Productivity and Wealth Dynamics under Financial Frictions: An Empirical Investigation of the Self-financing Channel

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Motivation

- Financial frictions prevent poor firms to invest at optimal scale
- If distributions of productivity and wealth are not aligned
  - Financial frictions → misallocation, low aggregate investment, TFP and income.
- Joint distribution of productivity and wealth drives quantitative results
- Self-financing channel: productive firms accumulate wealth and build collateral
  - Mitigate the effects of financial frictions on TFP (Moll 2014, Midrigan & Xu 2014)

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- Strength of self-financing depends on the productivity process and is reflected on how policy functions respond to productivity shocks
- Two observations on previous literature
  1. Lack of micro evidence on policy functions and their response to productivity
  2. Prevalent methods to identify productivity fail under financial constraints

This Paper

- Characterizes and estimates the investment and wealth accumulation policy functions under financial fictions using firm-level data.

  → Examine the transmission of productivity shocks to investment and wealth accumulation and explore the self-financing channel using micro data.

- **Uses rich micro data on firms' balance sheet to**

  1. Consistently estimate firm productivity process
     - How persistent and volatile are productivity shocks?
  2. Document the transmission of productivity shocks to firm decisions
     - How do these responses vary along the wealth distribution?
  3. Empirically explores the strength of the self-financing channel
     - How fast the MPKs of two firms with different levels of wealth converge?

New empirical approach

- Recover productivity from firm production function and estimate nonlinear policies that depends on key state variables
  - Consistent with heterogeneous-firm models with collateral constraints
    - (Moll 2014; Midrigan Xu 2014; Buera Kaboski Shin 2015, etc)
  - Not require specifying functional forms for preference and productivity

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→ Challenge: productivity is not observable
  - Literature recovers productivity by estimating production function
    - Proxy Variable: Olley & Pakes 96; Levinsohn & Petrin 2003; etc
  - OP delivers biased estimates under financial frictions
  - Not well-suited to estimate policy functions

- Combine the insights of the self-financing channel with developments in nonlinear panel data to show non-parametric identification of the empirical model
  - (Hu & Schennach 2008, Arellano, Blundell & Bonhomme 2017)

Literature Review

Financial Frictions, Self-financing and Firms' Dynamics:

- Banerjee Moll (2010); Buera Shin (2011); Buera Shin (2013); Buera Kaboski Shin (2013); Moll (2014); Midrigan Xu (2014); Buera Kaboski Shin (2015); Buera Kaboski Townsend (2021); Kaboski (2021) Drechsel (2022); di Giovanni Garcia-Santana Jeenas, Moral-Benito Pijoan-Mas (2022) ; Cavalcanti, Kaboski, Martin, Santos (2022)
- Banerjee Duflo (2005); Hall Jones (1999); Caggese (2007); Caggese Cunat (2008); Restuccia Rogerson (2008); Alburquerque Hoppenhayn (2004); Hsieh and Klenow (2009)

Production Function Estimation and Nonparametric Identification:

- Olley-Pakes (1996); Levinsohn Petrin (2003); Ackerberg, Caves, Frazer (2015); Gandhi, Navarro, Rivers (2020); Shenoy (2020)
- Hu Schennach (2008); Hu Shum (2012); Arellano, Blundell, Bonhomme (2017); Arellano Bonhomme (2017); Hu, Huang, Sasaki (2020), Cunha, Heckman, Schennach (2010)

Outline

- ✓ Introduction
- ▶ A simple model with Financial Frictions and Olley Pakes
- 3. Empirical framework
- 4. Results
- 5. Concluusion

A simple model (Buera, Kaboski & Shin 2015)

- Heterogenous entrepreneurs with initial net worth/wealth A_{it} and prod Z_{it}

$$\max_{A_{it+1}, K_{it+1}, L_{it}, C_{it}} \sum_{t=1}^{\infty} \beta^t E\left[u(C_{it})\right]$$

$$\text{s.t.} \quad \begin{aligned} C_{it} + K_{it+1} - (1-\delta)K_{it} &= Y_{it} - W_t L_{it} + B_{it+1} - (1+r_t)B_{it}, \\ Y_{it} &= Z_{it} K_{it}^{\beta_k} L_{it}^{\beta_l} \\ Z_{it} &= \rho Z_{it-1} + \eta_{it} \\ A_{it} &= K_{it} - B_{it} \end{aligned}$$

- Financial friction: $B_{it} \leq \kappa(K_{it}, Z_{it}) \Rightarrow K_{it} \leq \tilde{\kappa}(A_{it}, Z_{it})$

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- Investment FOC

$$C_k E(Z_{it+1}|Z_{it})^{\frac{1}{1-\beta_l}} K_{it+1}^{\frac{\beta_k}{1-\beta_l}-1} = \beta(r+\delta) + \mu(A_{it}, Z_{it})$$

$\Rightarrow$ $I_{i,t} = h_t\left(Z_{it}, A_{it}\right)$

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- Self-financing (Euler):

$$u'(C_t) = \beta\left[(1+r_{t+1}) + E_t\left[\kappa_A \widehat{\mu}(A_{t+1}, Z_{t+1})\right]\right] u'(C_{t+1})$$

$\Rightarrow \; A_{t+1} = g_{t+1}\left(Z_{it}, A_{it}\right)$

Olley-Pakes under financial frictions

- Goal: Estimate the (log) production function:

$$y_{it} = \beta_l l_{it} + \beta_k k_{it} + z_{it} + \varepsilon_{it}$$

  - endogeneity problem: $cov(k_{i,t}, z_{i,t}) \neq 0$, $cov(l_{i,t}, z_{i,t}) \neq 0$

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- With financial constraints: $i_{i,t} = h(z_{it}, a_{it})$
  - In our simple model:

$$y_{it} = \beta_l l_{it} + \beta_k k_{it} + \frac{1}{\rho}(1 - \beta_k - \beta_l) i_{it} + (1 - \beta_l)\tilde{\mu}(a_{it}) + \varepsilon_{it}$$

    - $\text{Cov}(k_{it}\tilde{\mu}(a_{it})) < 0 \rightarrow \beta_k^{OP} < \beta_k$
    - High $\tilde{\mu}(a_t) \rightarrow z^{OP} < z$;
    - $\text{Cov}(l_{it}\tilde{\mu}(a_{it})) > 0 \rightarrow \beta_l^{OP} > \beta_l$

Outline

- ✓ Introduction
- ✓ A simple model of financial frictions and Olley-Pakes
- ▶ Empirical Framework

4. Results
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Empirical model under financial frictions

- Value added (log) production function for firms $i = 1 \cdots N$, $t = 1 \cdots T$

$$y_{it} = \beta_l l_{it} + \beta_k k_{it} + z_{it} + \varepsilon_{it}$$
$$z_{it} = \varphi\left(z_{it-1}\right) + \eta_{it}$$

  - The innovation η_{it} is independent of z_{it-1}
  - k_{it} is dynamic but predetermined: $k_{it} = (1-\delta)k_{t-1} + i_{it-1}$

- **Non-linear policy functions**

$$i_{it} = h_t\left(z_{it}, k_{it}, a_{it}, \mu_{it}\right)$$
$$a_{it+1} = g_t\left(z_{it}, k_{it}, a_{it}, \nu_{it+1}\right)$$

  - $h\left(.\right)$ and $g\left(.\right)$ allow for rich interactions between a_{it} (wealth) and z_{it}
  - μ_{it} and ν_{it+1} are scalar i.i.d shocks independent of state variables at t
  - The joint distribution of z_{i1}, k_{i1} and a_{i1} is left unrestricted

Objects of interest: marginal derivative effects

- Investment propensity in response to productivity shocks:

$$\Phi_t^h (a, k, z) = E_\mu \left[\frac{\partial h_t (z, k, a, \mu)}{\partial z} \right]$$

- Wealth accumulation propensity in response to productivity shocks:

$$\Phi_t^g (a, k, z) = E_\nu \left[\frac{\partial g_t (z, k, a, \nu_{t+1})}{\partial z} \right]$$

- Propensities reflect how firms reacts to productivity shocks

- Propensities are heterogenous and vary with a_{it} and z_{it}
    - Evidence on collateral constraint: $\Phi_t^h (a, k, z)$ increasing in a
    - Evidence on self-financing: $\Phi_t^g (a, k, z) > 0$ and decreasing in a

Identification Assumptions

- **Goal**: show that β_k, β_l, $\varphi(z_{it-1})$, h_t, g_t are identified given that:
  - y_{it}, k_{it}, l_{it}, i_{it}, a_{it} are observed for $i = 1 \cdots N$, $t = 1 \cdots T$
  - z_{it}, ε_{it}, ν_{it+1}, μ_{it} are not observed
  - z_{it} is correlated with X_{it}=(l_{it}, a_{it}, k_{it}).
- Stochastic and Markovian assumptions
  - ε_{it}, η_{it}, μ_{it} and ν_{it+1} are independent over time
  - y_{it}, i_{it} and a_{it+1} are independent given $\{k_{it}, a_{it}, z_{it}, l_{it}\}$
  - a_i^{t+1} and i_i^t are indep of $\left(a_i^{t-1}, k_i^{t-1}, z_i^{t-1}\right)$ cond on (a_{it}, k_{it}, z_{it})

Nonparametric Identification

- Sequential identification scheme
  - Identification of β_k and β_l
  - Given β_k and β_l we identify the productivity process and the policy functions

Intuition in linear model

- If $f\left(a_{t+1} \mid z_t, X_t\right)$. and $f\left(i_t \mid z_t, X_t\right)$ are normal distributed

$$a_{it+1} = g_z z_{it} + g_a a_{it} + \nu_{it+1}$$
$$i_{it} = h_z z_{it} + h_a a_{it} + \mu_{it}$$

  - Self-financing $\Rightarrow g_z \neq 0$
  - Proxy variable: $z_{it} = \pi_1 i_{it} + \pi_2 a_{it} + \pi_3 \mu_{it}$

- Replacing the proxy variable in the prod function:

$$y_{it} = \beta_l l_{it} + \beta_k k_{it} + \pi_1 i_{it} + \pi_2 a_{it} + \varepsilon_{it} + \pi_3 \mu_{it}$$

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- Use a_{it+1} as an instrument for i_{it} in an IV estimation
  - Relevance: a_{it+1} is correlated with i_{it} through z_{it} if $g_z \neq 0$ (self-financing)
  - Exogeneity: $E(i_{it}\mu_{it+1}) = 0$ since $E(\nu_{it+1}\mu_{it}) = 0$
- If a firm experiences a positive productivity shock it increases **simultaneously** investment and wealth accumulation

Non parametric identification (Hu Schennach 2008)

- Identify $f\left(y_{it} \mid z_{it}, k_{it}, l_{it}\right) \rightarrow E\left[y_{it} \mid z_{it}=0, k_{it}, l_{it}\right]=\beta_{l} l_{it}+\beta_{k} k_{it}$
- Challenge: recover $f\left(y_{it} \mid z_{it}, k_{it}, l_{it}\right)$ from $f\left(y_{it}, i_{it}, a_{it+1}, a_{it}, k_{it}, l_{it}\right)$.

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- Given the conditional independence assumption:

$$f(y_t, i_t \mid a_{t+1}, X_t) = \int f(y_t \mid z_t, k_t, l_t)\, f(i_t \mid z_t, X_t)\, f(z_t \mid a_{t+1}, X_t)\, dz$$

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- Defining $L_{y|z,k,l}$ and $L_{a|z,X}$ as the integral operators associated with $f\left(y_t \mid z_t, k_t, l_t\right)$ and $f\left(a_{t+1} \mid z_t, X_t\right)$ and integrating over i_{it}

$$L_{y;I|a,X} L_{y|a,X}^{-1}=L_{y|z,k,l} D_{I;z|X} L_{y|z,k,l}^{-1}$$

- Observed object $L_{y;I|a,X} L_{y|a,X}^{-1}$ admits eigenvalue-eigenfunction decomp.

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- Observed object $L_{y;I|a,X} L_{y|a,X}^{-1}$ admits eigenvalue-eigenfunction decomp.
    - $L_{y|a,X} = L_{y|z,k,l} L_{z|a,X}$ has to be invertible
    - Relevance: $L_{z|a,X}$ is invertible if $f(a_{t+1} \mid z_t, X_t)$ is complete

Identification Productivity Process

- Once we identify β_l and β_k, we have the net income process:

$$\tilde{y}_{it} = y_{it} - \beta_l l_{it} - \beta_k k_{it} = z_{it} + \varepsilon_{it}$$

  - additive model with two independent latent variables z_{it} and ε_{it}
  - Hidden Markov: $\{\tilde{y}_{it-2}, \tilde{y}_{it-1}, \tilde{y}_{it}\}$ are independent given z_{it-1}.
  - the joint distribution of $(\varepsilon_{i2,\cdots,}\varepsilon_{iT-1})$ and $(z_{i2,\cdots,}z_{iT-1})$ are identified from the autocorrelation structure of $(\tilde{y}_{i1,\cdots,}\tilde{y}_{iT})$ for $T \geq 3$

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- Linear Process:

$$z_{it} = \rho_z z_{it-1} + \eta_{it}$$

$$\rightarrow \tilde{y}_{it} = \rho_z \tilde{y}_{it-1} + \eta_{it} + \varepsilon_{it} - \rho_z \varepsilon_{it-1}$$

- Use $\tilde{y}_{it-2}$ as an instrument for $\tilde{y}_{it-1}$ to identify ρ_z
- σ_η^2 and σ_ε^2 are identify from the following moment conditions:

$$E\left(\tilde{y}_{it}\tilde{y}_{it-1}\right) = \rho_z E\left(\tilde{y}_{it-1}\tilde{y}_{it-1}\right) - \rho_z \sigma_\varepsilon^2$$
$$E\left(\tilde{y}_{it}\tilde{y}_{it}\right) = \rho_z^2 E\left(\tilde{y}_{it-1}\tilde{y}_{it-1}\right) + \sigma_\eta^2 + \left(1 - \rho_z^2\right)\sigma_\varepsilon^2$$

Policy Functions: Linear case

- Policy rules depends on unobserved z_{it}:

$$a_{it+1} = g_z z_{it} + g_a a_{it} + g_k k_{it} + \nu_{it+1}$$

$$\rightarrow a_{it+1} = g_z \tilde{y}_{it} + g_a a_{it} + g_k k_{it} + \nu_{it+1} - g_z \varepsilon_{it}$$

- $E\left(\tilde{y}_{it}\varepsilon_{it}\right) \neq 0$: OLS regression of a_{it+1} on $\tilde{y}_{it}$, a_{it} and k_{it} do not work
- Use $\tilde{y}_{it-1}$ as an instrument for $\tilde{y}_{it}$
  - Relevance: Markovian productivity ensures: $E\left(\tilde{y}_{it}\tilde{y}_{it-1}\right) \neq 0$
  - Exogeneity: $E\left(\tilde{y}_{it-1}\varepsilon_{it}\right) = 0$

Outline

- ✓ Introduction
- ✓ Brief example: Olley-Pakes under financial frictions
- ✓ Empirical framework
- ▶ Results
- 5. Conclusion

Data

- Chilean tax annual administrative data from 2005-2016.
- Approx. 4800 firms (manufacturing), 13500 obs.
- Form 22 (income tax/ reflect dividend policies, balance sheet):
    - y_{it}: *value added* .
    - k_{it}: *physical capital.*
    - a_{it}: *wealth/ net worth.*
- Form 29 (monthly report of expenditures/flows):

- i_{it} : *investment*

- DJ 1887:
    - l_{it}: Number of workers

Results: Production Function with Chilean data

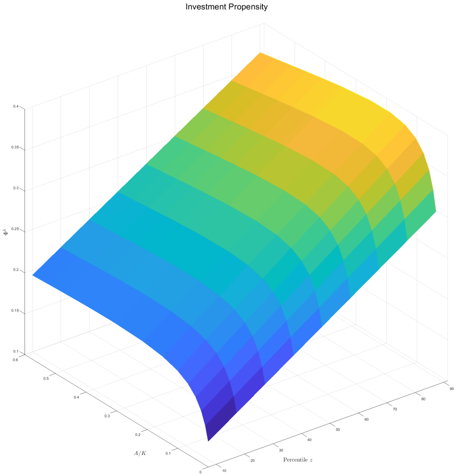
	OP	Proxy-IV
β_k	0.35	0.43
	0.05	*0.01*
β_l	0.67	0.46
	0.008	*0.01*
σ_ϵ	0.68	0.22
ρ_z	0.53	0.87
	0.01	*0.01*
σ_η	0.18	0.31
Observations	13516	13516
Firms	4867	4867

Table: Production Function Estimates from Microdata

Note: The table shows the Production function estimates from administrative data for Chile, using alternative methodologies: Olley and Pakes 1996 (OP), and two estimators that control for financial friction, Proxy-IV and SEM.

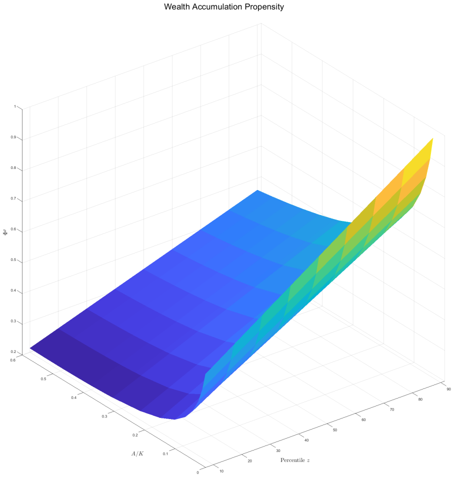
Investment Policy Rule: Nonlinear effects

- Investment propensity evaluated at different quantiles of a and z
  - High heterogeneity in propensities (0.08-0.6)
  - Increasing in a and z
  - Sensitivity to a depends on z (low z increases in 200%, high z in 30%)



Wealth Accumulation Policy Rule: Nonlinear

- Wealth accumulation propensity evaluated at different quantiles of a and z
  - High heterogeneity in propensities (0.2 - 0.98)
  - Decreasing in a and increasing z
  - Low a - high z firms benefit the most from an additional unit of wealth



How strong is self-financing?

- Simulate MPKs for two firms with the same z but different a
- MPK of low a firm is 3 times MPK of high a (same as Banerjee and Moll 2010)
- Convergence takes more than 50 years (7 years in Banerjee and Moll 2010)

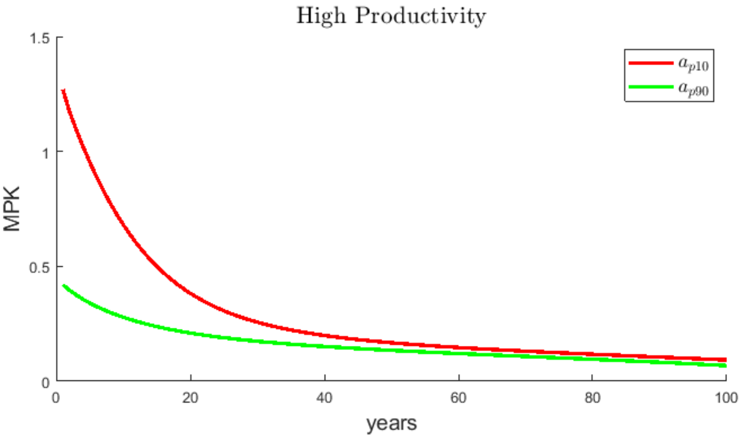


Figure: Convergence in MPKs

Conclusions

- Flexible non-linear framework to jointly model and estimate the firm wealth accumulation dynamics and the unobservable productivity process under collateral constraints.
- Reduce bias of prevalent strategies to estimate production functions and productivity.
- New results on policy functions: heterogenous responses of investment and wealth
  - Investment propensity is increasing in wealth and productivity
  - Wealth accumulation propensity is increasing in productivity and decreasing in wealth
- Self-financing is active in the data but its impact is limited
- Our estimates might inform structural models
  - Direct estimates of production parameters
  - new elasticitiesto indirectly estimate key deep parameters

Estimated propensities

- The figure exhibits how the investment propensity and the wealth accumulation propensity vary along the distribution of $\frac{A}{K}$ in the micro data.
- Each point represents the propensity of each particular firm evaluated at its actual value of a, k and z.

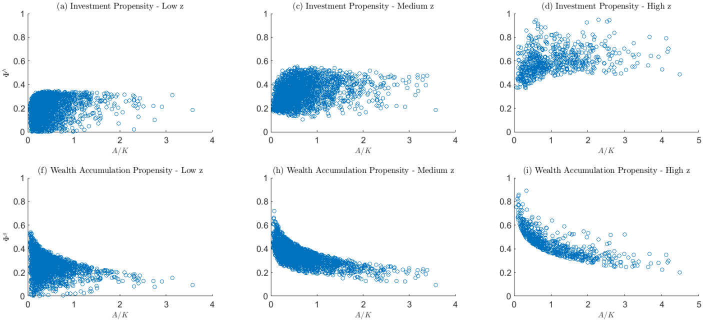


Figure: Estimated propensities

Correlation between policy shocks

- For different values of β_l and β_k generate $\tilde{y}_{it}$:

$$\tilde{y}_{it} = y_{it} - \beta_l l_{it} - \beta_k k_{it} = z_{it} + \varepsilon_{it}$$

- Estimate using IV the following policies:

$$i_{it} = h_t\left(\tilde{y}_{it}, k_{it}, a_{it}\right) + \mu_{it}$$
$$a_{it+1} = g_t\left(\tilde{y}_{it}, k_{it}, a_{it}\right) + \nu_{it+1}$$

- Run OLS regression between μ_{it} and ν_{it+1}

β_k/β_l	0.05	0.15	0.26	0.36	0.47	0.57	0.68	0.78	0.89	0.99
0.05	-0.00005	0.00333	0.00797	0.01277	0.01563	0.01522	0.01329	0.0132	0.01593	0.01998
	(0.00611)	(0.0061)	(0.00613)	(0.00623)	(0.00635)	(0.00645)	(0.00648)	(0.00646)	(0.00644)	(0.00645)
0.15	0.00037	0.00272	0.00559	0.00762	0.00692	0.00353	0.00108	0.00311	0.00889	0.0155
	(0.0061)	(0.0061)	(0.00615)	(0.00626)	(0.0064)	(0.00651)	(0.00651)	(0.00646)	(0.00643)	(0.00644)
0.26	0.00033	0.00128	0.00187	0.00073	-0.0031	-0.0075	-0.0078	-0.0022	0.00643	0.01462
	(0.00609)	(0.00609)	(0.00614)	(0.00626)	(0.0064)	(0.00647)	(0.00645)	(0.0064)	(0.00638)	(0.00639)
0.36	0.00032	0.0006	0.00045	-0.0012	-0.0047	-0.0075	-0.0056	0.00127	0.00998	0.0178
	(0.00608)	(0.00608)	(0.00613)	(0.00622)	(0.00632)	(0.00637)	(0.00634)	(0.00631)	(0.00631)	(0.00635)
0.47	0.00089	0.00124	0.0011	-0.0007	-0.0041	-0.006	-0.003	0.00454	0.01334	0.021
	(0.00609)	(0.00608)	(0.00611)	(0.00619)	(0.00626)	(0.00628)	(0.00626)	(0.00625)	(0.00627)	(0.00632)
0.57	0.00156	0.00185	0.00128	-0.0011	-0.0047	-0.0058	-0.0017	0.00665	0.01591	0.02373
	(0.0061)	(0.00608)	(0.0061)	(0.00616)	(0.0062)	(0.00621)	(0.00619)	(0.00619)	(0.00623)	(0.00629)
0.68	0.00235	0.00237	0.00116	-0.002	-0.0058	-0.0063	-0.0011	0.00824	0.01817	0.02626
	(0.0061)	(0.00608)	(0.00609)	(0.00612)	(0.00615)	(0.00615)	(0.00613)	(0.00614)	(0.00619)	(0.00626)
0.78	0.00334	0.00295	0.00095	-0.0031	-0.0073	-0.0073	-0.0009	0.00972	0.02043	0.02881
	(0.0061)	(0.00607)	(0.00607)	(0.00609)	(0.00609)	(0.00608)	(0.00606)	(0.00608)	(0.00614)	(0.00623)
0.89	0.00443	0.0035	0.00054	-0.0046	-0.0092	-0.0085	-0.0006	0.01134	0.02284	0.03146
	(0.0061)	(0.00606)	(0.00605)	(0.00604)	(0.00603)	(0.006)	(0.00598)	(0.00602)	(0.00609)	(0.00619)
0.99	0.00553	0.00393	-0.0002	-0.0065	-0.0115	-0.0098	-0.0002	0.01322	0.02547	0.03426
	(0.00609)	(0.00604)	(0.00602)	(0.00599)	(0.00595)	(0.00591)	(0.0059)	(0.00594)	(0.00604)	(0.00615)

Empirical model under financial frictions

- At the beginning of $t = 1$ firm starts with fixed and financial assets K_1, A_1^f and debt D_1 (decided in $t = 0$)
- Firm's net worth at the beginning of $t = 1$ $A_1 = A_1^f + K_1 - D_1$
- Productivity Z_1 is realized:
  - Firm hires L_1 and produce with predetermined $K_1 \rightarrow Y_1$
  - There is time to built (is that it takes a full period for new capital to be ordered, delivered, and installed) so firm decides investment today to produce next period:
  - Use internal assets A_1^f and/or ask for a loan: $K_2 \rightarrow I_1 = h(Z_1, A_1) \rightarrow \Delta D$

    I_1 is a production decision for $t = 2$ but depends on A_1decided at $t = 0$
- At that point in time net worth is unchanged ($\mathrm{I}_1 = \Delta D$)
- At the end of $t = 1$ firm decide to retain earnings or pay dividends $\Delta A^f = \tilde{Y_1} - C_1 = \tilde{g}(E(u(A_2, Z_2)) = \tilde{g}(A_2, Z_1)$
- net worth in $t = 2$ (accumulated at the end of $t = 1$) is used for I_2 to produce in $t = 3$
- is the time to built assumption and the dividend payment policy at the end of the period that creates the timing that separates I_1 from A_2?

Investment Policy Rule: Linear effects

- Marginal effect of productivity and wealth on investment function evaluated at different quantiles of distribution of wealth

$$i_{it} = h_{1t}\left(k_{it}, a_{it}\right) + h_{2t}\left(k_{it}, a_{it}\right) z_{it} + \nu_{it}$$

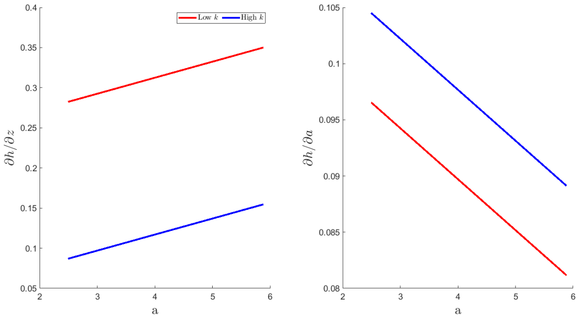


Figure: Marginal effect of productivity on investment

Materials Policy Rule: Linear effects

- Marginal effect of productivity on materials policy function evaluated at different quantiles of distribution of wealth

$$m_{it} = h_{1t}(k_{it}, a_{it}) + h_{2t}(k_{it}, a_{it}) z_{it} + \nu_{it}$$

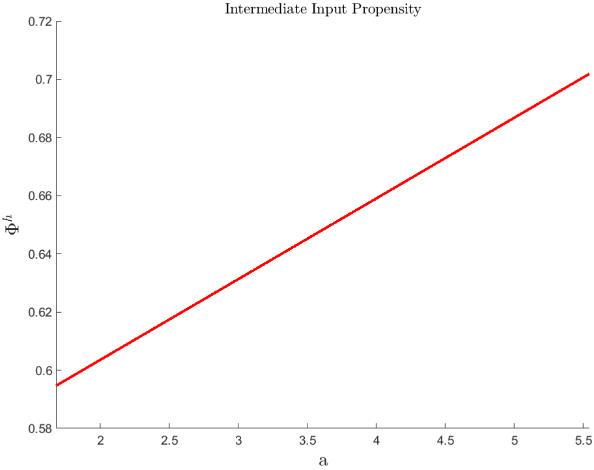


Figure: Marginal effect of productivity on materials

Wealth Accumulation Policy Rule: Linear effects

- Marginal effect of productivity and wealth on wealth function evaluated at different quantiles of distribution of wealth

$$a_{it+1} = g_{1t}\left(k_{it}, a_{it}\right) + g_{2t}\left(k_{it}, a_{it}\right) z_{it} + \mu_{it+1}$$

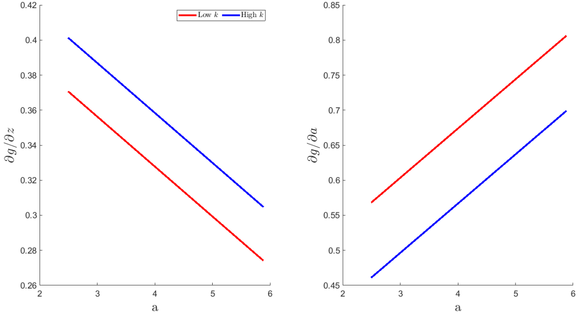


Figure: Marginal effect of productivity on investment

Results: Productivity Process

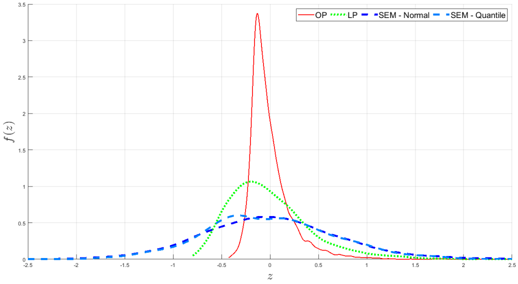


Figure: Estimated distribution of productivities

Notes: The figure shows the estimated distribution of firm-level productivities using administrative microdata for Chile, under alternative methodologies: OP , LP, the SEM algorithm using Normal shocks and the SEM algorithm using a quantile model (ABB) .

Results: Productivity Process

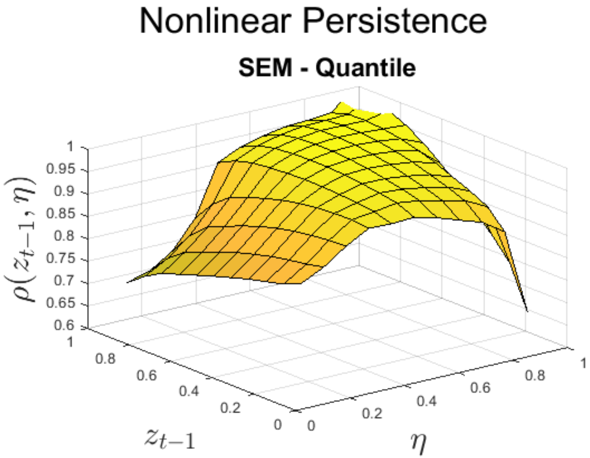


Figure: Estimated distribution of productivities

Notes: The figure shows the estimated persistence of firm-level productivities using administrative microdata for Chile, using Normal shocks and the SEM algorithm using a quantile model (ABB) .

Resuls: Production Function with simulated data

	Parameter	OP	Proxy-IV	SEM
β_l	0.44	0.515	0.443	0.442
β_k	0.43	0.387	0.424	0.431

Table: Production Function Estimates Using Simulated Data

Note: Production function estimates from simulated data using alternative methodologies: Olley and Pakes 1996 (OP), and two estimators that control for financial friction, Proxy-IV and SEM.

Results: Production Function with Chilean data

	OP	Proxy-IV	SEM
β_l	0.67	0.44	0.46
	0.008	*0.01*	*0.003*
β_k	0.35	0.42	0.43
	0.05	*0.01*	*0.007*
σ_ϵ	0.68	0.22	0.20
Observations	13516	13516	13516
Firms	4867	4867	4867

Table: Production Function Estimates from Microdata

Note: The table shows the Production function estimates from administrative data for Chile, using alternative methodologies: Olley and Pakes 1996 (OP), and two estimators that control for financial friction, Proxy-IV and SEM.

Results: Productivity Process

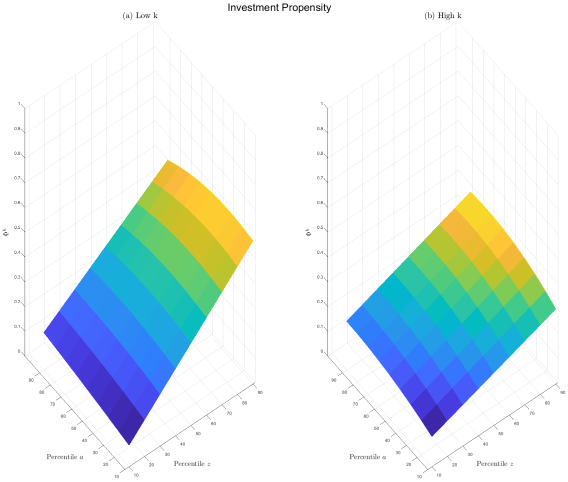
$$z_{it} = \rho z_{it-1} + \eta_{it}$$

	OP	IV	SEM
ρ_z	0.53	0.87	0.85
	0.01	*0.01*	*0.01*
σ_η	0.18	0.31	0.39
Observations	13516	13516	13516
Firms	4867	4867	4867
R^2	0.37	-	0.70

Investment Policy Rule: Nonlinear effects

- Marginal effect of productivity on nonlinear investment function evaluated at different quantiles of distribution of wealth

$$i_{it} = h\left(z_{it}, k_{it}, a_{it}, \mu_{it}\right)$$



From Micro to Macro Development (Buera, Kaboski Townsend, 2021)

- Buera, Kaboski Townsend (JEL 2021) Over the same period of time, a macro economic literature has made advances in building and solving models incorporating rich micro-structure, that is, with well-defined agent problems, with heterogeneity, and with contracting and market frictions. However this line of work has tended to rely on strong structural assumptions, e.g., assumptions on functional forms and distributions of unobservables, and on somewhat stylized calibration strategies, and thus economists often view it as disconnected from micro empirical research.

back

A simple model with Financial Frictions

- Entrepreneur, with initial wealth A_t, capital K_t and productivity Z_t, maximize the discounted value of distributed profits, D_t:

$$\begin{aligned} V(A_t, K_t, Z_t) &= \max_{A_{t+1}, K_{t+1}, L_t} D_t + \beta E\left[V(A_{t+1}, K_{t+1}, Z_{t+1}) | Z_t\right] \\ st. \qquad D_t &= Z_t K_t^{\beta_k} L_t^{\beta_l} - (r_t^b + \delta) K_t - w_t L_t - \frac{g(A_{t+1})}{1 + r_t^a} + A_t \\ K_{t+1} &\leq \lambda A_t \end{aligned}$$

where $g(\cdot)$ is a cost function and λ is the financial friction parameter

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where $g(\cdot)$ is a cost function and λ is the financial friction parameter

- FOCs

$$\begin{aligned} \beta_l Z_t K_t^{\beta_k} L_t^{\beta_l - 1} &= w + \omega_t \\ C_k E(Z_{t+1} | Z_t)^{\frac{1}{1-\beta_l}} K_{t+1}^{\frac{\beta_k}{1-\beta_l} - 1} &= r + \delta + \mu(A_t) \\ g'(A_{t+1}) &= \beta \left(1 + \lambda E\left[\mu(A_{t+1})\right]\right) \end{aligned}$$

where $\mu(A_t)$ is the multiplier of the financial constraint

Estimating Capital Elasticity

- From the FOC for investment we obtain an implicit function for z_t (after taking logs and assuming a Markov process for z_t)

$$z_t = c_z + \frac{1}{\rho}\left((1-\beta_k-\beta_l)k_{t+1} + (1-\beta_l)\tilde{\mu}(a_t)\right)$$

- Following OP we replace this into the production function

$$\begin{aligned} y_t &= \beta_l l_t + \beta_k k_t + z_t + \varepsilon_t \\ &= \beta_l l_t + \beta_k k_t + \frac{1}{\rho}(1-\beta_k-\beta_l)k_{t+1} + (1-\beta_l)\tilde{\mu}(a_t) + \varepsilon_t \end{aligned}$$

- OP: investment (k_{t+1} here) controls for unobservable productivity
- But with collateral constraints $E(k_t\tilde{\mu}(a_t)) < 0 \rightarrow \beta_k^{OP} < \beta_k$

Estimating Labor Elasticity

- We can express the FOC for labor as

$$
\begin{aligned}
l_t &= c_n + \frac{1}{1-\beta_l}\left(\beta_k k_t + \tilde{\omega}_t + z_t\right) \\
&= \tilde{c}_n + \frac{1}{1-\beta_l}\left(\beta_k k_t + \tilde{\omega}_t\right) + \frac{(1-\beta_k-\beta_l)}{\rho(1-\beta_l)} k_{t+1} + \frac{1}{\rho}\tilde{\mu}(a_t)
\end{aligned}
$$

- Recall the equation for estimating the production function

$$
\begin{aligned}
y_t &= \beta_l l_t + \beta_k k_t + z_t + \varepsilon_t \\
&= \beta_l l_t + \beta_k k_t + \frac{1}{\rho}(1-\beta_k-\beta_l)k_{t+1} + (1-\beta_l)\tilde{\mu}(a_t) + \varepsilon_t
\end{aligned}
$$

- With collateral constraints $E(l_t\tilde{\mu}(a_t)) > 0 \rightarrow \beta_l^{OP} > \beta_l$

Estimation: Policies without shocks

- OPA: Augmented OP

$$i_{it} = h\left(z_{it}, k_{it}, a_{it}\right) \Rightarrow z_{it} = h^{-1}\left(i_{it}, k_{it}, a_{it}\right)$$

$\Rightarrow$

$$y_{it} = \beta_l l_{it} + \beta_k k_{it} + h^{-1}\left(i_{it}, k_{it}, a_{it}\right) + \varepsilon_{it},$$

  - OLS of y_{it} on l_{it} and a polynomial on k_{it}, i_{it} and a_{it}

Estimation: Policies without shocks

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$$i_{it} = h\left(z_{it}, k_{it}, a_{it}\right) \Rightarrow z_{it} = h^{-1}\left(i_{it}, k_{it}, a_{it}\right)$$

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  - OLS of y_{it} on l_{it} and a polynomial on k_{it}, i_{it} and a_{it}

- Proxy-Wealth

$$a_{it+1} = g\left(z_{it}, k_{it}, a_{it}\right) \Rightarrow z_{it} = g^{-1}\left(a_{it+1}, k_{it}, a_{it}\right)$$

$\Rightarrow$

$$y_{it} = \beta_l l_{it} + \beta_k k_{it} + g^{-1}\left(a_{it+1}, k_{it}, a_{it}\right) + \varepsilon_{it},$$

  - OLS of y_{it} on l_{it} and a polynomial on k_{it}, a_{it+1} and a_{it}

Policies with shocks: Proxy-IV

- Consider the following quasi-linear wealth accumulation policy:

$$a_{it+1} = g_1(k_{it}, a_{it}) + g_2(k_{it}, a_{it}) z_{it} + w_{it+1}$$

$\Rightarrow$

$$z_{it} = \pi_1(k_{it}, a_{it}) + \pi_2(k_{it}, a_{it}) a_{it+1} + \omega_{it+1}$$

Policies with shocks: Proxy-IV

- Consider the following quasi-linear wealth accumulation policy:

$$a_{it+1} = g_1(k_{it}, a_{it}) + g_2(k_{it}, a_{it}) z_{it} + w_{it+1}$$

$\Rightarrow$

$$z_{it} = \pi_1(k_{it}, a_{it}) + \pi_2(k_{it}, a_{it}) a_{it+1} + \omega_{it+1}$$

- Replacing the proxy variable in the production function:

$$y_{it} = \beta_l l_{it} + \phi(k_{it}, a_{it}) + \pi_2(k_{it}, a_{it}) a_{it+1} + \omega_{it+1} + \varepsilon_{it}$$

where $\phi(k_{it}, a_{it}) = \beta_k k_{it} + \pi_1(k_{it}, a_{it})$

  - IV using $\pi_2(k_{it}, a_{it}) i_{it}$ as the instrument for $\pi_2(k_{it}, a_{it}) a_{it+1}$

Results: Production Function

	OP	OPA	Proxy-Wealth	Proxy-IV	SEM
β_l	0.65	0.51	0.48	0.43	0.44
	0.008	*0.007*	*0.006*	*0.01*	*0.002*
β_k	0.35	0.41	0.43	0.45	0.43
	0.05	*0.04*	*0.04*	*0.01*	*0.001*
σ_ϵ	0.68	0.57	0.52	0.22	0.20
Observations	13516	13516	13516	13516	13516
Firms	4867	4867	4867	4867	4867

- OP: inverting the investment equation
- OPA: inverting the investment equation but controlling for a_{it}
- Proxy-Wealth: inverting the wealth accumulation equation
- Proxy-IV: uses investment and wealth equation through an IV regression
- SEM: non-linear approach that uses the full information of the model

Results: Production Function with Chilean data

	OP	LP	Proxy-IV	SEM
β_l	0.67	0.81	0.44	0.46
	0.008	*0.007*	*0.01*	*0.003*
β_k	0.35	0.33	0.42	0.43
	0.05	*0.04*	*0.01*	*0.007*
σ_ϵ	0.68	0.62	0.22	0.20
Observations	13516	13516	13516	13516
Firms	4867	4867	4867	4867

Table: Production Function Estimates from Microdata

Note: The table shows the Production function estimates from administrative data for Chile, using alternative methodologies: Olley and Pakes 1996 (OP), Levinsohn and Petrin 2003 (LP), and two estimators that control for financial friction, Proxy-IV and SEM.

Results: Productivity Process

	OP	OPA	Proxy-Wealth	Proxy-IV	SEM
ρ_z	0.53	0.70	0.83	0.87	0.82
	0.01	*0.01*	*0.01*	*0.01*	*0.01*
σ_η	0.16	0.31	0.26	0.28	0.42
Observations	13516	13516	13516	13516	13516
Firms	4867	4867	4867	4867	4867
R^2	0.37	0.53	0.74	-	0.70

- OP: inverting the investment equation
- OPA: inverting the investment equation but controlling for a_{it}
- Proxy-Wealth: inverting the wealth accumulation equation
- Proxy-IV: uses investment and wealth equation through an IV regression
- SEM: non-linear approach that uses the full information of the model

Estimator

- Unfeasible moment condition:

$$E\left[\sum_{t=1}^{T}\left(y_{it}-\beta_l l_{it}-\beta_k k_{it-1}-z_{it}\right)k_{it-1}\right]=0$$

- Applying the law of iterated exp, obtain the integrated moment conditions:

$$E\left[\int\left(\sum_{t=1}^{T}\left(y_{it}-\beta_l l_{it}-\beta_k k_{it-1}-z_{it}\right)k_{it-1}\right)f\left(z_{it}\mid z_{it-1},y_{it},k_{it-1},l_{it}\right)dz\right]=0$$

where $f\left(z_{it}\mid z_{it-1},y_{it},k_{it-1},l_{it}\right)$ is the posterior distribution of z_{it} given data.

- Draw $\left\{z_{it}^{(1)}\ldots z_{it}^{(M)}\right\}$ M realizations of z_{it} from $f\left(z_{it}\mid .\right)$

$$\sum_{i=1}^{N}\sum_{m=1}^{M}\sum_{t=1}^{T}\left(\left(y_{it}-\beta_l l_{it}-\beta_k k_{it-1}-z_{it}^{(m)}\right)k_{it-1}\right)$$

First step: posterior distributions

- Let X_i and Z_i the history of observables and productivity for firm i.
- Posterior distribution of z_{it}

$$
\begin{aligned}
f\left(z_{i1} \ldots z_{iT}, X_i\right) = & \prod_{t=1}^{T} f\left(y_{it} \mid k_{it}, l_{it}, z_{it}\right) \times \\
& \prod_{t=1}^{T} f\left(i_{it} \mid k_{it}, z_{it}, a_{it}\right) \times \\
& \prod_{t=2}^{T} f\left(a_{it} \mid z_{it-1}, k_{it-1}, i_{it-1}, a_{it-1}\right) f\left(a_{i1} \mid z_{i1}\right) \times \\
& \prod_{t=1}^{T} f\left(z_{it} \mid z_{it-1}\right) f\left(z_{i1}\right)
\end{aligned}
$$

Second step: Integrated GMM

- After drawing M realizations of $Z_i^{(m)}$ from the joint posterior distribution, compute the integrated-GMM estimator of the parameters:

$$\sum_{i=1}^{N}\sum_{t=2}^{T}\sum_{m=1}^{M}\left(y_{it}-\beta_l l_{it}-\beta_k k_{it}-z_{it}^{(m)}\right)^2$$

$$\sum_{i=1}^{N}\sum_{t=2}^{T}\sum_{m=1}^{M}\left(i_{it}-\tilde{h}\left(z_{it}^{(m)},k_{it},a_{it}\right)\right)^2$$

$$\sum_{i=1}^{N}\sum_{t=2}^{T}\sum_{m=1}^{M}\left(a_{it+1}-\tilde{g}\left(z_{it}^{(m)},a_{it},k_{it},i_{it}\right)\right)^2$$

$$\sum_{i=1}^{N}\sum_{t=2}^{T}\sum_{m=1}^{M}\left(z_{it}^{(m)}-\tilde{\varphi}\left(z_{it-1}^{(m)}\right)\right)^2$$

where $\tilde{h}(.)$, $\tilde{g}(.)$, $\widetilde{\varphi(.)}$, $\tilde{\pi}(.)$ are the polynomial approx. of $h(.)$, $g(.)$, $\varphi(.)$, $\pi(.)$.

Estimation: No shocks in the policies

- OPA: Augmented OP (including a_{it} in investment)

$$z_{it} = h^{-1}\left(i_{it}, k_{it}, a_{it}\right)$$

Estimation: No shocks in the policies

- OPA: Augmented OP (including a_{it} in investment)

$$z_{it} = h^{-1}\left(i_{it}, k_{it}, a_{it}\right)$$

  - First Stage: Replace proxy variable in the production function

$$y_{it} = \beta_l l_{it} + \phi\left(i_{it}, k_{it}, a_{it}\right) + \varepsilon_{it},$$

    where $\phi\left(i_{it}, k_{it}, a_{it}\right) = \beta_k k_{it} + h^{-1}\left(i_{it}, k_{it}, a_{it}\right)$.

    - OLS of y_{it} on l_{it} and a polynomial identifies β_l and $\phi\left(i_{it}, k_{it}, a_{it}\right)$

Estimation: No shocks in the policies

- OPA: Augmented OP (including a_{it} in investment)

$$z_{it} = h^{-1}(i_{it}, k_{it}, a_{it})$$

  - First Stage: Replace proxy variable in the production function

$$y_{it} = \beta_l l_{it} + \phi(i_{it}, k_{it}, a_{it}) + \varepsilon_{it},$$

    where $\phi(i_{it}, k_{it}, a_{it}) = \beta_k k_{it} + h^{-1}(i_{it}, k_{it}, a_{it})$.

    - OLS of y_{it} on l_{it} and a polynomial identifies β_l and $\phi(i_{it}, k_{it}, a_{it})$

  - Second Stage: Use markovian model of productivity

$$\phi_t(i_{it}, k_{it}, a_{it}) = \beta_k k_{it} + \rho_z \phi_t(i_{it-1}, k_{it-1}, a_{it-1}) - \rho_z \beta_k k_{it-1} + \eta_{it} + \varepsilon_{it}$$

    - OLS identifies β_k

Estimation: No shocks in the policies

- OPA: Augmented OP (including a_{it} in investment)

$$z_{it} = h^{-1}\left(i_{it}, k_{it}, a_{it}\right)$$

  - First Stage: Replace proxy variable in the production function

$$y_{it} = \beta_l l_{it} + \phi\left(i_{it}, k_{it}, a_{it}\right) + \varepsilon_{it},$$

    where $\phi\left(i_{it}, k_{it}, a_{it}\right) = \beta_k k_{it} + h^{-1}\left(i_{it}, k_{it}, a_{it}\right)$.

    - OLS of y_{it} on l_{it} and a polynomial identifies β_l and $\phi\left(i_{it}, k_{it}, a_{it}\right)$

  - Second Stage: Use markovian model of productivity

$$\phi_t\left(i_{it}, k_{it}, a_{it}\right) = \beta_k k_{it} + \rho_z \phi_t\left(i_{it-1}, k_{it-1}, a_{it-1}\right) - \rho_z \beta_k k_{it-1} + \eta_{it} + \varepsilon_{it}$$

    - OLS identifies β_k

- Proxy-Wealth: Invert the wealth accumulation policy function

$$z_{it} = g^{-1}\left(a_{it+1}, k_{it}, a_{it}\right)$$

Estimation: Policy functions with shocks

- Proxy-IV: Consider the following quasi-linear wealth policy function

$$a_{it+1} = g_1\left(k_{it}, a_{it}\right) + g_2\left(k_{it}, a_{it}\right) z_{it} + w_{it+1}$$

$\Rightarrow$

$$z_{it} = \pi_1\left(k_{it}, a_{it}\right) + \pi_2\left(k_{it}, a_{it}\right) a_{it+1} + \omega_{it+1}$$

Estimation: Policy functions with shocks

- Proxy-IV: Consider the following quasi-linear wealth policy function

$$a_{it+1} = g_1 (k_{it}, a_{it}) + g_2 (k_{it}, a_{it}) z_{it} + w_{it+1}$$

$\Rightarrow$

$$z_{it} = \pi_1 (k_{it}, a_{it}) + \pi_2 (k_{it}, a_{it}) a_{it+1} + \omega_{it+1}$$

  - First Stage:

$$y_{it} = \beta_l l_{it} + \phi (k_{it}, a_{it}) + \pi_2 (k_{it}, a_{it}) a_{it+1} + \omega_{it+1} + \varepsilon_{it}$$

    where $\phi(k_{it}, a_{it}) = \beta_k k_{it} + \pi_1 (k_{it}, a_{it})$

    - IV using $\pi_2 (k_{it}, a_{it}) i_{it}$ as the instrument for $\pi_2 (k_{it}, a_{it}) a_{it+1}$

Estimation: Policy functions with shocks

- Proxy-IV: Consider the following quasi-linear wealth policy function

$$a_{it+1} = g_1(k_{it}, a_{it}) + g_2(k_{it}, a_{it})\, z_{it} + w_{it+1}$$

$\Rightarrow$

$$z_{it} = \pi_1(k_{it}, a_{it}) + \pi_2(k_{it}, a_{it})\, a_{it+1} + \omega_{it+1}$$

  - First Stage:

$$y_{it} = \beta_l l_{it} + \phi(k_{it}, a_{it}) + \pi_2(k_{it}, a_{it})\, a_{it+1} + \omega_{it+1} + \varepsilon_{it}$$

    where $\phi(k_{it}, a_{it}) = \beta_k k_{it} + \pi_1(k_{it}, a_{it})$

    - IV using $\pi_2(k_{it}, a_{it})\, i_{it}$ as the instrument for $\pi_2(k_{it}, a_{it})\, a_{it+1}$

  - Second Stage:

$$z_{it} = \rho_z \pi_1(k_{it-1}, a_{it-1}) + \rho_z \pi_2(k_{it-1}, a_{it-1})\, a_{it} + \rho_z \omega_{it} + \eta_{it},$$