Deconstructing the Missing Middle: Informality and Growth of Firms in Sub-Saharan Africa

K. Abreha X. Cirera E. Davies R. Fattal–Jaef H. Maemir

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Introduction

- Distortions as drivers of low *TFP*
  - Misallocation, barriers to entry
- How to measure distortions? Information in Size Distributions
  1. Direct inference: theory of efficiency + rich data → back out distortion (Hsieh-Klenow)
  2. Indirect inference: salient differences in employment-based size distributions (Tybout, Hsieh-Olken)

This paper follows indirect approach

- Characterize employment size distribution in 4 Sub-Saharan African countries
- Proposes model and distortions that rationalize patterns and quantify *TFP*

Preview of Findings

- Pervasive missing middle in employment size distribution
- Accounted for by informal firms
    - Formal firms' distribution resembles USA
- Robust to removing FDI and SOEs
- Does not emerge at birth:
    - Right tail emerges as formal firms get larger over life-cycle
- Model with size-dependent distortions to formal firms rationalizes patterns
    - Without distortions, formal firms grow faster and employ the informal

Missing middle useful metric to infer distortions when lacking comprehensive richer data for direct inference

Controversy in the Literature

- Conflicting results on indirect approach to identifying distortions
  - Tybout coined missing middle as pervasive feature of under-development
  - Hsieh-Olken: no bimodality in firm size distribution, artifact of bin aggregation

Our take

- Employment-based size distribution conceptually better to identify distortions
- Revisit controversy in sub-Saharan Africa: clear evidence of missing-middle

Data source

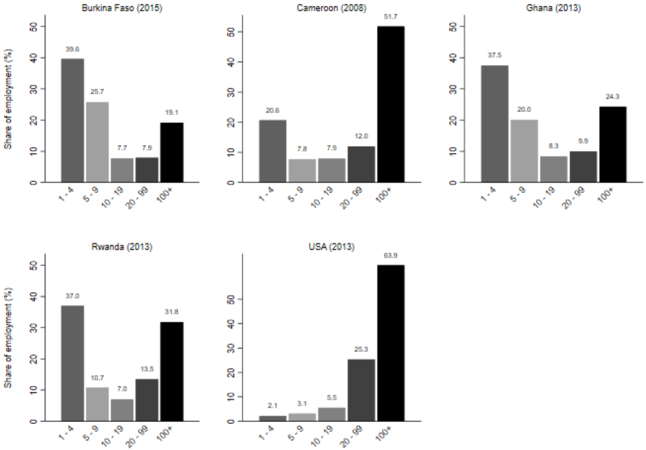
Country	Source	Census year	# Establishments	Coverage
Burkina Faso	INSD	2016	99,261	Non-agricultural sector and institutional agriculture. Businesses with fixed location in 45 provincial capitals and 4 other cities.
Cameroon	INS	2009	88, 144	The entire national territory and targets all establishments in operating in a fixed location; keeping operational account; business in markets; others (e.g., construction operators) except for activities carried out without a fixed place (e.g., informal agricultural, taxi drivers, small repair activities).
Ghana	GSS	2014	638,234	Non-agricultural sector and institutional agriculture. Businesses with fixed location, signposts of business activities except for mobile traders.
Rwanda	NISR	2014	154,236	Complete account of every operating establishment with a fixed location and is practicing a specific economic in the enumeration areas. It covered all regions and districts.

stats

Result:

- The employment based size-distributions in Sub-Saharan Africa is characterized by a missing middle

The Missing Middle

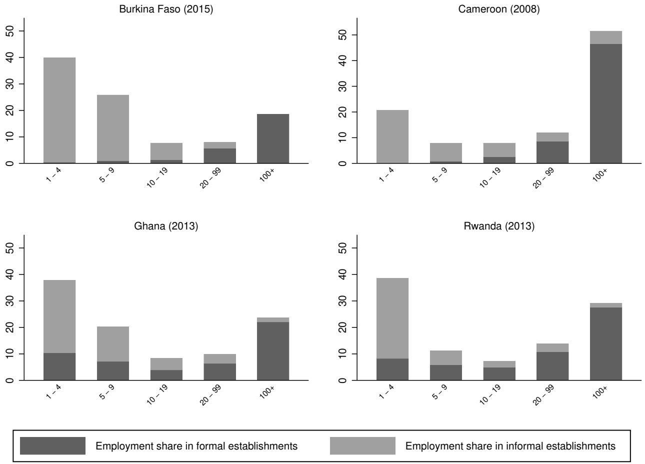


economy-wide

Results:

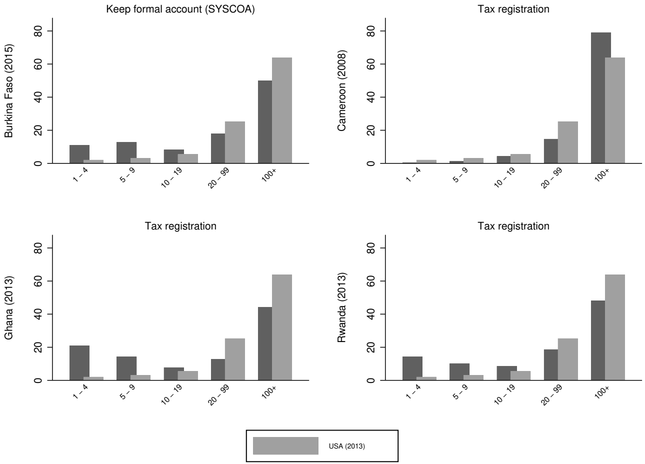
- The Missing Middle is driven by informality
- The formal firms' employment distribution resembles the U.S.'

The Missing Middle is Driven by Informality



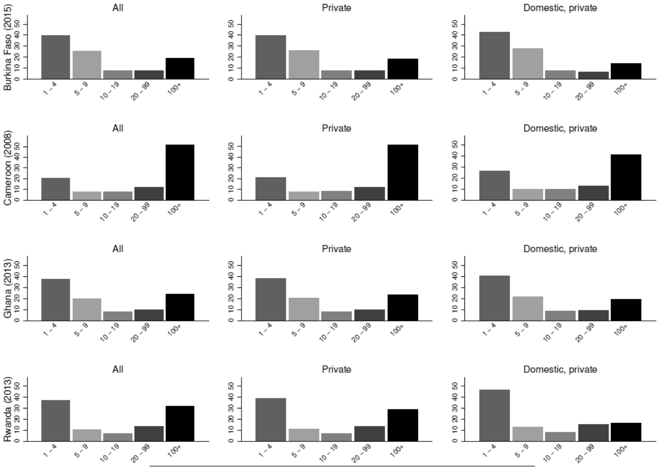
formality-definition

The Formal Firms' Employment Distribution Resembles the U.S.



Result: Missing middle persists after removing SOEs or foreign owned firms

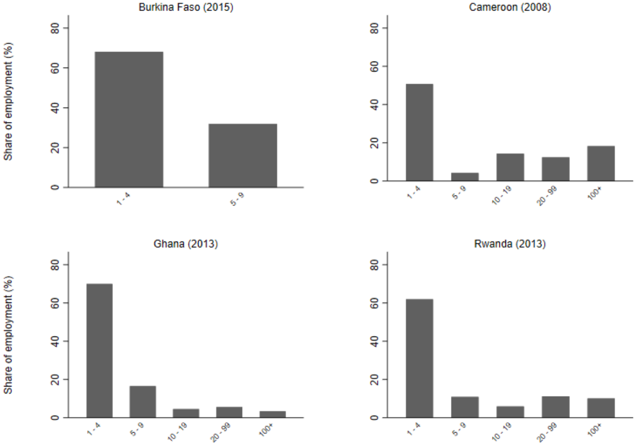
SOEs and FDI Play Minor Role in Missing Middle



Result:

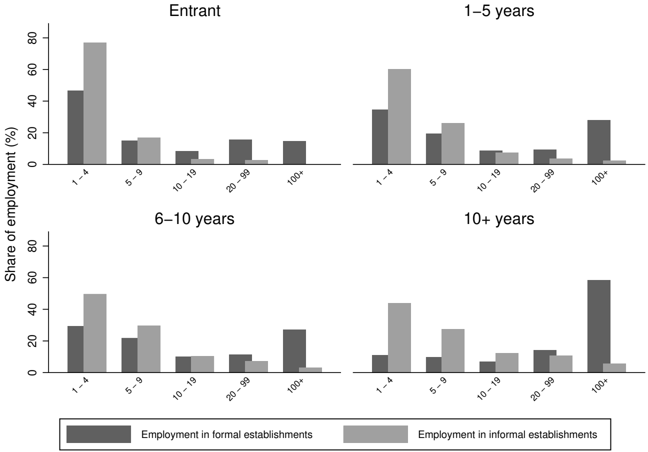
- the missing middle does not emerge at entry
- formal firms grow large over life-cycle conditional on survival

The missing middle does not emerge at entry



▸ life_cycle

Employment Distributions by Age Cohort: Ghana



▸ cameroon ▸ rwanda

Recap

What we learned:

- Employment distribution characterized by missing middle
- Informality is the driver
- Formal firms behave closer to advanced economies

Next Step

- Can formal sector distortions generate missing middle?
  - Idiosyncratic distortions (financial frictions, size-dependent policies)
  - Barriers to firm entry
- Propose model to asses story and quantify macro effects

A Model of Formal Sector Distortions and Informality

Model Environment

- 2 sectors, formal and informal

$$C = \left[Q_{formal}^{\frac{\lambda-1}{\lambda}} + Q_{informal}^{\frac{\lambda-1}{\lambda}} \right]^{\frac{\lambda}{\lambda-1}}$$

  - Imperfect substitutes: grocery store food vs informal mini-market
- Endogenous entry to each sector
- Endogenous firm dynamics in formal sector, exogenous for the informal
- Formal sector distortions: barriers to entry and idiosyncratic distortions
  - e.g.: registration costs, size-dependent taxation, financial frictions

Intuition of Mechanisms

- Calibrate to undistorted economy with small informal sector
- Idiosyncratic distortions generate:
  - misallocation among given set of formal firms
  - reduces innovation incentives
  - So TFP in formal sector goes down $\Longrightarrow$ relative price goes up
  - Encourages entrants to shift to the informal sector
- On top of this, entry barriers:
  - increase the cost of formalization
  - further increase the relative price of formal goods
  - another push to informality
- Elasticity of substitution λ shapes responsiveness of informality
- Effect of distortions on size distributions, informalty, and *TFP*?

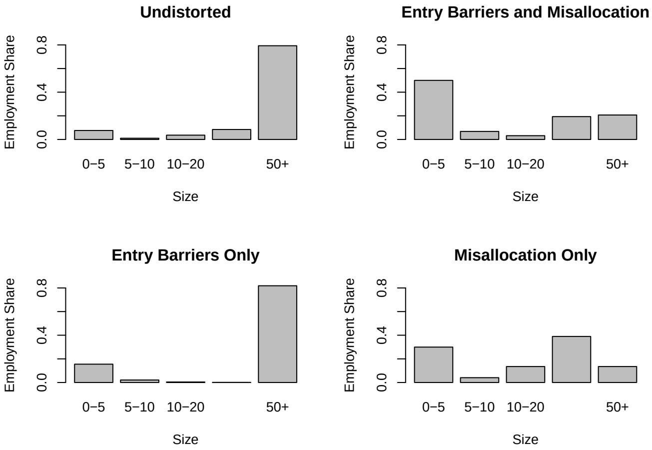
▸ technologies ▸ static ▸ dynamics ▸ entry ▸ equilibrium

Quantitative Analysis

- Calibrate USA undistorted, 8% informal employment
- Feed formal sector distortions:
  - Productivity dependent idiosyncratic distortions (Hsieh-Klenow)
  - Entry barriers
- Size Distributions?
- Aggregate Effects?

distortions

Employment Size Distributions in the Model



Aggregate Results

	Undistorted	Barrier+Misalloc.	Barrier	Misalloc
Informal Empl. Share	0.09	0.58	0.18	0.35
TFP	1.00	0.88	0.95	0.90
Formal GDP	1.00	0.41	0.84	0.65

Concluding Remarks

- Formal sector Census useful to compare with WB Enterprise Surveys
- Informal sector Census can also be used to compare with the WB Informal Surveys
- WB Informal surveys key to uncover the nature of informality
  - Repressed gazelles or otherwise workers?
  - Indirect evidence so far favors the latter

▸ surveys

APPENDIX

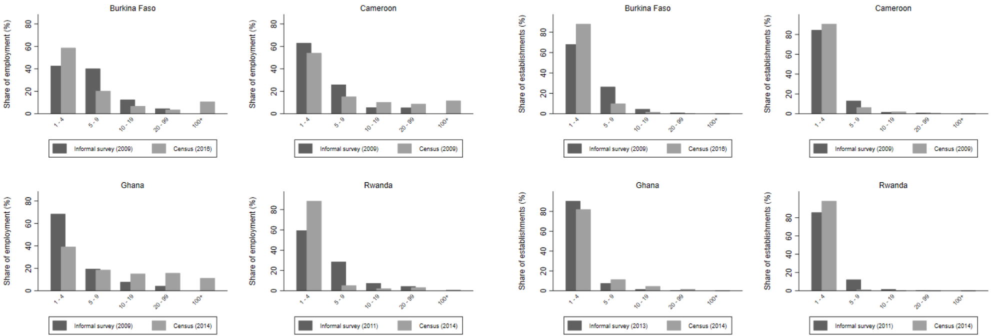
Data description

Country	Establishments				Employment			
		Formality indicator				Formality indicator		
	#	Taxation	Formal account	Census	#	Taxation	Formal account	Census
Burkina Faso (2015)	16, 705	3.89	9.03	3.71	53,116	27.48	10.05	25.27
Cameroon (2008)	10, 922	5.26	8.02	8.12	82,502	58.65	61.39	61.49
Ghana (2013)	99, 437	28.12	4.29	2.95	437,316	50.07	33.37	31.38
Rwanda (2013)	10,742	21.47	9.04	3.77	39, 708	59.11	48.21	37.94

Note: Census definition of formality slightly differs across the statistical offices of the sample countries. In Burkina Faso, an establishment is considered formal it has registration number (with mobile trade and mobile credit, tax identification number) and uses SYSCOA as an accounting standard. Additionally, there is no data on tax registration, approximated by whether an establishment use SYSCOA accounting standard. In Cameroon, formal establishments are the ones that prepare a Statistical and Fiscal Declaration (DSF) or those that do not prepare DSF but which keep an operating account and a partial account of balance sheets. In Ghana, formal establishments are defined as those that are registered with the Registrar General's Department (RGD) and keep formal accounts. In Rwanda, an establishment is considered to operate in the formal sector if it is registered at Rwanda Revenue Authority (RRA) and maintains regular operational accounts.

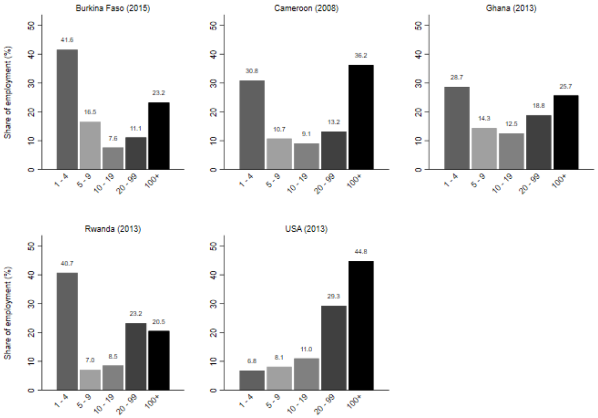
data

Employment Size Distributions: WB Surveys vs Census



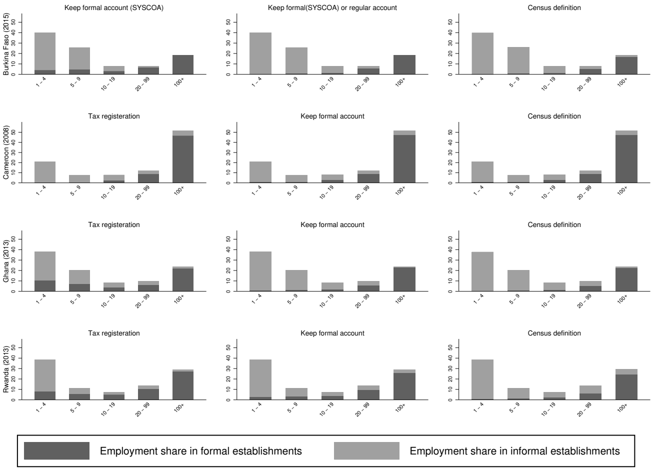
conclusion

The Missing Middle, Economy Wide



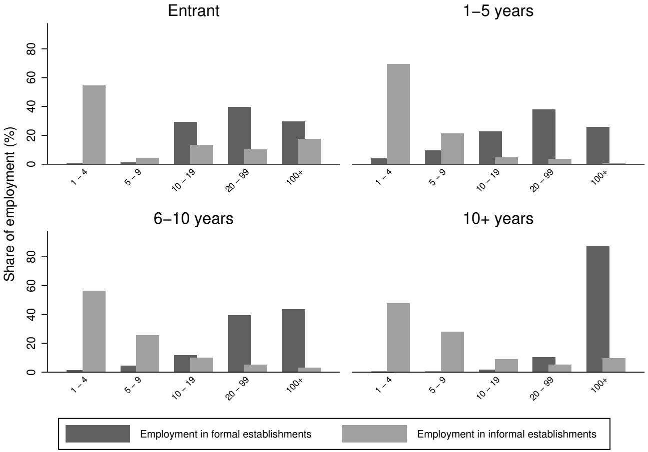
manuf

The Missing Middle is Driven by Informality: Various Definitions



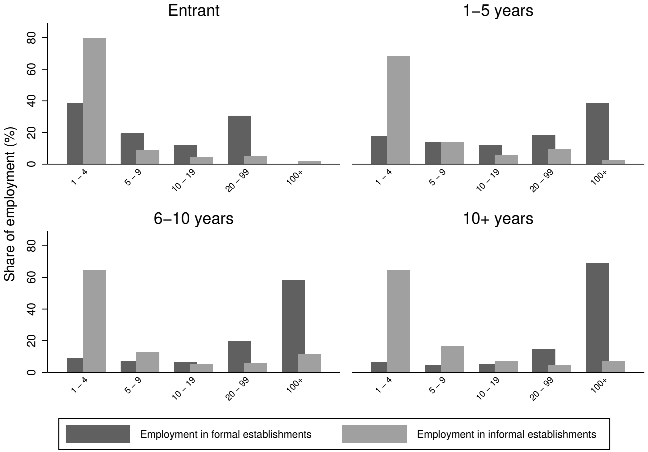
formality

Employment Distributions by Age Cohort: Cameroon



ghana

Employment Distributions by Age Cohort: Rwanda



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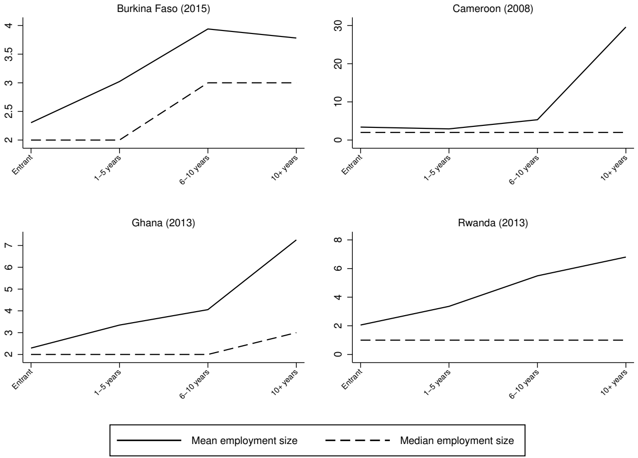
Life-Cycle Growth from the Cross-Section



age entrants

Technologies

- Final good

$$C = \left[Q_{formal}^{\frac{\lambda-1}{\lambda}} + Q_{informal}^{\frac{\lambda-1}{\lambda}} \right]^{\frac{\lambda}{\lambda-1}}$$

- Endogenous number of formal and informal firms

$$Q_i = \left[\int q_i\left(\omega\right)^{\frac{\theta-1}{\theta}} dM\left(\omega\right) \right]^{\frac{\theta}{\theta-1}}$$

  - $\theta > \lambda$: if Nike sneakers become too price, easier to switch to Reebok, ultimately if all formal production of sneakers is pricey consumers switch to the fake ones

- Varieties' technology

$$y_i\left(\omega\right) = \left(A_i\right)^{\frac{1}{\theta-1}} \left(e^{\omega}\right)^{\frac{1}{\theta-1}} l\left(\omega\right)$$

  - $\left(A_i\right)^{\frac{1}{\theta-1}}$ sector wide productivity
  - $\left(e^{\omega}\right)^{\frac{1}{\theta-1}}$ idiosyncratic productivity

mechanisms

Static Optimization

- Competitive final good and factor markets, monopolistic competition in varieties
- Demand function for varieties:

$$q_1(\omega) = \left(\frac{p_1(\omega)}{P_1}\right)^{-\theta} \left(\frac{P_1}{P}\right)^{-\lambda} Q$$

- Intermediate producers' profit maximization

$$Profits = max_{p_i(\omega),l_i(\omega)} \{(1-\tau_\omega) Revenue - Wage\ Bill\}$$

$$MRPL[1-\tau(\omega)] = w$$

$$Labor\ Demand \propto e^{\omega} A_i (1-\tau_\omega)^{\theta}$$

- Idiosyncratic distortions $(1-\tau_\omega)$ only in formal sector (empirically motivated)

mechanisms

Firm Dynamics in Formal Sector: Innovation

- **Formal sector: firm dynamics driven by innovation**

$$e^{\omega} \rightarrow \left\{ \begin{array}{ll} e^{\omega+\Delta} & \text{proba } q_t(\omega) \\ e^{\omega-\Delta} & \text{proba } [1-q_t(\omega)] \end{array} \right\}$$

- Innovation=investing in increasing probability of upgrade $q_t(\omega)$, subject to convex cost

$$FirmValue = \max_{q_t(\omega), Exit/Stay} \left\{ \begin{array}{c} Profits - Innovation\ Cost - Fixed\ Cost \\ + Expected\ Present Value \end{array} \right\}$$

▸ mechanisms

Firm Dynamics in Informal Sector: Exogenous Growth

- Exogenous growth μ and exogenous exit δ

$$e^{\omega(a)} = e^{\mu * a}$$

$$f(a) = e^{-\delta a}$$

- $e^{\omega(a)}$ productivity at age a, $f(a)$ surviving firms cohort age a
- Calibrate growth and exit rate to size-distribution informal firms
- So, shape of size distribution of informal firms will be matched by construction. The question is can we capture the shape of the formal firms', plus can we capture the overall share of informal employment.

mechanisms

Entry

- Free entry into formal (1) and informal sectors (2)

$$wf_{e1}(1+\tau^e) = \mathit{Expected\ Profits}_1(\tau_\omega, \mathit{innovation})$$

$$wf_{e2} = \mathit{Expected\ Profits}(\mu, \delta)$$

  - $f_{e1} > f_{e2}$: without distortions, harder to come up with formal sector idea
  - $(1+\tau^e)$ is the entry barrier

mechanisms

General Equilibrium

- Inelastic supply of labor $\overline{L}$
- Markets clear:

$$Q^{D}_{formal} = Q^{S}_{formal}$$

$$Q^{D}_{informal} = Q^{S}_{informal}$$

$$L_{D} = \overline{L}$$

- Equilibrium objects
  1. $\frac{P_1}{P_2}$: relative price of formal and informal
  2. $M_i\left(e^{\omega}\right)$: distribution of firms across productivity

mechanisms

Quantitative Analysis

Entry Barriers, Idiosyncratic Distortions, and Informality

- Idiosyncratic distortions with productivity dependent feature

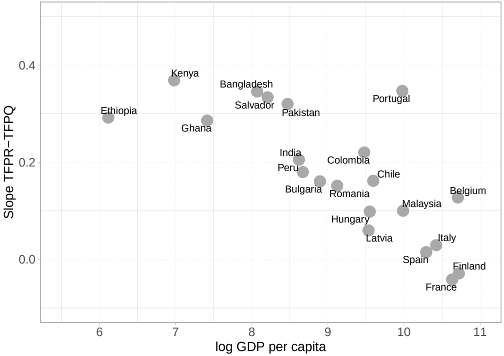
$$(1-\tau_\omega) = \left[(e^\omega)^{\frac{1}{\theta-1}}\right]^{-\gamma}$$

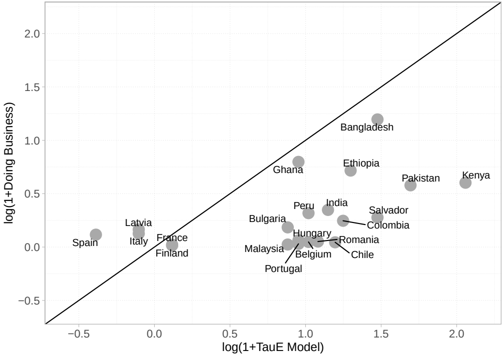
  - Hsieh and Klenow (2009)

$$MRPL[1-\tau(\omega)] = w$$

  - estimate γ from micro-data on formal firms (Cirera et.al.)
- Entry barriers: come up with some estimate of τ^e(doing business, Fattal-Jaef 2022, etc)
- Quantitative Exercise: feed distortions and solve for stationary equilibrium
  - informal employment share? firm-size distributions? GDP effects?

Distortions





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