$ , we have the following property

**Proposition 4.4.** *Assume that  $\mathbb{I} = \{1, 2\}$ . the value function  $v(k) = \tilde{v}_1(k)1_{\{x_1 < k < k_{12}\}} + \tilde{v}_2(k)1_{\{k_{12} \leq k\}}$ , where  $k_{12}$  is defined by (4.31). In addition, the corresponding optimal consumption is given by  $c(k) = c_1^*(k)1_{\{x_1 < k < k_{12}\}} + c_2^*(k)1_{\{k_{12} \leq k\}}$ .*

In the case that the economy has three industrial structures,  $\mathcal{Y}_i = \{f_i\}$ ,  $i \in \mathbb{I} = \{1, 2, 3\}$ . The competitive equilibrium of the economy in this case is a little more complicated than that with two industrial structures, as the social planner in this economy needs to choose not only when to transform the industrial structure, but where to transform as well.

**Proposition 4.5.** Suppose that  $\mathbb{I} = \{1, 2, 3\}$ . Then  $v(k) = v_1(k) = v_2(k) = v_3(k)$  is given by the following.

- (1) If  $k_{12} < \min\{k_{23}, k_{13}\}$ ,  $v(k) = \tilde{v}_1(k)1_{\{x_1 < k < k_{12}\}} + \tilde{v}_2(k)1_{\{k_{12} \geq k < k_{23}\}} + \tilde{v}_3(k)1_{\{k_{23} \leq k\}}$ . In this case, if the economy's capital per capita starts at  $k \leq k_{12}$ , the order of industrial transformation is 1, 2, and 3, and the corresponding optimal consumption is given by  $c_i^*$ , respectively.
- (2) If  $k_{13} < k_{12}$ ,  $v(k) = \tilde{v}_1(k)1_{\{x_1 < k < k_{13}\}} + \tilde{v}_3(k)1_{\{k_{13} \leq k\}}$ . In this case, if the economy's capital per capita starts at  $k \leq k_{12}$ , the order of industrial transformation is 1 and 3, and the corresponding optimal consumption is given by  $c_i^*$ , respectively.

## 5 Complex economic structures and stagewise development

The extended Ramsey model with EST can be extended to incorporate more complicated structures. The mathematical underpinnings also provides us a general framework to “paste together” models of economic development and growth at different development stages via transformation of economic structures, so that various economic issues and different economic ideas can be discussed on the same platform. In this section, we explain this idea by discussing how to incorporate complex economic substructures, including composite production structures, composite consumer preference, choice of exogenous technology, switching between technology adoption and R&D, and institutional structures.

## 5.1 Intermediate goods and hierarchical production structures

We first consider an economy in which intermediate goods and a unique final good are produced and the final good uses intermediate goods as inputs. Using a variant of the non-balanced growth model (Acemoglu and Guerrieri, 2008) as building blocks, the economy experiences stage-wise economic growth with EST.

### Intermediate goods and technology

The economy is similar to that in section 3, except intermediate goods are also produced. The final good is produced competitively by combining  $m$  intermediate goods with elasticity of substitution  $\epsilon \in [0, \infty)$ , that is,

$$Y(t) = F[Y_1(t), \dots, Y_m(t); \omega] = \left( \sum_{j=1}^m w_j Y_j(t; \omega)^{\frac{\epsilon-1}{\epsilon}} \right)^{\frac{\epsilon}{\epsilon-1}}, \quad (5.1)$$

where  $w_1 > 0, \dots, w_m > 0$ ,  $w_1 + \dots + w_m = 1$ , and  $\omega$  represents the vector of parameters in the production functions and will be specified later. The intermediate goods  $Y_1, \dots, Y_m$  are produced competitively with production functions

$$Y_j(t; \omega) = F_j(K_j, L_j, A_j; \omega) = A_j(t) L_j(t)^{1-\alpha_j} K_j(t)^{\alpha_j}, \quad (5.2)$$

where  $A_j(t)$  is the technology of the  $j$ th intermediate good at time  $t$ . Technological progress in all sectors is exogenous and takes the form

$$\dot{A}_j(t) = \vartheta_j A_j(t), \quad \vartheta_j > 0, j = 1, \dots, m. \quad (5.3)$$

For convenience, we assume that  $\alpha_1 \leq \dots \leq \alpha_m$ , that is, the sectors with larger  $\alpha$ 's are more capital intensive. Capital and labor market clearing requires that at each time

$$K_1(t) + \dots + K_m(t) \leq K(t), \quad L_1(t) + \dots + L_m(t) \leq L(t), \quad (5.4)$$

where  $K$  denotes the aggregate capital stock and  $L$  is total population.  $L_j(t)$  and  $K_j(t)$  ( $j = 1, \dots, m$ ) are nonnegative. Labor  $L(t)$  is supplied inelastically and follows the process  $L(t) = L(0) \exp(\pi t)$ .

Let  $P_j(t; \omega)$  be the price of  $Y_j$  good  $j$  ( $j = 1, \dots, m$ ) at time  $t$  for the given industrial structure  $\omega$ . We normalize the price of the final good,  $P$ , to one at all points, so that

$$1 \equiv P(t; \omega) = \left( \sum_{j=1}^m w_j^\epsilon P_j(t; \omega)^{1-\epsilon} \right)^{1/(1-\epsilon)}. \quad (5.5)$$

Denote the rental price of capital, wage rate, and interest rate by  $R(t; \omega)$ ,  $w(t; \omega)$ , and  $r(t; \omega)$ , respectively.

## Production structures

We now represent the production structure of the economy, using the notation introduced in section 3. Since  $m$  intermediate goods and a final good are produced, production and its structure can be expressed as  $(\mathcal{Y}, \mathcal{Y}(\omega))$ , where  $\mathcal{Y} = \{(Y(t), Y_1(t), \dots, Y_m(t))\}$  and  $\mathcal{Y}(\omega) = \{F, F_1, \dots, F_m\}$ , and  $\omega = (w_1, \dots, w_m, \alpha_1, \dots, \alpha_m)$  is an element of the set  $\Omega_{\mathcal{Y}} = \{w_1, \dots, w_m \geq 0, \sum_{j=1}^m w_j = 1, 0 \leq \alpha_1 \leq \dots < \alpha_m < 1\}$ . Here,  $\mathcal{Y}(\omega)$  describes the composition and organization of industrial structures, producing the final good and intermediate goods. Note that  $w_j = 0$  implies that intermediate goods  $j$  are not produced in the economy. Hence, the change of  $w_j$  from  $w_j = 0$  to  $w_j > 0$  suggests that the composition of intermediate goods in the economy is changed.

Exogenous technology and its structure of production are expressed as  $(\mathcal{A}, \mathcal{A})$ , where  $\mathcal{A} = \{(A_1(t), \dots, A_m(t))\}$ ,  $\mathcal{A}(\tilde{\vartheta}) = \{(A_{1,\vartheta_1}(\cdot), \dots, A_{m,\vartheta_m}(\cdot)) | A_j(\cdot) \text{ are given by (5.3)}\}$ , and  $\tilde{\vartheta} = (\vartheta_1, \dots, \vartheta_m) \in \Omega_{\mathcal{A}} = \{\tilde{\vartheta} | \vartheta_j > 0, j = 1, \dots, m\}$ . Since at most  $m$  intermediate goods are produced, labor allocation in production and its structure are given by  $(\mathcal{L}, \mathcal{L})$ , where  $\mathcal{L} = \{(L(t), L_1(t), \dots, L_m(t))\}$  and  $\mathcal{L} = \{(L(\cdot), L_1(\cdot), \dots, L_m(\cdot)) | L_1(\cdot) + \dots + L_m(\cdot) \leq L(\cdot), L(\cdot) \text{ satisfies (3.1)}\}$ . Similarly, the allocation of capital stock and its structure are expressed as  $(\mathcal{K}, \mathcal{K})$ , where  $\mathcal{K} = \{(K(t), K_1(t), \dots, K_m(t)) | K_1(t) + \dots + K_m(t) \leq K(t)\}$  and  $\mathcal{K} = \{(K(\cdot), K_1(\cdot), \dots, K_m(\cdot)) | K_1(\cdot) + \dots + K_m(\cdot) \leq K(\cdot), K(\cdot) \text{ satisfies the budget constraint (3.4)}\}$ . Goods and factor prices and their structure can be represented as  $(\mathcal{P}, \mathcal{P})$ , where  $\mathcal{P} = \{(R(t), w(t), P_1(t), \dots, P_m(t), P(t))\}$  and  $\mathcal{P} = \{\text{factor prices are determined by firms to maximize their profit, and } P_1(t), \dots, P_m(t), P(t) \text{ satisfy (5.5)}\}$ .

Then with  $(\mathcal{H}, \mathcal{H})$ ,  $(\mathcal{F}, \mathcal{F})$ ,  $(\mathcal{M}, \mathcal{M})$ ,  $(\mathcal{C}, \mathcal{C})$ , and  $(\mathcal{U}, \mathcal{U})$  defined as those in section 3, agents' behavior and economic activities and the economic structure in the economy can still be expressed as  $(\mathcal{E}, \mathcal{E})$ . To highlight the structures of production and exogenous technological progress, which are described by  $\omega \in \Omega_{\mathcal{Y}}$  and  $\tilde{\vartheta} \in \Omega_{\mathcal{A}}$ , respectively, the economic structure of the economy may be expressed as  $\mathcal{E}(\omega, \tilde{\vartheta})$ .

## Social planner's objective and competitive equilibrium

Suppose the social planner's information set of economic structures is  $\mathcal{I}_{\mathcal{Y} \times \mathcal{A}} = \{\mathcal{E}_i | \mathcal{E}_i = \mathcal{E}(\omega_i, \tilde{\vartheta}_i), i \in \mathbb{I}\}$ . The social planner can choose an economic structure for the economy and allocate resources under the chosen economic structure. Assume that an economic structure  $\mathcal{E}_i \in \mathcal{I}_{\mathcal{Y} \times \mathcal{A}}$  (or equivalently,  $i \in \mathbb{I}$ ) is chosen for the economy. The aggregate resource constraint, which is equivalent to the budget constraint of the representative household, has the same form as (3.4). Define the shares of capital and labor allocated to industry  $i$  as  $\xi_j(t) \equiv \frac{K_j(t)}{K(t)}$ , and  $\lambda_j(t) \equiv \frac{L_j(t)}{L(t)}$ . Then output per capita at time  $t$  can be expressed as

$$y(t; \omega) \equiv f(t, k(t); \omega_i) \equiv \left\{ \sum_{j=1}^m w_{ij} [k(t)^{\alpha_{ij}} A_j(t) \xi_j(t)^{\alpha_{ij}} \lambda_j(t)^{1-\alpha_{ij}}]^{\frac{\epsilon-1}{\epsilon}} \right\}^{\frac{\epsilon}{\epsilon-1}}. \quad (5.6)$$

At each time  $t$ , the social planner in the economy must optimally choose an eco-

conomic structure  $\theta(t) = \mathcal{E}_i \in \mathcal{I}_{\mathcal{Y} \times \mathcal{A}}$ , consumption  $c(t)$ , and allocations of capital and labor  $(\xi_j(t), \lambda_j(t))$ . The aggregate resource constraint per capita is

$$\begin{cases} \dot{k}(t) = f_{\kappa_n}(t, k(t); \omega_i) - (\delta + \pi)k(t) - c(t), & \tau_n \leq t < \tau_{n+1}, \\ k(\tau_n-) = k(\tau_n), & t = \tau_{n+1}, n = 1, 2, \dots \end{cases} \quad (5.7)$$

The representative household's total utility, starting at time  $t_0$ , has the same form as (3.9) and is given as

$$J_i(t_0, k; \{c(t), \xi\}) = \int_{t_0}^{\infty} e^{-(\rho-\pi)t} u(c(t)) dt \quad (5.8)$$

The social planner's objective is to solve the maximization problem

$$\begin{aligned} V_i(t_0, k) &= \max_{\{c(t), \xi\}} J_i(t_0, k; \{c(t), \xi\}) \\ &\text{subject to (5.7) and } k(t_0) = k \in \mathbb{R}^+, \theta(t_0) = \mathcal{E}_i \in \mathcal{I}_{\mathcal{Y} \times \mathcal{A}}. \end{aligned} \quad (5.9)$$

Given the above specification, the *competitive equilibrium* of the economy consists of paths for factor and intermediate goods prices  $\{r(t; \theta(t)), w(t; \theta(t)), p_j(t; \theta(t))\}$ , employment and capital allocation  $\{\xi_j(t; \theta(t)), \lambda_j(t; \theta(t))\}$ , consumption and savings decisions  $\{c(t), K(t)\}$ , and economic structures  $\{\theta(t)\}$  such that the utility of the representative household is maximized, firms maximize profits, and markets clear. To characterize the competitive equilibrium mathematically, we first express the maximization problem in the form of the combined optimal control and optimal switching problem in section 4.1 in which

$$\mu_i(t, k, c) = f(t, k(t); \omega_i) - (\delta + \pi)k(t) - c(t), \quad \tau_n \leq t < \tau_{n+1},$$

and the control space

$$\begin{aligned} \mathfrak{U} \equiv \{ & (c(t), \xi_1(t), \dots, \xi_I(t), \lambda_1(t), \dots, \lambda_I(t)) \mid c(t) : [0, \infty) \rightarrow U, \xi_i(t) : [0, \infty) \rightarrow [0, 1], \\ & \lambda_i(t) : [0, \infty) \rightarrow [0, 1] \text{ are Lebesgue measurable functions} \}. \end{aligned}$$

Using the method in Xing (2021), we have the following:

**Proposition 5.1.** *For each  $i \in \mathbb{I}$ ,  $V_i(t, k)$  defined by (3.10) is a viscosity solution to*

$$\begin{aligned} \max \left\{ \sup_{c(t), \lambda_j(t), \xi_j(t)} \left[ \frac{\partial V_i}{\partial t}(t, k) + [f_i(t, k) - (\delta + \pi)k - c] \frac{\partial V_i}{\partial k}(t, k) + e^{-(\rho-\pi)t} u(c) \right], \right. \\ \left. \max_{j \neq i} V_j(t, k) - V_i(t, k) \right\} = 0, \end{aligned} \quad (5.10)$$

and such solutions are unique. In particular, the static and dynamic equilibria of the economy are characterized by the supremum in (5.10), and the structural equilibrium is characterized by the maximum over  $j \neq i$  in (5.10). Furthermore, the path of optimal production structures in the overall economic structure can be determined as in Theorem ??.

Our model extends the two-sector nonbalanced growth model in Acemoglu and Guerrieri (2008) to the case of an  $m$ -sector stagewise nonbalanced growth model with EST. The static and dynamic equilibria in (5.10) can be analyzed similarly as in Acemoglu and Guerrieri (2008).

## Economic implication

The extended EST model of stagewise growth characterizes an economy's endogenous transformation among various production structures and has different variants if more detailed assumptions about the economy can be provided. We now briefly discuss several EST scenarios of the model.

First, consider a problem of economic development in which the economy is at the early development stage and produces only a few intermediate goods. For example, assume the economy produces two intermediate goods, that is,  $\omega = (w_1, w_2, 0, \dots, 0)$ , with  $0 < \alpha_1 < \alpha_2 < 1$ , so the production structure is  $\mathcal{Y}(\omega)$ . This is the case of non-balanced growth studied by Acemoglu and Guerrieri (2008). When the number of intermediate goods in the world is greater than two, that is,  $I > 2$ , development of the economy can be achieved by transforming the production structures from two to three intermediate products. Suppose the economy transforms from  $\mathcal{Y}(\omega)$  to  $\mathcal{Y}(\omega')$ , where  $\omega' = (w'_1, w'_2, w'_3, 0, \dots, 0)$  and  $0 < \alpha_1 < \alpha_2 < \alpha_3 < 1$ . Usually two issues are involved in the transformation: one is when to start producing the third intermediate good and the other is which intermediate goods should be chosen. Using the extended EST model and competitive equilibrium argument here, these two issues can be solved.

Second, consider a problem of economic transition in which the country gives priority to development of capital intensive heavy industries when capital in the country is scarce. Since the optimal economic structure is dependent on the level of capital intensity, the process of economic transition is equivalent to that of structural transformation in which more economic resources need to be allocated to less capital intensive industries, and the path of optimal economic structures characterizes the process of transition of production structures. Such transition process also involves transformation of institutional structures, which will be discussed briefly in section 6.5.

Third, the extended model here also sheds a light on poverty trap and/or middle-income trap problems. Most poverty trap models in the literature argue the existence of poor and nonpoor equilibria and discuss the possibilities of moving from the poor to the non-poor equilibria. They seldom study the poverty trap problem by equilibrium arguments on transformation of economic structures. The extended EST model and its competitive equilibrium can be used to characterize the poor and nonpoor (static and dynamic) equilibria under different economic structures and the transition process of economic structures (or structural equilibrium). In addition to the poverty trap problem, the middle-income trap problem can be similarly discussed when the economy in the EST model is further extended to incorporate countries' trade structures in open economies.

## 5.2 Structures of consumer preference

As an alternative to the EST with hierarchical production structures, we consider an EST model in which production and consumption structures are involved in the transformation.

The model here is extended from a benchmark approximately balanced growth model in Herrendorf, Rogerson, and Valentinyi (2014), which describes proportional structural transformation at the sector level and an economy with stagewise structural transformation with approximately balanced development and growth.

### Composite consumption and production of investment and consumption goods

Assume that consumption at time  $t$ ,  $c(t)$ , is a composite of  $m$  consumption goods,

$$c(t; \omega_c) := c(c_1(t), \dots, c_m(t)) = \left( \sum_{i=1}^m w_i (c_i(t) + \bar{c}_i)^{\frac{\epsilon-1}{\epsilon}} \right)^{\frac{\epsilon}{\epsilon-1}} \quad (5.11)$$

where  $\omega_c = (w_1, \dots, w_m) \in \Omega_{\mathcal{E}}$ ,  $\Omega_{\mathcal{E}} = \{\omega_c | w_1, \dots, w_m \geq 0, w_1 + \dots + w_m = 1\}$ . As discussed in Herrendorf, Rogerson, and Valentinyi (2014, section 3.2), (5.11) can capture two features on the demand side: how household demand reacts to changes in income and relative prices. The term  $\bar{c}_i$  allows for the period utility function to be non-homothetic and, hence, changes in income may lead to changes in expenditure shares even if relative prices are constant.

We assume the following Cobb-Douglas production function, for each of the  $m$  consumption goods and a unique investment good:

$$c_j(t) = k_j(t)^\alpha (A_i(t) n_j(t))^{1-\alpha}, \quad j = 1, \dots, m, \quad x(t) = k_x(t)^\alpha (A_x(t) n_x(t))^{1-\alpha}, \quad (5.12)$$

in which  $k_j(t)$  and  $n_j(t)$  are the capital and labor allocated for good  $j = 1, \dots, m$  and  $x$ . Technological progress in all sectors is exogenous and takes the form

$$\dot{A}_j(t) = \vartheta_j A_j(t), \quad \vartheta_j > 0, j = 1, \dots, m, x. \quad (5.13)$$

We assume that capital and labor are freely mobile between the  $m+1$  goods, so that feasibility requires that in each period:

$$k(t) = k_x(t) + \sum_{j=1}^m k_j(t), \quad 1 = n_x(t) + \sum_{j=1}^m n_j(t). \quad (5.14)$$

Total output is given by

$$y(t; \omega) = p_x(t)x(t) + \sum_{j=1}^m p_j(t)c_j(t), \quad (5.15)$$

where  $p_x(t)$  and  $p_j(t)$  are prices of the investment good and the  $j$ th consumption good, respectively. Since all consumption goods are consumed at time  $t$ , capital accumulates in the form

$$\dot{k}(t) = p_x(t)x(t) - \delta k(t), \quad (5.16)$$

where  $\delta \in (0, 1)$  denotes the depreciation rate.

## Economic structures

To represent the economic structure of this economy, we modify the composition of  $(\mathcal{E}, \mathcal{E})$  as follows. Let  $\mathcal{C} = \{(c(t), c_1(t), \dots, c_m(t))\}$  and  $\mathcal{C}(\omega_c) = \{c(\cdot; \omega_c)\}$ , where  $\omega_c \in \Omega_{\mathcal{C}}$ . The consumption preference and its structure are expressed as  $(\mathcal{C}, \mathcal{C}(\omega_c))$ . The production and its structure are then given by  $(\mathcal{Y}, \mathcal{Y})$ , where  $\mathcal{Y} = \{(c_1(t), \dots, c_m(t), x(t))\}$  and  $\mathcal{Y}(\alpha) = \{(c_1(\cdot), \dots, c_m(\cdot), x(\cdot)) | c_1(t), \dots, c_m(t) \text{ and } x(t) \text{ are given by (5.12)}\}$  for  $\alpha \in (0, 1)$ .

Provided that  $m$  consumption goods and one investment good are produced, the technology in production and its structure can be expressed as  $(\mathcal{A}, \mathcal{A}(\tilde{\vartheta}))$ , where  $\mathcal{A} = \{(A_1(t), \dots, A_m(t), A_x(t))\}$ ,  $\mathcal{A}(\tilde{\vartheta}) = \{(A_1(\cdot), \dots, A_m(\cdot), A_x(\cdot)) | A_1(\cdot), \dots, A_m(\cdot) \text{ and } A_x(\cdot) \text{ are given by (5.13)}\}$ , and  $\tilde{\vartheta} = (\vartheta_1, \dots, \vartheta_m, \vartheta_x) \in \Omega_{\mathcal{A}} = \{\vartheta | \vartheta_1, \dots, \vartheta_m, \vartheta_x > 0\}$ . Labor allocation in production and its structure are expressed as  $(\mathcal{L}, \mathcal{L})$ , where  $\mathcal{L} = \{(n_1(t), \dots, n_m(t), n_x(t))\}$  and  $\mathcal{L} = \{(n_1(\cdot), \dots, n_m(\cdot), n_x(\cdot)) | n_1(\cdot) + \dots + n_m(\cdot) + n_x(\cdot) = 1\}$ . Capital allocation in production and its structure can be similarly expressed. Specifically, let  $\mathcal{K} = \{(k(t), k_1(t), \dots, k_m(t), k_x(t))\}$  and  $\mathcal{K} = \{(k_1(\cdot), \dots, k_m(\cdot), k_x(\cdot)) | k_1(\cdot) + \dots + k_m(\cdot) + k_x(\cdot) = k(t), k(t) \text{ satisfies (5.16)}\}$ . Capital allocation and its structure are then given by  $(\mathcal{K}, \mathcal{K})$ . Goods and factor prices and their structure can be represented as  $(\mathcal{P}, \mathcal{P})$ , where  $\mathcal{P} = \{(R(t), w(t), p_1(t), \dots, p_m(t), p_x(t))\}$  and  $\mathcal{P} = \{\text{factor prices are determined by firms to maximize their profit, and } p_1(t), \dots, p_m(t), p(t) \text{ satisfy (5.14)}\}$ .

Then with  $(\mathcal{H}, \mathcal{H})$ ,  $(\mathcal{F}, \mathcal{F})$ ,  $(\mathcal{M}, \mathcal{M})$ ,  $(\mathcal{C}, \mathcal{C})$ , and  $(\mathcal{U}, \mathcal{U})$  defined as in section 3, agents' behavior and economic activities and the economic structure in the economy are expressed as  $(\mathcal{E}, \mathcal{E})$ . The structures of production, exogenous technological progress, and consumption involve parameters  $\alpha \in (0, 1) =: \Omega_{\mathcal{Y}}$ ,  $\tilde{\vartheta} \in \Omega_{\mathcal{A}}$ , and  $\omega_c \in \Omega_{\mathcal{C}}$ , respectively, and the economic structure of the economy may be expressed as  $\mathcal{E}(\alpha, \tilde{\vartheta}, \omega_c)$ .

## Objective of the social planner and the competitive equilibrium

Suppose the social planner's information set of economic structures is  $\mathcal{I}_{\mathcal{Y} \times \mathcal{A} \times \mathcal{C}} = \{\mathcal{E}_i, i \in \mathbb{I} | \mathcal{E}_i = \mathcal{E}(\alpha_i, \tilde{\vartheta}_i, \omega_{c,i}), \alpha_i \in \Omega_{\mathcal{Y}}, \tilde{\vartheta}_i \in \Omega_{\mathcal{A}}, \omega_{c,i} \in \Omega_{\mathcal{C}}\}$ . At each time  $t$ , the social planner of the economy must choose an economic structure  $\mathcal{E}_i \in \mathcal{I}_{\mathcal{Y} \times \mathcal{A} \times \mathcal{C}}$  and determine the allocation of total income between total consumption and savings and total consumption expenditure between  $m$  consumption goods, that is,  $e(t) := (c_1(t), \dots, c_m(t), x(t), k_1(t), \dots, k_m(t), n_1(t), \dots, n_m(t), k_x(t), n_x(t))$ . Denote  $\xi = \{\tau_n, \kappa_n\}_{n \geq 1}$  the sequence of decisions on "when to transform" and "where to transform," and let  $\mathbf{A}$  be the set of all such sequences. The aggregate resource constraint is given by

$$\begin{cases} \dot{k}(t) = p_x(t)x(t) - \delta k(t), & \tau_n \leq t < \tau_{n+1}, \\ k(\tau_{n+1}) = k(\tau_{n+1}^-), & t = \tau_{n+1}, n = 0, 1, 2, \dots \end{cases} \quad (5.17)$$

The representative household's total utility, starting at time  $t$ , is given by

$$J_i(t, k; \{\xi, \{e(s)\}\}) = \int_t^{\infty} e^{-\rho s} u(c(s)) ds, \quad (5.18)$$

in which  $k(t) = k$  and  $\theta(t-) = \mathcal{E}_i$ . The social planner must solve the maximization problem

$$V_i(t, k) = \max_{\{\xi, \{e(s)\}\}} J_i(t, k; \{\xi, \{e(s)\}\}) \quad (5.19)$$

subject to (5.17) and  $k(t) = k, \theta(t-) = \mathcal{E}_i$ .

Given the above argument, the *competitive equilibrium* of the economy consists of paths of consumption and savings decisions  $\{c(t), \dot{k}(t)\}$ , consumption expenditure of  $m$  consumption goods and the investment good  $\{c_1(t), \dots, c_m(t), c_x(t)\}$ , and economic structures  $\{\theta(t)\}$ , such that the utility of the representative household (5.18) is maximized, firms maximize profits, and markets clear. To characterize the competitive equilibrium, the economy can be expressed using the notation from section 4.1 as

$$\mu_i(t, k, c) = p_x(t)x(t) - \delta k(t), \quad i \in \mathbb{I}$$

and the control space  $\mathfrak{U} \equiv \{(c_1(t), \dots, c_m(t), x(t)) \mid x(t) : [0, \infty) \rightarrow U, c_j(t) : [0, \infty) \rightarrow U \text{ are Lebesgue measurable functions}\}$ . Using the method in Xing (2021), we obtain the following result:

**Proposition 5.2.** *For every  $i \in \mathbb{I}$ , the value function  $V_i(t, k)$  defined by (5.19) is a viscosity solution to*

$$\max \left\{ \sup_{c_1, \dots, c_m, c_x} \left\{ \frac{\partial V_i}{\partial t}(t, k) + (p_x(t)x(t) - \delta k) \frac{\partial V_i}{\partial k}(t, k) + e^{-\rho t} u(c(t)) \right\}, \right. \quad (5.20)$$

$$\left. \sup_{j \neq i} V_j(t, k) - V_i(t, k) \right\} = 0.$$

*and such solutions are unique. Furthermore, the static and dynamic equilibria of the economy are characterized by the supremum in (5.20), and the structural equilibrium is determined by the maximum over  $j \neq i$  in (5.20).*

A nice property in Herrendorf, Rogerson, and Valentinyi's (2014) balanced growth model is that it allows analytical expressions for many economic variables in the economy. For example, factor and goods prices can be computed analytically as  $R(t) = \alpha k(t)^{\alpha-1} A_x(t)^{1-\alpha}$ ,  $w(t) = (1 - \alpha)K(t)^\alpha A_x(t)^{1-\alpha}$ , and  $p_i(t) = (A_x(t)/A_i(t))^{1-\alpha}$  for  $i = 1, \dots, m$  and  $x$ . We skip the discussion on this and, instead, highlight that the extended EST model describes a stagewise approximately balanced development and growth process.

### 5.3 Structures of technological progress

Economic history shows that technological progress in countries on the world's production possibility frontier is usually achieved via R&D, whereas technological progress in countries inside the frontier may be attained via technology adoption and/or R&D. As previous EST models assume exogenous technological progress, we extend the EST to endogenize structural transformation of different types of technological progress and discuss its economic implications.

## Production with variety and quality ladders

The literature usually considers two types of technological change, namely process and product innovation, or the introduction of a new product and innovations to reduce the costs of production of existing products. Two commonly used canonical models for these changes are product-variety models (Romer, 1990) and Schumpeterian models (Aghion and Howitt, 1992). For convenience, we extend production function (5.1) and assume that the unique final good is produced competitively by combining the output of  $m(t)$  sectors with elasticities of substitution  $\epsilon \in [0, \infty)$ , that is,

$$Y(t; \omega) = F[Y_1(t), \dots, Y_{m(t)}(t)] = \left( \int_0^{m(t)} w(j, t) q(j, t) Y(j, t)^{\frac{\epsilon-1}{\epsilon}} dj \right)^{\frac{\epsilon}{\epsilon-1}}, \quad (5.21)$$

where the weights  $w(j, t)$  satisfy  $w(j, t) \geq 0$  for all  $t$  and  $j$  and  $\int_0^{m(t)} w(j, t) dj = 1$ ,  $m(t)$  denotes the number of varieties of inputs at time  $t$ ,  $q(j, t)$  represents the “quality ladder” for machine type  $j$ , and  $\omega$  is a vector of parameters related to the production functions and will be specified later. The intermediate goods of machine type  $j$ ,  $Y(j, t)$ , are produced by the production function

$$Y(j, t) := F(j, t) = K(j, t)^{\alpha(j, t)} L(j, t)^{1-\alpha(j, t)}. \quad (5.22)$$

The technology level of machine type  $j$  is represented by the quality ladder  $q(j, t)$ ; hence, there is no need to keep the term  $A(j, t)$  in (5.22). Let  $\xi(j, t; \omega) = K(j, t; \omega)/K(t)$  and  $\lambda(j, t; \omega) = L(j, t; \omega)/L(t)$ . The output per capita at time  $t$  is

$$f(t, k(t); \omega) = \left\{ \int_0^{m(t)} w(j, t) q(j, t) [k(t)^{\alpha(j, t)} \xi_j(t)^{\alpha(j, t)} \lambda_j(t)^{1-\alpha(j, t)}]^{\frac{\epsilon-1}{\epsilon}} dj \right\}^{\frac{\epsilon}{\epsilon-1}}. \quad (5.23)$$

Capital and labor market clearing requires that

$$\int_0^{m(t)} K(j, t) dj \leq K(t), \quad \int_0^{m(t)} L(j, t) dj \leq L(t). \quad (5.24)$$

Then production and its structure of the economy can be expressed as  $(\mathcal{Y}, \mathcal{Y}(\omega))$ , where  $\mathcal{Y} = \{Y(t), \{Y(j, t)\}_{0 \leq j \leq m(t)}\}$  describes the output levels of the intermediate and final goods,  $\mathcal{Y}(\omega) = \{F(\cdot), \{F(j, \cdot)\}_{0 \leq j \leq m(t)}\}$  represents the composition and organization of all production, and  $\omega = (\{w(j, \cdot), \alpha(j, \cdot)\}_{0 \leq j \leq m(t)})$  is an element of the set  $\Omega_{\mathcal{Y}} = \{\omega | w(j, t) \geq 0, \int_0^{m(t)} w(j, t) dj = 1, 0 < \alpha(j, t) < 1 \text{ for all } j\}$ .

## Endogenous technological progress

We first consider endogenous technological progress via technology adoption. Suppose  $\tilde{m}(t)$  and  $\tilde{q}(j, t)$  are the number of machines and quality ladder for machine type  $j$  on the world's

production possibility frontier at time  $t$ , respectively. By definition,  $\tilde{m}(t)$  and  $\tilde{q}(j, t)$  are exogenously determined and monotonically nondecreasing over  $t$ . The economy in our study has  $m(t-)$  machines and machine type  $j$  is on the quality ladder  $q(j, t-)$  prior to time  $t$ . Obviously,  $m(t-) \leq \tilde{m}(t)$  and  $q(j, t-) \leq \tilde{q}(j, t)$ .

At time  $t$ , the social planner of the economy decides whether the number of machine types and the quality of each machine type should be improved via technology adoption.<sup>14</sup> If yes, the number of machine types  $m(t)$  and the quality ladder  $q(j, t)$  of machine type  $j$  at time  $t$  should be chosen from the sets  $(m(t-), \tilde{m}(t)]$  and  $(q(j, t-), \tilde{q}(j, t)]$ , respectively. Then the level of technology is given by

$$\mathcal{A} = \{(m(t), \{q(j, t)\}_{0 \leq j \leq m(t)}), \text{ where } m(t) \in (m(t-), \tilde{m}(t)] \text{ and } q(j, t) \in (q(j, t-), \tilde{q}(j, t)]\},$$

and the technological structure in production is expressed as  $\mathcal{A}_{\text{adopt}}(\epsilon) = \{\epsilon = (\epsilon_m, \{\epsilon_j\}_{0 \leq j \leq \epsilon_m}), \epsilon_m = m(\cdot), \epsilon_j = q(j, \cdot)\}$ , which is an element of the information set  $\mathcal{I}_{\mathcal{A}, \text{adopt}} = \{\mathcal{A}_{\text{adopt}}(\epsilon) | \epsilon \in \Omega_{\mathcal{A}, \text{adopt}}\}$  and  $\Omega_{\mathcal{A}, \text{adopt}} = \{(\epsilon_m, \{\epsilon_j\}_{0 \leq j \leq \epsilon_m}) | \epsilon_m \in (m(t-), \tilde{m}(t)], \epsilon_j \in (q(j, t-), \tilde{q}(j, t)]\}$ . A mathematical concern here is whether such a decision process leads to continuous variation of  $m(t)$  and  $q(j, t)$ , which indicates that the technological structure violates the durability attribute. This concern only arises in theoretical analysis, since the improvement from  $m(t-)$  to  $m(t)$  and/or from  $q(j, t-)$  to  $q(j, t)$  in reality involves some cost, which prevents the social planner from choosing  $m(t)$  and  $q(j, t)$  continuously. In theoretical analysis, the diminishing marginal returns of production functions ensure durability and transformality. Hence, the functionals  $\mathcal{A}_{\text{adopt}}$ , chosen optimally by the social planner, are still piecewise constant. Then given production  $(\mathcal{Y}, \mathcal{Y}(\omega))$  and technology  $(\mathcal{A}, \mathcal{A}_{\text{adopt}})$ , the resource constraint of the economy at time  $t$  is

$$\dot{K}(t) = Y(t; \omega) - \delta K(t) - C(t). \quad (5.25)$$

Accordingly, the allocation of capital stock and its structure are expressed as  $(\mathcal{K}, \mathcal{K}_{\text{adopt}}(\omega, \epsilon))$ , where  $\mathcal{K}$  is defined similarly as in section 6.1 and  $\mathcal{K}_{\text{adopt}}(\omega, \epsilon) = \{K(j, \cdot) | \int_0^{m(t)} K(j, \cdot) dj \leq K(\cdot), K(\cdot)$  satisfies (5.25)\}.

In addition to technology adoption, the social planner may also decide to improve the technology level via R&D. To explain how to describe the structure of technological progress via R&D, we suppose that  $Z(t)$  is expenditure on R&D at time  $t$ , and the number of machine types  $m(t)$  and the quality of the  $j$ th machine type  $q(j, t)$  at time  $t$  satisfy the following

$$\dot{m}(t) = \iota_m Z(t), \quad \lim_{\Delta t \rightarrow 0} \frac{q(j, t + \Delta t) - q(j, t)}{\Delta t} = \iota_j Z(t) \quad \text{for all } j \text{ and } t, \quad (5.26)$$

where  $\iota_m$  and  $\iota_j$  are nonnegative parameters and satisfy the constraint  $\iota_m + \int_0^{m(t)} \iota_j dj = 1$ . Then the technology level of production is  $\mathcal{A} = \{(m(t), \{q(j, t)\}_{0 \leq j \leq m(t)}) | m(t)$  and  $q(j, t)$  satisfy (5.26)\}. To represent the technological structure, let  $\iota = (\iota_m, \{\iota_j\})$  and  $\Omega_{\mathcal{A}, \text{r\&d}} = \{\iota | \iota_m \geq$

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<sup>14</sup>Recall that the social planner is defined as the social elite of an economy, which can collect information and make decision on production and other economic activities.

$0, \iota_j \geq 0$  for all  $j, \iota_m + \int_0^{m(t)} \iota_j dj = 1$ . Then the structure of technology can be expressed as  $\mathcal{A}_{r\&d}(\iota)$ , which is an element of the set of functionals  $\mathcal{I}_{\mathcal{A},r\&d} = \{(m(\cdot), \{q(j, \cdot)\}_{0 \leq j \leq m(\cdot)}) | \iota \in \Omega_{\mathcal{A},r\&d}, m(\cdot) \text{ and } q(j, \cdot) \text{ satisfy (5.26)}\}$ . Provided production  $(\mathcal{Y}, \mathcal{Y}(\omega))$  and technology  $(\mathcal{A}, \mathcal{A}_{r\&d}(\iota))$ , the resource constraint of the economy at time  $t$  is

$$\dot{K}(s) = Y(t; \omega) - \delta K(t) - C(t) - Z(t). \quad (5.27)$$

Consequently, the allocation of capital stock and its structure are expressed as  $(\mathcal{K}, \mathcal{K}_{r\&d}(\omega, \iota))$ , where  $\mathcal{K}$  is defined similarly as in section 6.1 and  $\mathcal{K}_{r\&d}(\omega, \iota) = \{K(j, \cdot) | \int_0^{m(t)} K(j, \cdot) dj \leq K(\cdot), K(\cdot) \text{ satisfies (5.27)}\}$ .

Therefore, the total information set of technology and its structure for the social planner is  $\mathcal{I}_{\mathcal{A}} := \mathcal{I}_{\mathcal{A},\text{adopt}} \cup \mathcal{I}_{\mathcal{A},r\&d}$ , and the corresponding total information set of capital allocation and its structure is  $\mathcal{I}_{\mathcal{K}} := \mathcal{I}_{\mathcal{K},\text{adopt}} \cup \mathcal{I}_{\mathcal{K},r\&d}$ . Once the social planner chooses structures of technology and capital allocation, technology level  $\mathcal{A}$  and capital allocation  $\mathcal{K}$  can be determined by corresponding mechanisms.

Other structures  $(\mathcal{H}, \mathcal{H}), (\mathcal{F}, \mathcal{F}), (\mathcal{M}, \mathcal{M}), (\mathcal{L}, \mathcal{L}), (\mathcal{P}, \mathcal{P}), (\mathcal{C}, \mathcal{C}),$  and  $(\mathcal{U}, \mathcal{U})$  can be defined similarly with the necessary modifications as in section 6.1. Then we may still use  $(\mathcal{E}, \mathcal{E})$  to represent agents' behavior and economic activities and their structures in the economy. To highlight production structure  $\omega \in \Omega_{\mathcal{Y}}$  and technological structure  $\epsilon \in \Omega_{\mathcal{A},\text{adopt}}$  or  $\iota \in \Omega_{\mathcal{A},r\&d}$ , one may express the economic structure of the economy as  $\mathcal{E}(\omega, \epsilon)$  or  $\mathcal{E}(\omega, \iota)$ .

### Social planner's maximization problem

Suppose the social planner's information set of economic structures is  $\mathcal{I}_{\mathcal{Y} \times \mathcal{K} \times \mathcal{A}} = \{\mathcal{E}_i | \mathcal{E}_i = \mathcal{E}(\omega_i, \tilde{\vartheta}_i), \tilde{\vartheta}_i \in \Omega_{\mathcal{A},\text{adopt}} \cup \Omega_{\mathcal{A},r\&d}, i \in \mathbb{I}\}$ . The social planner can choose an economic structure for the economy and allocate resources under the given economic structure. When  $\mathcal{E}(\omega_i, \tilde{\vartheta}_i)$  ( $\tilde{\vartheta}_i \in \Omega_{\mathcal{A},\text{adopt}}$ ) is chosen, equation (5.25) implies that the resource constraint per capita is

$$\dot{k}(t) = f(t, k(t); \omega_i) - (\delta + \pi)k(t) - c(t), \quad (5.28)$$

and when  $\mathcal{E}(\omega_i, \tilde{\vartheta}_i)$  ( $\tilde{\vartheta}_i \in \Omega_{\mathcal{A},r\&d}$ ) is chosen, equation (5.27) implies that the resource constraint per capita is

$$\dot{k}(t) = f(t, k(t); \omega) - (\delta + \pi)k(t) - c(t) - z(t). \quad (5.29)$$

Suppose the economy starts with economic structure  $\mathcal{E}(\omega_i, \tilde{\vartheta}_i)$  at time  $t_0$ , Then the social planner's total utility starting at time  $t_0$  is given by

$$J_i(t_0, k; \{c(t), \xi\}) = \int_{t_0}^{\infty} e^{-(\rho-\pi)t} u(c(t)) dt.$$

The social planner's objective is to solve the maximization problem

$$\begin{aligned} V_i(t_0, k) &= \max_{\{c(t), \xi\}} J_i(t_0, k; \{c(t), \xi\}) \quad \text{subject to (5.28) or (5.29)} \\ &\text{and } k(t_0) = k \in \mathbb{R}^+, \theta(t_0) = \mathcal{E}_i \in \mathcal{I}_{\mathcal{Y} \times \mathcal{K} \times \mathcal{A}}. \end{aligned} \quad (5.30)$$

Given the above specification, the competitive equilibrium of the economy consists of paths for factor and intermediate goods prices, employment and capital allocation  $\{\xi_j(t; \theta(t)), \lambda_j(t; \theta(t))\}$ , consumption and savings decisions  $\{c(t), \dot{K}(t)\}$ , and structures of production, capital allocation, and technological progress  $\{\theta(t)\}$  such that the utility of the representative household is maximized, firms maximize profits, and markets clear. Using the method in Xing (2021), we can show the following.

**Proposition 5.3.** *For each  $\mathcal{E}_i = \mathcal{E}(\omega_i, \tilde{\vartheta}_i) \in \mathcal{I}_{\mathcal{Y} \times \mathcal{X} \times \mathcal{A}}$ , let  $V_i(t, k)$  be defined by (5.9). Then for  $\tilde{\vartheta}_i \in \Omega_{\mathcal{A}, \text{adopt}}$ ,  $V_i(t, k)$  is a unique viscosity solution to*

$$\begin{aligned} \max \left\{ \sup_{e(t)} \left[ \frac{\partial V_i}{\partial t}(t, k) + [f_i(t, k) - (\delta + \pi)k - c] \frac{\partial V_i}{\partial k}(t, k) + e^{-(\rho - \pi)t} u(c) \right], \right. \\ \left. \max_{j \neq i} \{V_j(t, k) | \tilde{\vartheta}_j \in \Omega_{\mathcal{A}, \text{r\&d}} \cup \Omega_{\mathcal{A}, \text{adopt}}\} - V_i(t, k) \right\} = 0, \end{aligned} \quad (5.31)$$

where  $e(t) = (c(t), \{\xi(j, t), \lambda(j, t)\})$ . For  $\tilde{\vartheta}_i \in \Omega_{\mathcal{A}, \text{r\&d}}$ ,  $V_i(t, k)$  is a unique viscosity solution to

$$\begin{aligned} \max \left\{ \sup_{e(t)} \left[ \frac{\partial V_i}{\partial t}(t, k) + [f_i(t, k) - (\delta + \pi)k - c - z] \frac{\partial V_i}{\partial k}(t, k) + e^{-(\rho - \pi)t} u(c) \right], \right. \\ \left. \max_{j \neq i} \{V_j(t, k) | \tilde{\vartheta}_j \in \Omega_{\mathcal{A}, \text{r\&d}} \cup \Omega_{\mathcal{A}, \text{adopt}}\} - V_i(t, k) \right\} = 0, \end{aligned} \quad (5.32)$$

where  $e(t) = (c(t), z(t), \{\xi(j, t), \lambda(j, t)\})$ . Furthermore, the static and dynamic equilibria of the economy are characterized by the supremum in the two equations, and the structural equilibrium is characterized by the second-line maximum of the two equations.

## Economic implications

Proposition 5.3 integrates several scenarios of endogenous technological progress into a single framework and has interesting implication for economic development and growth. We next briefly discuss scenarios and implications of Proposition 5.3.

The first scenario is countries on the global production possibility frontier and their technological progress. Since these countries or economies are on the world production possibility frontier, the social planners of these economies will rule out the possibility of technological adoption and the information set of technological structure reduces from  $\mathcal{I}_{\mathcal{A}, \text{adopt}} \cup \mathcal{I}_{\mathcal{A}, \text{r\&d}}$  to  $\mathcal{I}_{\mathcal{A}, \text{r\&d}}$ . Hence, the HJB-QVI system (5.32) for the value function of the representative household in Proposition 5.3 becomes that, for  $\tilde{\vartheta}_i \in \Omega_{\mathcal{A}, \text{r\&d}}$ ,

$$\begin{aligned} \max \left\{ \sup_{e(t)} \left[ \frac{\partial V_i}{\partial t}(t, k) + [f_i(t, k) - (\delta + \pi)k - c - z] \frac{\partial V_i}{\partial k}(t, k) + e^{-(\rho - \pi)t} u(c) \right], \right. \\ \left. \max_{j \neq i} \{V_j(t, k) | \tilde{\vartheta}_j \in \Omega_{\mathcal{A}, \text{r\&d}}\} - V_i(t, k) \right\} = 0. \end{aligned} \quad (5.33)$$

This indicates that R&D is the only way to achieve technological progress for countries on the global production possibility frontier, and the structure of technological progress via R&D is characterized by the rate  $\iota = (\iota_m, \{\iota_j\}) \in \Omega_{\mathcal{A}, \text{r\&d}}$ . If a rate of R&D is fixed with  $\iota$  and no transformation of the R&D structure is needed, then (5.33) further reduces to

$$\sup_{e(t)} \left[ \frac{\partial V_i}{\partial t}(t, k) + [f_i(t, k) - (\delta + \pi)k - c - z] \frac{\partial V_i}{\partial k}(t, k) + e^{-(\rho - \pi)t} u(c) \right] = 0, \quad (5.34)$$

which is in the form of technological progress via R&D in the neoclassical economy.

The second scenario is developing countries that are inside the global production possibility frontier. Two situations might occur. One is that the developing country is well connected with the world's economy, so that the information of the world's technology can be accessed by the economy freely or with low cost. In such case, the social planner of the economy would naturally consider improving the country's technology via adoption instead of R&D, as inside-the-frontier indigenous innovation is not efficient. Thus, the HJB-QVI system (5.32) for the value function of the representative household in Proposition 5.3 becomes that, for  $\tilde{\vartheta}_i \in \Omega_{\mathcal{A}, \text{adopt}}$ ,

$$\begin{aligned} \max \left\{ \sup_{e(t)} \left[ \frac{\partial V_i}{\partial t}(t, k) + [f_i(t, k) - (\delta + \pi)k - c] \frac{\partial V_i}{\partial k}(t, k) + e^{-(\rho - \pi)t} u(c) \right], \right. \\ \left. \max_{j \neq i} \{ V_j(t, k) | \tilde{\vartheta}_j \in \Omega_{\mathcal{A}, \text{adopt}} \} - V_i(t, k) \right\} = 0. \end{aligned} \quad (5.35)$$

Some discussion starts from the perspective of the social planners in developed countries and focuses on technology diffusion. The social planners in these discussions play a passive role and accept the technology diffused from developed countries without the process of choosing "appropriate" technology levels (or structures). Equation (5.35) avoids this issue and highlights the active role of social planners in developing countries in choosing "appropriate" technological structures. Another situation in the second scenario is that the developing country is somehow isolated from the world economy, so that inside-the-frontier innovation is necessary. In such case, the value function of the representative household in the isolated economy is characterized by the HJB-QVI system (5.34).

The third scenario is countries that are inside but near the global production possibility frontier. As the technology level in those countries is near the global production possibility frontier, due to the fear of competition, the countries on the global production possibility frontier may embargo their technology knowhow. Therefore, R&D must be carried out for further economic development. Then the main issue for the social planners in these countries is when to switch from technology adoption to R&D. To fix the idea, suppose the social planner's information set of technological structures is  $\mathcal{I}_{\mathcal{A}} = \{\mathcal{E}(\omega_1, \tilde{\vartheta}_1), \mathcal{E}(\omega_2, \tilde{\vartheta}_2)\}$  with  $\tilde{\vartheta}_1 \in \Omega_{\mathcal{A}, \text{adopt}}$  and  $\tilde{\vartheta}_2 \in \Omega_{\mathcal{A}, \text{r\&d}}$ . Then Proposition 5.3 implies that the value functions  $V_1(t, k)$  and  $V_2(t, k)$  of the representative household with corresponding initial economic structures  $\mathcal{E}(\omega_1, \tilde{\vartheta}_1)$  and  $\mathcal{E}(\omega_2, \tilde{\vartheta}_2)$ , respectively, and times of switching from the mode of

technology adoption to that of R&D are characterized by the following HJB-QVI system

$$\left\{ \begin{array}{l} \max \left\{ \sup_{e(t)} \left[ \frac{\partial V_1}{\partial t}(t, k) + [f_1(t, k) - (\delta + \pi)k - c] \frac{\partial V_1}{\partial k}(t, k) + e^{-(\rho - \pi)t} u(c) \right], \right. \\ \quad \left. V_2(t, k) - V_1(t, k) \right\} = 0, \quad e(t) = (c(t), \{\xi(j, t), \lambda(j, t)\}); \\ \max \left\{ \sup_{e(t)} \left[ \frac{\partial V_2}{\partial t}(t, k) + [f_2(t, k) - (\delta + \pi)k - c - z] \frac{\partial V_2}{\partial k}(t, k) + e^{-(\rho - \pi)t} u(c) \right], \right. \\ \quad \left. V_1(t, k) - V_2(t, k) \right\} = 0, \quad e(t) = (c(t), z(t), \{\xi(j, t), \lambda(j, t)\}). \end{array} \right. \quad (5.36)$$

## 5.4 Infrastructure and economic institutions

The discussion so far has focused on structures of production and consumption and their transformation in the process of economic development. In general, in addition to the process of industrial upgrading and technological progress, which involves households' and firms' decisions on the supply and demand of factors of production, an economy's development also involves the production of public goods and infrastructure, which are supplied by governments or require collective actions and cannot be internalized in the decisions of individual households or firms. We now consider production structures of public goods and infrastructure.

Infrastructure includes hard infrastructure and soft infrastructure. Hard infrastructure consists of the physical infrastructure of highways, port facilities, airports, telecommunication systems, electricity grids, and other public utilities. Soft infrastructure consists of institutions, regulations, social capital, and other social and economic arrangements. Most hard infrastructure and almost all soft infrastructure is exogenously provided to individual firms in the form of public goods and cannot be internalized in their production decisions. To illustrate the idea, we consider an approach to model infrastructure as public goods available to firms and a government's tax policy as a simplified economic institution.<sup>15</sup>

### Production, infrastructure, and economic institution

Suppose the economy has a unique final good, and the representative firm produces output  $Y_t$  according to the following production function:

$$Y(t) = F[K(t), L(t), A(t), G(t)], \text{ or } y(t) := f(t, k(t)) = F_i[k(t), 1, A(t), G(t)], \quad (5.37)$$

where  $G(t)$  is the aggregate stock of public goods (or infrastructure) available to all firms at time  $t$ . The public good  $G(t)$  is a common external input to each firm's production

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<sup>15</sup>Economic institutions have different meanings in different contexts, and we refer to them as taxes, the security of property rights, contracting institutions, and other economic arrangements. This is different from the political institutions discussed in the next subsection, which refer to the rules and regulations affecting political decision making.

function. The government impose a tax rate  $\tau \in [0, 1)$  on output. Hence, equation for capital accumulation per capita is given by

$$\dot{k}(t) = (1 - \tau)f(t, k(t)) - (\delta + \pi)k(t) - c(t). \quad (5.38)$$

Accordingly, the accumulation of public goods is described by

$$\dot{G}(t) = \tau f(t, k(t)), \quad (5.39)$$

and firms hire capital and labor to maximize  $(1 - \tau)f(t, k(t)) - w(t) - R(t)k(t)$ .

### Economic structures

In addition to the components discussed in the previous sections, the economic structure of the economy includes structures (functionals) of infrastructure investment and economic institutions. We characterize the level of infrastructure investment and its structure by  $(\mathcal{I}, \mathcal{S})$ , where  $\mathcal{I} = \{G(t)\} \in \mathbb{R}$  and  $\mathcal{S} := \{G(\cdot) \text{ satisfies (5.39)}\}$ . Infrastructure investment is determined by the tax rate  $\tau$ , and its level and structure can be described by the pair  $(\mathcal{T}, \mathcal{S})$ , where the tax level  $\mathcal{T} = \{\tau f(t, k(t))\}$  and the structure (or the functional) of the tax  $\mathcal{S} := \{\tau f(\cdot, \cdot)\}$ . Provided  $(\mathcal{I}, \mathcal{S})$  and  $(\mathcal{T}, \mathcal{S})$ , the production structure  $(\mathcal{Y}, \mathcal{Y})$  must be modified accordingly. Thus, activities in the economy can be summarized as  $\mathcal{E} := (\mathcal{H}, \mathcal{F}, \mathcal{M}, \mathcal{Y}, \mathcal{L}, \mathcal{K}, \mathcal{A}, \mathcal{P}, \mathcal{C}, \mathcal{U}, \mathcal{I}, \mathcal{T})$  and its structure can be represented by  $\mathcal{E} := (\mathcal{H}, \mathcal{F}, \mathcal{M}, \mathcal{Y}, \mathcal{L}, \mathcal{K}, \mathcal{A}, \mathcal{P}, \mathcal{C}, \mathcal{U}, \mathcal{I}, \mathcal{T})$ . Then, given a set of specific structures of infrastructure investment and economic institution  $\{\mathcal{S}_i, \mathcal{T}_i\}$ ,  $(i \in \mathbb{I})$  at an initial time, the discussion on the social planner's maximization problem and its solution is analogous to that in the previous sections and hence is skipped here.

### Economic implications

In contrast to infrastructure that can be represented as public goods produced via explicit production functions, most soft infrastructure cannot be described in this way. However, when the mechanism of the impact of soft infrastructure on economic activities can be described via explicit functions, the EST approach can be extended to characterize the transformation of soft infrastructure and the corresponding competitive equilibrium and optimal infrastructure. This would help us understand the diversity of certain soft infrastructure in different countries. Take a country's financial structure as an example. Modern financial institutions, such as the stock market, venture capital, corporate bonds, and large banks have the functions of mobilizing large amounts of capital and/or diversifying risks. The industrial structure of developed countries consists of large-scale capital intensive industries that rely on risky R&D for achieving endogenous technological innovation. Hence, modern financial institutions are appropriate for serving the needs of the real sector in developed countries. However, such financial institutions may not be appropriate for developing countries, due to the differences in the structures of industries and technological progress between

developing and developed countries. Since developing countries possess mostly small-scale, labor-intensive industries and rely on the adoption of mature technologies for technological progress, small local or community-based financial institutions may be better suited (Lin, 2011).

For other types of soft infrastructure or institutions, such as economic policies, protection of intellectual property, and other regulations and rules, the EST framework can embed them and their transformation into an economy's development process, which provides the following implications for studies of economic policies and institutions.

First, most studies of economic policies focus on the design, implementation, and evaluation of specific policies and seldom deal with their dynamic changes. As the time scales of economic policies (or institutions) are usually larger than those of economic variables and even some economic substructures, transformations of economic institutions and their competitive equilibria can be characterized via the EST framework. In particular, as many models in the neoclassical sense concentrate on static and dynamic equilibria under a given economic policy or institution, the EST framework characterizes the structural equilibrium of policies and institutions in the development process.

Second, the transformation or birth and decay process of a specific economic institution has been largely discussed by economic historians, but it has not been studied via theoretical models. When different development stages of a particular economic institution are modeled as different economic structures, the EST framework helps us understand the structures of economic policies and institutions and their transformations, for example, the optimal entry and exit times of specific policies and institutions.

Third, recent studies show that economic policies can be categorized as structural and nonstructural policies (Abdel-Kader, 2013). Since structural and nonstructural policies serve different purposes in an economy's development process, it is difficult to study both types of policies with the existing economic models. The EST framework can overcome this difficulty by modeling structural and non-structural policies as different institutional structures and characterizing transformations among structural and nonstructural policies.

## 5.5 Political regimes and institutions

Another related and interesting issue is the impact of political institutions on economic development. As institutions have different meanings in different contexts, we define political institutions as a system of laws on the organizational form and methods of political decision making, such as the organization of state power, structural form of the state, political party system, and so on. It is not difficult to see that the structures of an economy's political institutions still have the attributes of durationality and transformality. Furthermore, political institutions change on a time scale much longer than that of economic activities and structures.

To provide an illustration of the transformation of political institutions via the EST framework, we extend the discussion as follows. Suppose that at initial time  $t$ , the initial

institutional structure and initial economic structure of the country are  $\mathcal{N}_{i_1}$  and  $\mathcal{E}_{i_2}$ , respectively, and the initial capital intensity is  $k$ . The dynamic equation for capital intensity under the given institutional and economic structures  $(\mathcal{N}_{i_1}, \mathcal{E}_{i_2})$  may be given by

$$\dot{k}(t) = f_{\mathcal{N}_{i_1}, \mathcal{E}_{i_2}}(t, k) - (\delta + \pi)k - c(t).$$

The total utility and social planner's objective are still defined by equations (5.8) and (5.9). The competitive equilibrium of the economy can be defined similarly as in earlier sections. Then the solution to the social planner's maximization problem satisfies the following.

**Proposition 5.4.** *For each institutional structure  $\mathcal{N}_{i_1} \in \mathcal{N}$  and each economic structure  $\mathcal{E}_{i_2} \in \mathcal{E}$ , the value function  $V_{\mathcal{N}_{i_1}, \mathcal{E}_{i_2}}(t, k)$  is a unique viscosity solution of the HJBQVI system*

$$\begin{aligned} \max \left\{ \sup_{c \in \mathcal{U}} \left[ \frac{\partial V_{\mathcal{N}_{i_1}, \mathcal{E}_{i_2}}}{\partial t}(t, k) - [f_{\mathcal{N}_{i_1}, \mathcal{E}_{i_2}}(t, k) - (\delta + \pi)k - c(t)] \frac{\partial V_{\mathcal{N}_{i_1}, \mathcal{E}_{i_2}}}{\partial k}(t, k) + e^{-\rho t} u(c) \right], \right. \\ \left. \max \left[ \max_{\mathcal{E}_{j_2} \neq \mathcal{E}_{i_2}} \{V_{\mathcal{N}_{i_1}, \mathcal{E}_{j_2}}(t, k)\}, \max_{\mathcal{N}_{j_1} \neq \mathcal{N}_{i_1}} \{V_{\mathcal{N}_{j_1}, \mathcal{E}_{i_2}}(t, k)\} \right] - V_{\mathcal{N}_{i_1}, \mathcal{E}_{i_2}}(t, k) \right\} = 0. \end{aligned} \quad (5.40)$$

The static and dynamic equilibria of the economy are still characterized by the supremum in (5.40), and the structural equilibrium is accounted for by the second line of the equation. Then, using an argument analogous to that for Proposition 5.2, we obtain the following.

**Proposition 5.5.** *The optimal institutional and economic structures at any given time  $t$  and with any given capital intensity  $k$  are characterized by the structural equilibrium (5.40) at  $(t, k)$ . Moreover, the optimal institutional and economic structures are given functions of  $t$  and  $k(t)$  and, hence, endogenous to the capital intensity (or more generally, the factor endowments of the economy) at time  $t$ .*

We next briefly discuss the implications of equation (5.40) for the transformation of institutional structures in different cases. The first case is for developed countries and is relatively simple. Since developed countries are mostly industrialized, and their income per capita is usually higher than that in developing countries, the social planner (or the economic and political elites) in developed countries will not consider it necessary to transform their political institutions. Or equivalently, the social planner in developed countries may have compared different types of institutional structures but concludes that the country's current institutional and economic structures are better than those in other countries. In such case, the structural equilibrium of equation (5.40) is not necessary and (5.40) degenerates to the equation of the supremum, which describes the static and dynamic equilibria of the economy with fixed institutional and economic structures.

The second case is developing countries focusing on “economic development” or “economic transition” and trying to figure out the optimal economic and political institutions for the economy. The structural equilibrium in equation (5.40) is

$$\max \left[ \max_{\mathcal{E}_{j_2} \neq \mathcal{E}_{i_2}} \{V_{\mathcal{N}_{i_1}, \mathcal{E}_{j_2}}(t, k)\}, \max_{\mathcal{N}_{j_1} \neq \mathcal{N}_{i_1}} \{V_{\mathcal{N}_{j_1}, \mathcal{E}_{i_2}}(t, k)\} \right] - V_{\mathcal{N}_{i_1}, \mathcal{E}_{i_2}}(t, k) = 0. \quad (5.41)$$

This implies that, for the social planner of the economy, there are two types of structural transformation involving institutional and economic structures. The first type is that the economic structure  $\mathcal{E}_{i_2}$  transforms into another economic structure, while the political institution  $\mathcal{N}_{i_1}$  remains the same, that is,

$$\max_{\mathcal{E}_{j_2} \neq \mathcal{E}_{i_2}} \{V_{\mathcal{N}_{i_1}, \mathcal{E}_{j_2}}(t, k)\} = V_{\mathcal{N}_{i_1}, \mathcal{E}_{i_2}}(t, k). \quad (5.42)$$

The second type is that political institution  $\mathcal{N}_{i_1}$  and economic structure  $\mathcal{E}_{i_2}$  transform into other structures in  $\mathcal{N} \times \mathcal{E}$ , which is characterized by

$$\max_{\mathcal{N}_{j_1} \neq \mathcal{N}_{i_1}} \{V_{\mathcal{N}_{j_1}, \mathcal{E}_{i_2}}(t, k)\} = V_{\mathcal{N}_{i_1}, \mathcal{E}_{i_2}}(t, k) \quad (5.43)$$

Equation (5.42) corresponds to a scenario in which a developing country develops the economy by reforming the economic structures, but the political institution remain invariant. By contrast, equation (5.43) indicates that the developing country transforms both the political institutions and economic structures.

The choice of equation (5.42) or (5.43) for the social planner (or the social elite) of the economy to solve may lead to different development paths. Take for example China's and the former Soviet Union's economic transition processes. China's economic reform process can be described as the process of solving equation (5.42), whereas the economic reform process in the former Soviet Union and other Eastern European countries can be characterized as the process of solving equation (5.43). Although these two processes are completely different, they can be described by a unified EST framework.

## 6 Concluding remarks

Structural transformation has been discussed intensively in the literature on economic growth and development over the past decades. Although sectoral structural transformation models have been developed to study changes in numerical economic variables across different sectors, a general theoretical framework is still missing to characterize a country's full process of structural transformation.

The EST framework proposed in this paper bridges this gap and makes the following contributions. First, three fundamental attributes of structures—struaturality, durationality, and transformality—are summarized from empirical observations and the literature of economic history. Second, with the necessary assumptions on the information set of economic structures, a theoretical framework is proposed to model the dynamics of economic activities and their structures in different time scales and characterize the endogenous transformation process of structures. Third, we solve the social planner's optimization problem in the EST model and establish the associated competitive equilibrium theory. We show that, in addition to the static and dynamic equilibria that constitute competitive equilibrium in neoclassical growth models, competitive equilibrium in the EST framework suggests

the existence of a third type of equilibrium, the structural equilibrium. To demonstrate the flexibility of the proposed EST framework, we have discussed extensions of the EST framework that deal with hierarchical production structures, composite structures of consumer preference, technological structures via adoption and R&D, changes in infrastructure and economic institutions, and switching of political institutions.

The EST framework provides a method to model the structural differences and endogeneity of those differences for countries at different levels of development and sheds new light on many interesting and oftentimes debated issues. We consider a few examples. The import-substitution strategy failed in most developing countries in the 1950s and 1960s, despite the coordination provided by their governments' big pushes (Murphy, Shleifer, and Vishny, 1989). The failure was due to the industrial structure targeted in the strategy being too capital intensive while capital was scarce in the countries. The growth driven by capital accumulation without total factor productivity in Singapore and other East Asian economies in their catching-up stage was sustainable instead of being doomed to fail, as predicted by Krugman (1994). This was because capital accumulation is required for upgrading industrial structure and technology adoption in the process of catching up and returns to capital will not diminish before they reach the global production possibility/technology frontier and switch to technological innovation through R&D and growth driven by total factor productivity. From the perspective of EST, the poverty trap or middle-income trap for many developing countries is a result of their inability to implement dynamic structural transformation due to the lack of sufficient capital accumulation to cross the capital thresholds required for structural transformation, or the lack of government facilitation to overcome coordination failures to make the required improvements in hard and soft infrastructure for the transformation.

The EST framework in this paper can be extended to incorporate other types of macroeconomic models with specific structures, such as models of heterogeneous households, heterogeneous firms, overlapping generations, trade structures in an open economy, stochastic growth and so forth. Mathematically, as long as the economic problem involves different types of structures, an extended version of the EST model can be obtained. Economically, the extended EST model and associated competitive equilibrium can be characterized by the mathematical method and theory developed here, and hence a structural equilibrium can be obtained for the extended EST problem. In addition, since the proposed EST framework can integrate different types of economic structures and turn a macroeconomic model with a single type of economic structures into one with multiple economic structures, the EST framework can integrate models of economic development and growth at different development stages into a stagewise development and growth model. Hence, the EST model provides a unified framework to account for a country's development and growth process with structural transformation.

The mechanism of structural transformation characterized in the EST framework assumes that the market is complete, information can be freely obtained, and the transformation is frictionless, which are certainly not true in the real world. Instead, in the real world, different kinds of market and information incompleteness and different structures re-

quire different hard infrastructure and economic as well as political institutions to unleash its economic potentials in practice. The EST cannot occur spontaneously. Instead, policy intervention is usually needed to facilitate the transformation of economic structures (Lin, 1989), which is a topic for further discussion in subsequent research.

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