STEG WORKING PAPER

SPATIAL PRODUCTION NETWORKS

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FEBRUARY 2023
STEG WP055

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January 31, 2023

Abstract

We use new theory and data to study how firms endogenously form production networks across regions and countries. Supplier and buyer relationships form depending on firms' productivity and geographic location. We characterize the normative and positive properties of the spatial distribution of economic activity and welfare in general equilibrium. We calibrate the model using domestic and international firm-to-firm trade data from Chile. Both iceberg trade costs and search and matching frictions are important for aggregate trade flows and production networks. Endogenous formation of production networks leads to larger and more dispersed effects of international and intra-national trade cost shocks.

Keywords: Production Networks, Global Value Chains, Spatial Equilibrium
JEL Classification: F10, R13

*We greatly appreciate comments from Pol Antràs, David Baqaee, Ariel Burstein, Vasco Carvalho, Thomas Chaney, Davin Chor, Jonathan Eaton, Inga Heiland, Rob Johnson, Sam Kortum, Isabelle Mejean, Andres Rodriguez-Clare, Felix Tintelnot, Jonathan Vogel, Horng Chern Wong, Daniel Xu, Yuan Zi, as well as conference and seminar participants for helpful comments. We are grateful to Sebastian Gallardo, Felipe Leal, and Adrian Kulesza, for excellent research assistance. This study was developed within the scope of the research agenda conducted by the Central Bank of Chile (CBC) in economic and financial affairs of its competence. The CBC has access to anonymized information from various public and private entities, by virtue of collaboration agreements signed with these institutions. The views expressed are those of the authors and do not necessarily represent the views of the CBC or its board members.

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1 Introduction

The modern economy is characterized by the geographic complexity of production networks. Producing clothes, automobiles, or smartphones requires a number of steps fragmented across countries, regions within a country, and firms within a region. This geographic complexity has deepened over the last few decades (Hummels, Ishii, and Yi 2001, Antras and Chor 2018). Policymakers advocate that successful integration into these global production networks, or "Global Value Chains," is key to countries' and regions' economic success (e.g., World Bank 2019). Reflecting its importance, a burgeoning academic literature has enhanced our understanding on the role of production networks both from the microeconomic and macroeconomic perspectives (see Johnson (2018) and Antràs and Chor (2021) for reviews). The microeconomic approach focuses on how firms endogenously form production networks given the economic environment surrounding them. The macroeconomic approach focuses on how countries' or regions' macroeconomic conditions are determined given the topography of production networks. Owing to the complexity of modeling firms' endogenous production network formation decisions across space and data limitations, we have limited understanding about how firms' endogenous network formation (highlighted by the microeconomic approach) affect aggregate trade flows and welfare across countries and regions (highlighted by the macroeconomic approach).[1]

We use new theory and data to study how firms endogenously form production networks in space and how these networks shape the spatial distribution of aggregate economic activity. Firms form supplier and buyer relationships across space facing iceberg trade costs and matching frictions. We characterize the normative and positive properties of how spatial frictions shape production networks, and in turn, how endogenous production network formation determines the spatial distribution of economic activity and aggregate welfare in general equilibrium. To quantify the importance of firms and geography for the endogenous formation of production networks, we combine our theory with newly constructed domestic and international firm-to-firm trade data from Chile. Using our quantitative framework, we demonstrate that accounting for the endogenous formation of production networks leads to larger (in absolute value) and more dispersed effects of both international and intra-national trade cost changes.

We start our analysis with a set of motivating facts on spatial production networks using domestic and international firm-to-firm data for the universe of firms in Chile. These stylized facts provide evidence that firms form production linkages depending on their fundamentals and geographic location. First, firms with a higher revenue tend to have more suppliers and buyers. Second, the number of supplier and buyer connections of each firm is also systematically related to the firm's geographic location. In particular, firms in a location with a higher population density tend to have more suppliers

[1] Recent examples of the macroeconomic approach include Yi (2003, 2010), Johnson and Noguera (2012), Caliendo and Parro (2015), Johnson and Moxnes (2019), Huo, Levchenko, and Pandalai-Nayar (2019) while examples of the microeconomic approach include Bernard, Moxnes, and Saito (2019), Oberfield (2018), Lim (2018), Huneeus (2018), Bernard, Dhyne, Magerman, Manova, and Moxnes (2022), Boehm and Oberfield (2020), Demir, Fieler, Xu, and Yang (2021), Dhyne, Kikkawa, Kong, Mogstad, and Tintelnot (2022), among others.

and buyers on average. Third, the number of supplier-to-buyer relationships (extensive margin) and the transaction volume per relationship (intensive margin) have a distinct spatial structure. Specifically, both margins contribute to the decay of aggregate trade flows in geographic distance between the regions, while the spatial decay for the extensive margin is faster than that for the intensive margin.

Motivated by these facts, we develop a microfounded model of spatial production networks. In this model the architecture of the aggregate production network endogenously arises from firm decisions that themselves depend on their productivity and location. Firms search for suppliers and buyers within and across locations to maximize the anticipated profit subject to location-pair-specific search costs. These supplier and buyer search turns into successful relationships with a certain probability determined by the matching technology and how many suppliers and buyers are searching in each pair of locations. Consistent with our empirical evidence, the model predicts that more productive firms form more supplier and buyer relationships and make higher revenues, yet both the number and intensity of these relationships depend on the geographic location of the firm and its connected counterparts.

We aggregate these firm decisions to obtain bilateral gravity equations for trade flows both at the extensive margin (number of supplier-to-buyer linkages) and at the intensive margin (transaction volume per linkage). There are two different types of bilateral frictions that affect trade flows: iceberg trade cost and the search and matching frictions. In particular, the extensive margin is driven by both types of spatial frictions, while the intensive margin is only driven by the iceberg costs. Given that these two types of frictions may be differentially related to geographic proximity, our model rationalizes the different spatial structures of intensive and extensive margins of trade flows as documented in the data.

These gravity equations facilitate the analysis of positive general equilibrium properties and welfare in the model. Despite the complexity of endogenous spatial linkages, we show that the equilibrium is characterized by two sets of equilibrium conditions corresponding to buyer access and supplier access analogous to the ones proposed in existing gravity trade models (Anderson and Van Wincoop 2003, Redding and Venables 2004, Donaldson and Hornbeck 2016). Indeed, our model nests a wide class of gravity trade models with roundabout intermediate goods and exogenous production networks as a special case (Eaton and Kortum 2002, Costinot and Rodríguez-Clare 2014, Caliendo and Parro 2015), a well-accepted benchmark model used to study macroeconomic implications of exogenous production networks models (Antràs and Chor 2021). This feature of our model allows for a formal theoretical and quantitative analysis of how endogenous network formation affects aggregate equilibrium. We establish sufficient conditions for equilibrium existence and uniqueness, characterize counterfactual equilibrium as a response to exogenous shocks, and provide a sufficient statistics expression for welfare changes from exogenous shocks. In particular, the sufficient statistics expression for a region's welfare changes depends not only on the aggregate import penetration of the region as in the traditional gravity trade models (Arkolakis, Costinot, and Rodríguez-Clare 2012) but also on an additional term summarizing the endogenous changes in production network architecture.

We also study the aggregate effects of exogenous shocks in the presence of endogenous production network formation. In particular, we follow Hulten (1978) and Baqaee and Farhi (2019b) to characterize the first-order and second-order approximation of the effects of a shock on aggregate welfare. In the first-order approximation, the endogenous formation of production networks amplifies the aggregate effects as long as search costs are directly affected by the shock. In the second-order approximation, in addition to the first-order effects, endogenous production network formation tends to amplify the aggregate welfare effects for a decrease of trade costs and dampen them for an increase of trade costs. Intuitively, under endogenous production network formation, firms tend to expand production networks in a region with a positive shock and cut back production networks in a region with a negative shock.

To evaluate the quantitative implications of endogenous production network formation, we calibrate our model to the observed domestic and international trade flows across municipalities and sectors within Chile and foreign countries. Using the calibrated model, we first assess how two types of spatial frictions – iceberg trade costs and search and matching frictions – contribute to the aggregate production networks and trade flows. We estimate these two types of friction for each sector and pair of locations from the bilateral trade flows in the extensive margin (number of supplier-to-buyer linkages) and the intensive margin (transaction volume per linkage). We find that both frictions contribute to the spatial architecture of the production networks and trade flows across municipalities and sectors in Chile. Therefore, solely focusing on iceberg trade costs, as typically done in gravity trade and spatial models, may yield a biased picture of regions' spatial linkages and economic activity.

Finally, we use the calibrated model to study how the endogenous formation of spatial production networks affects the outcome of inter- and intra-national trade cost shocks with two different counterfactual exercises. In our first counterfactual simulation, we evaluate the impact of the recent tariff liberalization of Chile with its two major international trading partners, the United States and China. In particular, we simulate the reversal of the observed tariff reductions that Chile experienced with these two countries over the last two decades. We find that reverting these tariff reductions decreases the aggregate welfare of Chile by 0.67 percent. Abstracting the endogenous formation of production networks, the welfare losses are instead estimated to be 0.32 percent, which is less than half of the effect in our baseline model. Furthermore, we find that accounting for endogenous production network formation leads to a larger dispersion of welfare gains across municipalities.

In our second counterfactual simulation, we study an improvement in domestic transportation infrastructure: a large-scale bridge between the mainland of Chile and Chiloé island, the biggest island in Chile. This bridge, planned to open in 2025 as the largest suspension bridge in South America, is expected to eliminate the travel time between the mainland and Chiloé island that takes 35 minutes by ferry. By calibrating the reduction in iceberg trade costs and search and matching frictions from this expected travel time reduction, we estimate that the opening of the bridge leads to a 0.25 percent increase in the aggregate welfare, and these gains are concentrated in a small subset of municipalities in and around Chiloé island. If we abstract the endogenous formation of production networks, the estimated aggregate welfare gains are 0.16 percent, which is only 62 percent of the effect in our baseline model. The difference in these predicted welfare gains is primarily driven by the municipalities in and around Chiloé island. Therefore, accounting for the endogenous formation of production networks leads to larger and more dispersed effects of both international and intra-national trade cost shocks.

This paper contributes to several strands of literature. First, as mentioned earlier, in the literature on global value chains and production networks, limited attempts have been made to analyze how firms' endogenous production network formation decisions across space (highlighted by the microeconomic approach) shape the spatial distribution of economic activity and welfare in general equilibrium (highlighted by the macroeconomic approach). One notable exception is Eaton, Kortum, and Kramarz (2022). They consider an environment where suppliers produce homogenous products and buyers select the least-cost supplier among the matched ones for each input. Together with the power law distribution of producers' productivity, they show that aggregate trade flows follow gravity equations, where the spatial structure of aggregate trade flows for each destination is entirely driven by the extensive margin (number of relationships). The key distinction between their formulation and our model is that we model endogenous search intensity for suppliers and buyers (Arkolakis 2010 and Demir, Fieler, Xu, and Yang 2021). This modeling choice allows us to flexibly capture the dispersion of suppliers across firms and geography (Fact 1 and 2) and the spatial structure of extensive and intensive margin trade flows (Fact 3), while at the same time allowing us to show a number of analytical properties of the general equilibrium. Nevertheless, our model cannot accommodate a finite number of realized relationships (or exactly zero suppliers thereof) as emphasized by Eaton, Kortum, and Kramarz (2022).[2]

Second, this paper contributes to the literature of micro-founded quantitative trade models based on gravity equations of trade flows. Various microfoundations of bilateral gravity equations have been proposed using Armington models (Anderson 1979), Ricardian models with or without input-output linkages (Eaton and Kortum 2002, Caliendo and Parro 2015), and models with firm heterogeneity and selective entry into trade (Melitz 2003, Chaney 2008, Eaton, Kortum, and Kramarz 2011). More recently, Arkolakis, Costinot, and Rodríguez-Clare (2012) show that these models with different microfoundations for gravity equations have common sufficient statistics expressions for the welfare gains from trade. We show that a special case of our model with exogenous production networks is isomorphic to these models. At the same time, allowing for endogenous production networks leads to different equilibrium properties and sufficient statistics expressions for welfare. We show theoretically and quantitatively how accounting for endogenous formation of production networks affects aggregate and heterogeneous effects from exogenous shocks.

Third, this paper contributes to the literature on the propagation of economic shocks through production networks. Theoretically, we characterize the first- and second-order effects of such shocks

[2] Miyauchi (2021) extends Eaton, Kortum, and Kramarz (2022) to consider dynamic search and matching between suppliers and buyers to study agglomeration economies and Panigraphi (2021) to consider multiple dimensions of firm heterogeneity. Antràs and De Gortari (2020) develop a model of sequential production in space, instead of roundabout production, where firms choose different locations for each stage of production. Bernard, Moxnes, and Ulltveit-Moe (2018) develop a model where production networks form subject to relationship-specific fixed costs and derive aggregate gravity equations under power law productivity distribution, but their analysis is limited to partial equilibrium.

in the presence of endogenous networks, extending the related work of Hulten (1978), Baqaee and Farhi (2019b) (second-order effects), Baqaee and Farhi (2020b,a) (imperfect competition and entry), and Atkeson and Burstein (2010) and Baqaee and Farhi (2019a) (trade under exogenous networks). Empirically, we relate to a literature that evaluates the propagation of economic shocks through production networks across firms, sectors, and regions, while we take into account the endogenous formation of production networks.[3]

The rest of the paper is organized as follows. Section 2 describes our main data set from Chile and presents salient patterns of spatial production networks. Section 3 presents our model. Section 4 presents theoretical results on our model's positive and normative predictions. Section 5 calibrates our model using Chilean data and provides estimates of the iceberg costs and search and matching frictions across space. Section 6 presents counterfactual simulation results of inter- and intra-national trade shocks in Chile. Section 7 concludes.

2 Data and Motivating Facts

In this section, we describe our main data set, the firm-to-firm transaction data from Chile. We also present a set of salient facts about spatial production networks.

2.1 Data

Our key data source is a firm-to-firm transaction-level data set that covers the universe of domestic trade between firms in Chile. All corporate entities in Chile are mandated to submit electronic receipts of all the transactions that occur across firms to the Chilean Internal Revenue Service, SII (for its acronym in Spanish). Reporting this information is mandatory for all corporate entities regardless of their revenue or the size of the transaction involved since 2018. Each receipt includes information on the supplier's and buyer's unique tax-ID, the day that the transaction occurred, and the total nominal amount of the transaction. In addition, they report the municipalities of the suppliers' and buyers' establishments where the transaction occurs. Unless otherwise specified, we treat the combination of tax-ID and the municipality as the unit of analysis for firms. For our main analysis, we use the pre-COVID-19 data from 2018 and 2019.

We merge this data set with balance sheet information (SII tax form 29) and labor information (SII tax

[3]Previous work provides empirical evidence of shock propagation across firms (e.g., Di Giovanni, Levchenko, and Méjean 2014, Carvalho, Nirei, Saito, and Tahbaz-Salehi 2021, Boehm, Flaaen, and Pandalai-Nayar 2019, Dhyne, Kikkawa, Mogstad, and Tintelnot 2021), across sectors (e.g., Acemoglu, Carvalho, Ozdaglar, and Tahbaz-Salehi 2012, Acemoglu, Akcigit, and Kerr 2016), and across regions or countries (e.g., Caliendo, Parro, Rossi-Hansberg, and Sarte 2018, Allen, Arkolakis, and Takahashi 2020, Adao, Arkolakis, and Esposito 2019). See also Lim (2018), Huneeus (2018), Taschereau-Dumouchel (2020), Miyauchi (2021) for evidence that firms adjust production networks as a response of shocks.

form 1887) at the tax-ID level.[4] We drop tax-IDs that report no value-added or employment and samples that report negative values of value-added, sales, or material inputs. After imposing these sample restrictions, the data set contains 36 million firm-to-firm-year supplier-to-buyer transactions with 28 million observations of unique firm pairs, which consists of 487 (1,763) thousand unique supplier-year (buyer-year) observations and 294 (1,158) thousand unique suppliers (buyers). We also use the balance sheet information to identify the main industry of the firm. When we calibrate our multiple sector model in Section 5, we use a 1-digit sector classification that includes: i) Agriculture and Fishing, ii) Mining, iii) Manufacturing, iv) Utilities, v) Construction, vi) Wholesale and Retail Trade, vii) Transport and Telecommunications, viii) Finance, Insurance, and Real Estate (FIRE), and iv) Other Services.

To study the interaction of domestic production networks with international trade, we also merge this data set with customs data at the tax-ID level. As is usual in other countries, this data set reports the export and import activity of each tax-ID, including information on the product being traded, the country of origin or destination, the transaction amount, and the unit value of the transaction. When we calibrate our multiple sector model in Section 5, we also use the World Input-Output Database (release 2016) from the Groningen Growth and Development Centre.

Lastly, we construct several key geographic variables. Our main spatial units are the 345 municipalities in Chile, which range over 16 states. First, we construct the bilateral travel time and travel distance using the fastest land and water transportation between all pairs of municipalities in Chile using Google Maps API. Second, we construct the population size of each municipality in Chile using census of population data from 2017. Panel A of Appendix Figure E.1 shows that there is a large dispersion of population density across municipalities. Panel B of Appendix Figure E.1 shows that different municipalities specialize in different sectors. For example, the densely populated regions around Santiago tend to specialize in manufacturing, retail and wholesale, and FIRE, while the less densely populated northern regions tend to specialize in mining. In contrast, regions south of Santiago that are also somewhat densely populated specialize relatively more in agriculture and fishing. Our multiple sector model in Section 5 captures this spatial heterogeneity in population density and sectoral specialization.

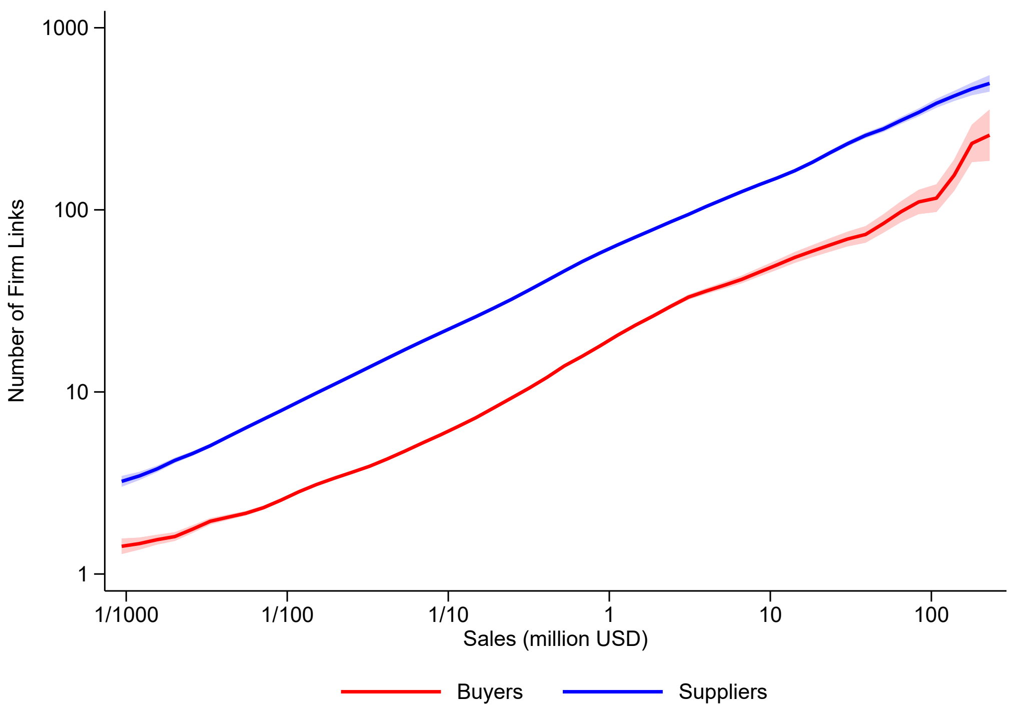
2.2 Motivating Facts on Spatial Production Networks

We begin by documenting a number of facts about spatial production networks in Chile. In particular, we argue that firm networks critically relate to firm and geographic characteristics. These facts motivate our model choices in the next section.

[4]We merge these data sets using unique tax IDs of firms that are common across sources. To secure the privacy of firms, the CBC mandates that the development, extraction and publication of the results should not allow for the identification, directly or indirectly, of natural or legal persons. All the analysis was implemented by the authors and did not involve nor compromise the Chilean IRS. Officials of the Central Bank of Chile processed the disaggregated data from the Chilean IRS. The information contained in the databases of the Chilean IRS is of a tax nature originating in self-declarations of taxpayers presented to the Service; therefore, the veracity of the data is not the responsibility of the Service.

Fact 1. Firms with a higher revenue tend to have more suppliers and buyers. Figure 1 presents the local linear regression plots of the number of domestic suppliers and buyers of a firm on the firm's total revenue. We find a strongly increasing and approximately log-linear relationship. The local linear regression line for the number of suppliers tends to be above that for the number of buyers, indicating that conditional on firm revenue, firms tend to have a larger number of suppliers than buyers.[5] These relationships between the number of supplier and buyer linkages and firm revenue are consistent with the findings in other contexts, such as in Japan (Bernard, Moxnes, and Saito 2019), Belgium (Bernard, Dhyne, Magerman, Manova, and Moxnes 2022), and the United States (Lim 2018). In Appendix Figure E.2, we document that the increasing relationships between the two variables hold within each sector, while the slopes are heterogeneous across sectors. In particular, we find that the sectoral heterogeneity of these slopes is larger for buyers than it is for suppliers. We will later interpret these differences as emanating from differences in the costs of searching for buyers and suppliers through the lens of the model.

Figure 1: Number of Domestic Suppliers and Buyers and Firm Size



Notes: This figure shows the local linear regression plots of the log number of suppliers and buyers of a tax ID on that tax ID's log total sales (excluding top 0.1 percentile). We produce this figure at the tax ID level, instead of tax ID and municipality level, because we do not observe total sales (including sales to final consumers) at the latter level. Shaded area indicates the 95% confidence intervals.

[5]Of course, the number of domestic suppliers and buyers coincide once we aggregate across all firms within Chile. The fact that the number of suppliers tends to be larger than that of buyers conditional on firm revenue implies that a small subset of extremely large firms has a disproportionally higher number of buyers than suppliers.

Fact 2. Firms in locations with higher population densities tend to have more suppliers and buyers. The second fact pertains to the relationship between supplier and buyer linkages and firms' geographic location. Panel (a) of Figure 2 shows the relationship between the average number of domestic suppliers and buyers per firm conditional on having at least one supplier and one buyer, respectively, and the population density at the municipality level. We find a large dispersion of the average numbers of supplier and buyer linkages across municipalities. Moreover, there is a strongly increasing relationship between the number of linkages and population density.

In Panel (b), we show that these relationships are statistically significant at the firm level and robust to controlling for firm size as well as other firm characteristics. Log numbers of suppliers and buyers are significantly positively related with the population density (Columns 1 and 5) and with firm revenue (Columns 2 and 6). Both of these relationships are robust to simultaneously including both variables as a regressor in the multiple regression framework (Columns 3 and 7). These patterns are further robust to controlling for sector fixed effects and other firm-level characteristics such as import and export intensity (Columns 4 and 8). These patterns are consistent with previous findings by Miyauchi (2021) using data from Japan and related to the findings of Eaton, Kortum, and Kramarz (2022) that the number of French exporters per importing firm in the destination market is systematically related to the market size of the destination country. In Appendix Figure E.3, we document that these relationships are broadly robust in each of the one-digit sectors. The only exception which exhibits negative correlation is the number of buyers in the mining sector, where the largest producers tend to locate in the northern part of Chile far from the country's economic center. The facts that both firm size and geographic location are related to firm linkage patterns is a key feature that we rationalize in our model below.

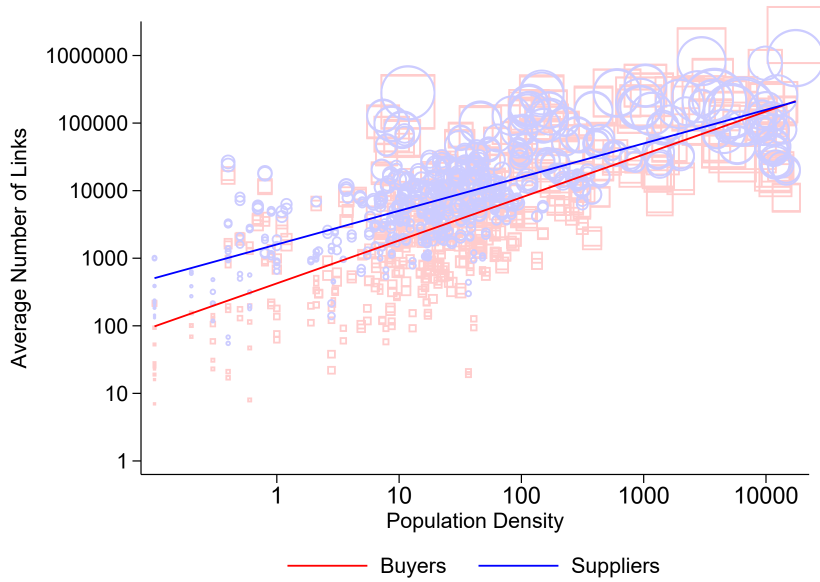
Fact 3. Both the number of supplier-to-buyer relationships (extensive margin) and the transaction volume per relationship (intensive margin) decay over geographic distance, while the spatial decay for the extensive margin is faster than that for the intensive margin. Another important aspect of geography is distance. To measure its importance for firm production networks, we estimate the following empirical gravity equation:

$$\log TradeFlow_{ijt} = \beta \log Dist_{ij} + \xi_{it} + \zeta_{jt} + \epsilon_{ijt}, \tag{1}$$

where $TradeFlow_{ijt}$ is the total firm-to-firm transaction volume from municipality i to municipality j in year t, $Dist_{ij}$ is the proxy for the geographic proximity (road or sea travel distance and travel time) between municipalities i and j, and ξ_{it} and ζ_{jt} indicate the origin-year and destination-year fixed effects, respectively. Furthermore, we also run the same set of regressions where we exactly decompose the dependent variable into the number of supplier-to-buyer relationships (extensive margin) and the transaction volume per relationship (intensive margin). Of course, the regression coefficients β from the extensive and intensive margins mechanically add up to that for the total transaction volume since the total trade flow is the product of the extensive and intensive margins.

Figure 2: Number of Domestic Suppliers and Buyers and Geography

(a) Number of Linkages and Population Density



(b) Number of Linkages by Geography and Firm Size

	Log Number of Buyers				Log Number of Suppliers			
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Log Density	0.039***		0.027***	0.018***	0.121***		0.109***	0.051***
	(0.001)		(0.001)	(0.001)	(0.002)		(0.002)	(0.002)
Log Sales		0.422***	0.421***	0.421***		0.449***	0.447***	0.412***
		(0.001)	(0.001)	(0.001)		(0.002)	(0.002)	(0.001)
R^2	0.012	0.461	0.462	0.541	0.019	0.198	0.207	0.419
Year FE	✓	✓	✓	✓	✓	✓	✓	✓
State FE	✓	✓	✓	✓	✓	✓	✓	✓
Industry FE				✓				✓
Other Controls				✓				✓
N	361142	361142	361142	361142	361886	361886	361886	361886

Notes: Panel (a) plots the average log number of domestic suppliers and buyers per firm (conditional on having at least one linkage) and log population density at the municipality level. The area of the circle represents the population size of each municipality. The straight lines represent the fit of the linear regressions between the two variables. Panel (b) presents the regression results at the tax ID level, where the dependent variable is the log number of domestic links per firm (with buyers in Columns 1-4 and suppliers in Columns 5-8). We run this regression at the tax ID level, instead of tax ID and municipality level, because we do not observe total sales (including sales to final consumers) at the latter level. The regression includes year and state fixed effects (16 states). Columns 4 and 8 add sector fixed effects at the most disaggregated level available in the SII (around 680 sectors) and additional controls including export and import intensity.

Table 1 presents the results. Column 1 shows that the coefficient on the log of distance is significant at -1.334, indicating that a 10% increase in travel distance is associated with a 13.34% decrease in aggregate trade flows. Column 2 shows that the coefficient on the log of travel time is significant at -1.571. We also find that both extensive margins (Columns 3 and 4) and intensive margins (Columns 5 and 6) are significantly negatively correlated with distance proxies, while the magnitude is substantially larger for the extensive margin (-0.929 for travel distance and -1.089 for travel time) compared to the intensive margin (-0.404 for travel distance and -0.482 for travel time).[6]

Table 1: Gravity Regression: Total Trade Flows, Intensive and Extensive Margin

	Total		Intensive		Extensive	
	(1)	(2)	(3)	(4)	(5)	(6)
Log Distance	-1.334***		-0.404***		-0.929***	
	(0.006)		(0.005)		(0.003)	
Log Time Travel		-1.571***		-0.482***		-1.089***
		(0.008)		(0.006)		(0.003)
R^2	0.639	0.639	0.312	0.313	0.818	0.816
Origin Municipality-Year FE	✓	✓	✓	✓	✓	✓
Destination Municipality- Year FE	✓	✓	✓	✓	✓	✓
Same Municipality- Year FE	✓	✓	✓	✓	✓	✓
N	134898	134898	134898	134898	134898	134898

Notes: This table presents the results of estimating the gravity regressions (1), where we regress the logarithm of the total transaction volume between a pair of municipalities on the logarithm of the proxies for distance, controlling for origin-year, destination-year, and the dummies for the same municipalities interacted with year fixed effects. The dependent variable corresponds to log total trade flow, log average trade flow (intensive margin), and the log number of links between municipalities (extensive margin). Distance (time travel) is measured with kilometers (minutes of time travel) between municipalities using the fastest land or water transportation method available within Chile. We impute distance (time travel) within municipalities as one kilometer (10 minutes).

These patterns suggest that the number of supplier-to-buyer relationships (extensive margin) – which determine the spatial architecture of production networks – and the transaction volume per relationship (intensive margin) have different spatial structures. In the next section, we microfound these patterns with the presence of two different types of spatial frictions, iceberg trade costs and search and matching frictions.

3 Model

This section develops a model of endogenous production network formation across space. We argue that our model rationalizes the patterns of spatial production networks documented in Section 2 and provides a number of theoretical properties in Section 4. For expositional purposes, we abstract from sectoral dimensions of production networks. In our quantitative analysis in Section 5, we operationalize an extension that incorporates multiple sectors.

We consider an economy that is partitioned by a finite number of locations $\mathcal{N}$.[7] In each location, there is an exogenous measure of workers, L_i, and each worker supplies one unit of labor and earns wage w_i.[8] We normalize the nominal aggregate GDP such that $\sum_i w_i L_i = 1$. There is also a continuum

[6]In Appendix Table E.1, we show that these results are robust to alternatively estimating the gravity equations with a Pseudo Poisson Maximum Likelihood (PPML) estimator to account for zero trade flows across municipalities (Silva and Tenreyro 2006, Dingel and Tiltenot 2020, Bernard and Zi 2022). In Appendix Figure E.4, we present the regression coefficients of gravity regression (1) sector-by-sector and find that the extensive margin decays with distance more strongly than the intensive margin for all sectors. In Appendix Figure E.5 we show that these relationships for the intensive and extensive margin are well approximated by a log-linear relationship as specified in equation (1).

[7]While we label $\mathcal{N}$ as "locations" for our focus on the spatial dimension of production networks, one can alternatively interpret $\mathcal{N}$ as any partition of firms within an economy.

[8]See Appendix C for the extension of our model to incorporate labor mobility.

of firms in each location i, indexed by $\omega \in \Omega_i$. We denote the entire set of firms by $\Omega \equiv \cup_i \Omega_i$. Each firm produces a distinct variety that can be used as both intermediate goods and final goods. It is endowed with a productivity z, which follows from the cumulative distribution function, $G_i(\cdot)$. We allow $G_i(\cdot)$ to flexibly depend on location i to accommodate differences in productivity across locations.

Production networks are connections between firms (for intermediate goods) and between firms and consumers (for final goods) on which transactions can occur. We denote by $\mathcal{S}(\omega) \subset \Omega$ the set of intermediate goods sellers that firm $\omega \in \Omega$ is connected with; and we denote by $\mathcal{S}(\omega^F) \subset \Omega$ the set of final goods sellers that final good consumer $\omega^F \in \Omega^F$ is connected with, where Ω^F is the set of final consumers in the economy. Therefore, the correspondence $\mathcal{S}(\cdot) : \Omega \cup \Omega^F \to \Omega$ describes the entire structure of production networks in this economy.

3.1 Production given Networks

We first describe the production structure taking the production network, $\mathcal{S}(\cdot)$, as given. Firms use labor and intermediate goods for production. These intermediate goods are imperfect substitutes with a constant elasticity of substitution, $\sigma \geq 1$. Labor and the composite of intermediate goods are combined in a Cobb-Douglas aggregator with labor share, β ($0 \leq \beta \leq 1$). Therefore, the unit cost of production for firm ω in location d, $c_d(\omega)$, is given by

$$c_d(\omega) = \frac{1}{z(\omega)} w_d^{\beta} \left(\int_{\upsilon \in \mathcal{S}(\omega)} p_{id}(\upsilon, \omega)^{1-\sigma} \, d\upsilon \right)^{\frac{1-\beta}{1-\sigma}}, \tag{2}$$

where $z(\omega)$ is firm ω's productivity; w_d is the wage at firm ω's production location d; $\mathcal{S}(\omega)$ is the set of intermediate goods producers that firm ω has access to; $p_{id}(\upsilon, \omega)$ is the intermediate goods price that supplier υ in location i charges to firm ω.

Intermediate goods are traded across regions, and a shipment from location u (supplier's production location) to location d (buyer's production location) requires an iceberg trade cost of $\tau_{ud} \geq 1$. Final goods are not traded across regions, and they are only provided by local firms.

Pricing and Market Structure Given production networks, suppliers determine the unit price for each connected buyer, and buyers decide the quantity to purchase. All firms are matched with a continuum of suppliers, and suppliers are under monopolistic competition to supply to each buyer. Thus, given the isoelastic intermediate goods demand (equation 2), suppliers charge a constant markup of their marginal cost net of the iceberg trade cost;

$$p_{id}(\upsilon, \omega) = \tilde{\sigma} c_i(\upsilon) \tau_{id}, \tag{3}$$

where $\tilde{\sigma} = \sigma / (\sigma - 1)$ is the markup ratio.

In addition to revenues from intermediate goods, firms also raise revenue by selling final goods to local consumers. The representative consumer has a constant elasticity of substitution (CES) utility function with an elasticity of substitution σ, which we assume is the same elasticity as that for the production of the intermediate goods. The monopolistic price charged to the final consumer is given by:

$$p_i^F(\upsilon) = \tilde{\sigma} c_i(\upsilon). \tag{4}$$

Notice that firms producing in the same location with the same productivity $z(\omega)$ charge the same prices and earn the same profit. Therefore, without risk of confusion, we index the cost function using z instead of ω, e.g., $c_i(z)$ instead of $c_i(\omega)$ for firm ω whose productivity is $z = z(\omega)$.

In the setup above we make a number of simplifying assumptions for expositional purposes. In particular, in our quantitative analysis in Section 5, we operationalize an extension that incorporates multiple sectors with different elasticities of substitution across sectors, different intermediate input intensities, different elasticities of substitution for final and intermediate goods sectors, and trade in final good sectors. The details of this extension are discussed in Section 5.1 and Appendix B.

3.2 Production Network Formation

Next we describe how the production network structure, $\mathcal{S}(\cdot)$, is endogenously determined through a search and matching process. Firms post advertisements to search for buyers and suppliers for each location depending on the anticipated profit and location-pair-specific search costs. These supplier and buyer searches turn into a successful relationship with a certain probability depending on the matching technology and how many suppliers and buyers are searching in each pair of locations. Our formalization of the buyer and supplier search decisions closely follows Demir, Fieler, Xu, and Yang (2021), who in turn build on the customer acquisition decisions by Arkolakis (2010). We additionally introduce spatial dimensions in search and matching.

Firm Search We first describe how firms search for final good consumers. Following Arkolakis (2010), we assume that firms have to post advertisements to reach out to consumers. Firms in region i that post $n_i^F \in \mathbb{R}_+$ measure of advertisements for final consumers pay an advertisement cost $e_i f_i^F \left(n_i^F\right)^{\gamma^F} / \gamma^F$; where e_i is the unit cost of advertisement services in region i, $\gamma^F > 1$ is a parameter that governs the curvature of the advertisement cost for buyer search, and f_i^F is the cost shifter for advertisement. There are no matching frictions in the final goods market, and hence all advertisement postings turn into a successful match with probability one. The average revenue from one unit of advertisement is given by

$$r_i^F(z) = (\tilde{\sigma} c_i(z))^{1-\sigma} D_i^F, \tag{5}$$

where D_i^F is the demand shifter net of the consumer price index, which is exogenous to the firm but endogenously determined in the general equilibrium.

Firms also search for intermediate goods buyers. Similarly to final goods consumers, to post $n_{id}^B \in$

$\mathbb{R}_+$ advertisements requires a payment of $e_i f_{id}^B \left(n_{id}^B\right)^{\gamma^B} / \gamma^B$; where f_{id}^B is the cost shifter for the location pair i and d. Unlike the search for final consumers, there is a matching friction, and only a fraction of advertisement turns into a successful match. In particular, each of the advertisements for intermediate goods buyers turns into a successful match with a random buyer in location d who posts a supplier advertisement to location i at rate m_{id}^B, where m_{id}^B is endogenously determined given matching technology as described in the next section. Given these assumptions, the average revenue for a match to buyers in d is given by:

$$r_{id}(z) = \left(\tilde{\sigma} c_i(z) \tau_{id}\right)^{1-\sigma} D_d, \tag{6}$$

where D_d is the average demand net of the price index averaged across all buyers.[9]

Finally, firms also search for suppliers. Firms in region i that post $n_{ui}^S \in \mathbb{R}_+$ measure of advertisements for suppliers in region u pay an associated advertisement cost $e_i f_{ui}^S \left(n_{ui}^S\right)^{\gamma^S} / \gamma^S$, where $\gamma^S > 1$ is a parameter that governs the curvature of the cost for supplier search, and f_{ui}^S is the cost shifter for the location pair u and i. Each of these advertisements turns into a successful match with a random supplier in location u who posts a supplier advertisement to location i at rate m_{ui}^S, which is endogenously determined given matching technology as described in the next section.[10]

Given the random matching with suppliers, and the cost function (2), the intermediate goods cost is given by:

$$c_i(z) = \frac{1}{z} w_i^\beta \left(\sum_{u \in \mathcal{N}} n_{ui}^S m_{ui}^S C_{ui}^{1-\sigma}\right)^{\frac{1-\beta}{1-\sigma}}, \tag{7}$$

where $C_{ui}^{1-\sigma} \equiv \int \left(\tilde{\sigma} c_u(z) \tau_{ui}\right)^{1-\sigma} dG_{ui}^B(z)$ is the CES aggregator of the price of a supplier producing in location u to supply to location i, and $G_{ui}^B(z)$ is the distribution of productivity weighted by the buyer search intensity.[11]

Putting these search decisions together, the net profit for a firm with productivity z in location i is:

$$\begin{aligned} \max_{n_i^F, \{n_{id}^B\}_d, \{n_{ui}^S\}_u} & \frac{1}{\sigma} n_i^F r_i^F(z) + \frac{1}{\sigma} \sum_{d \in \mathcal{N}} m_{id}^B n_{id}^B r_{id}(z) \\ & - e_i \left\{ f_i^F \frac{\left(n_i^F\right)^{\gamma^F}}{\gamma^F} + \sum_{d \in \mathcal{N}} f_{id}^B \frac{\left(n_{id}^B\right)^{\gamma^B}}{\gamma^B} + \sum_{u \in \mathcal{N}} f_{ui}^S \frac{\left(n_{ui}^S\right)^{\gamma^S}}{\gamma^S} \right\}, \end{aligned} \tag{8}$$

subject to marginal cost (7). The first two terms inside the max operator represent the final goods and

[9]Formally, denote the input demand of a buyer in location d with productivity z (net of the input price index) by $\mathcal{D}_d(z)$. Then $D_d \equiv \int \mathcal{D}_d(z)\, dG_d^S(z)$, where $G_d^S(z)$ is the CDF of the intensity of supplier search by firms in location d with productivity z, i.e., $dG_d^S(z) = dG_{id}^S(z) = n_{id}^S(z)\, dG_d(z) / \int n_{id}^S(z')\, dG_d(z')$, where $n_{id}^S(z)$ is the equilibrium intensity of supplier search by firms with productivity z in d that purchase from i. Note that, given the solution to $n_{id}^S(z)$ in Proposition 1, $dG_{id}^S(z)$ does not depend on origin location i.

[10]Whenever the equilibrium variables involve two locations with an upstream and downstream relationship (e.g., n_{id}^B, n_{ui}^S), we adopt the convention of denoting the subscripts in the order of upstream and then downstream locations.

[11]Formally, $dG_{ui}^B(z) = n_{ui}^B(z)\, dG_u(z) / \int n_{ui}^B(z')\, dG_u(z')$, where $n_{ui}^B(z)$ is equilibrium intensity of buyer search by firms with productivity z in u to sell in i. Note that, given the solution to $n_{ui}^B(z)$ in Proposition 1, $dG_{ui}^B(z)$ does not depend on origin location i, i.e., $dG_{ui}^B(z) = dG_i^B(z)$.

intermediate goods variable profit, and the last term is the advertisement cost as discussed above. We impose a parameter restriction that $1-\frac{1}{\gamma^B}-\frac{1-\beta}{\gamma^S}>0$, which guarantees that firms make positive sales and profit. In addition, we consider the case $\gamma^B=\gamma^F$ that attains a tractable characterization, since in general this problem does not have a closed-form solution:[12]

Proposition 1. *If $\gamma^B=\gamma^F$ and $1-\frac{1}{\gamma^B}-\frac{1-\beta}{\gamma^S}>0$, the solution to firm's search problem (8) takes the following form:*

$$n_i^F(z)=a_i^F z^{\frac{\delta_1}{\gamma^B}};\ n_{id}^B(z)=a_{id}^B z^{\frac{\delta_1}{\gamma^B}};\ n_{ui}^S(z)=a_{ui}^S z^{\frac{\delta_1}{\gamma^S}}, \tag{9}$$

where $\delta_1\equiv(\sigma-1)/\left(1-\frac{1}{\gamma^B}-\frac{1-\beta}{\gamma^S}\right)$, and $\{a_i^F, a_{id}^B, a_{ui}^S\}$ are functions of $\{m_{ui}^S, m_{ui}^B, w_i, C_{ui}, D_d, D_i^F, f_{ui}^S, f_{id}^B, f_i^F, e_i\}$ as explicitly given by equations (A.5), (A.6), and (A.7) in Appendix A.1. Furthermore, the marginal cost under optimal search, $c_i(z)$, is given by

$$c_i(z)=C_i^* z^{-\frac{\delta_1}{\gamma^S}\frac{1-\beta}{\sigma-1}-1},\quad (C_i^*)^{1-\sigma}\equiv w_i^{\beta(1-\sigma)}\left(\sum_{u\in\mathcal{N}} a_{ui}^S m_{ui}^S C_{ui}^{1-\sigma}\right)^{1-\beta}, \tag{10}$$

and firm revenue in location i with productivity z, $r_i(z)$, is given by

$$r_i(z)=(\tilde{\sigma})^{1-\sigma} D_i^*(C_i^*)^{1-\sigma}(z)^{\delta_1},\quad D_i^*=a_i^F D_i^F+\sum_d m_{id}^B a_{id}^B D_d(\tau_{id})^{1-\sigma} \tag{11}$$

The proof of Proposition 1 and the remaining propositions of this paper are relegated to Appendix A.

The proposition illustrates that the optimal search decisions, $\{n_i^F(z), n_{id}^B(z), n_{ui}^S(z)\}$, are multiplicatively separable between the location-pair-specific components, $\{a_i^F, a_{id}^B, a_{ui}^S\}$, and firm-specific components that depend on productivity, z. It is consistent with the observation that both location-specific components and firm-specific components are relevant for the connection with suppliers and buyers, as documented in Fact 1 and 2 of Section 2.2.[13] Furthermore, the unit cost of firms, $c_i(z)$, is also multiplicatively separable between a location-specific component, C_i^*, and a firm-specific component, $z^{-\frac{\delta_1}{\gamma^S}\frac{1-\beta}{\sigma-1}-1}$. Notice that $c_i(z)$ decays at a faster rate than z^{-1}; more productive firms search for suppliers more intensively (equation 9), which leads to disproportionately lower production cost.

Matching Technology The matching rates between suppliers and buyers, m_{ui}^S and m_{ui}^B, are determined for each pair of locations. We follow a long tradition in the literature of labor search and match-

[12]Alternatively, one can obtain a similar analytical solution by assuming that firms do not raise profit from final goods sales and consumers can purchase all locally-produced varieties at their marginal cost. The model's implications remain broadly unchanged except for the expression for the consumer price index.

[13]While we assume that firms' search cost shifters, $\{f_i^F, f_{id}^B, f_{ui}^S\}$, do not vary across firms within the same location, one can incorporate such heterogeneity by indexing i as the combination of locations and the values of these search cost shifters.

ing (Diamond 1982, Mortensen 1986, Pissarides 1985) and assume that only a fraction of supplier and buyer advertisements lead to a successful match. The measure of total matches created for each pair of locations is determined by the matching function that takes the aggregate supplier and buyer postings as arguments. The aggregate measure of supplier advertisement posting by buyers in location d for suppliers in location u is given by:

$$\overline{M}_{ud}^{S} = N_d \int n_{ud}^{S}(z) dG_d(z) = N_d a_{ud}^{S} \mathbb{M}_d \left(\frac{\delta_1}{\gamma^S}\right), \tag{12}$$

where we define $\mathbb{M}_d(\chi) \equiv \int z^{\chi} dG_d(z)$. Similarly, the aggregate measure of buyer advertisement postings by suppliers in location u for buyers in location d is given by:

$$\overline{M}_{ud}^{B} = N_u \int n_{ud}^{B}(z) dG_u(z) = N_u a_{ud}^{B} \mathbb{M}_u \left(\frac{\delta_1}{\gamma^B}\right). \tag{13}$$

The aggregate measure of successful matches between a pair of locations, M_{ud}, is determined by the following Cobb-Douglas matching function:

$$M_{ud} = \kappa_{ud} \left(\overline{M}_{ud}^{S}\right)^{\lambda^S} \left(\overline{M}_{ud}^{B}\right)^{\lambda^B}, \tag{14}$$

where λ^S, $\lambda^B \geq 0$ denote the elasticities of total matches created for the pair of regions with respect to the supplier and buyer advertisement postings, respectively, and κ_{ud} is the parameter governing the efficiency of matching technology that can flexibly depend on the location pairs, $\underline{u}$ and d. Given M_{ud}, the matching rates m_{ud}^{S} and m_{ud}^{B} are defined by:

$$m_{ud}^{S} = \frac{M_{ud}}{\overline{M}_{ud}^{S}}, \quad m_{ud}^{B} = \frac{M_{ud}}{\overline{M}_{ud}^{B}}. \tag{15}$$

Aggregate Production Networks and Trade Flows The analytical characterization of the firm search decision combined with the Cobb-Douglas matching technology yields a tractable expression for the aggregate production networks and trade flows. In particular, the measure of supplier-to-buyer relationships from supplier location u to buyer location d (extensive margin), M_{ud}, and the average transaction volume per relationship (intensive margin), $\overline{r}_{ud}$, are given by the following gravity equations:

$$M_{ud} = \varrho^{E} \chi_{ud}^{E} \zeta_{u}^{E} \xi_{d}^{E}, \quad \overline{r}_{ud} = \varrho^{I} \chi_{ud}^{I} \zeta_{u}^{I} \xi_{d}^{I}, \tag{16}$$

with bilateral resistance shifters

$$\chi_{ud}^{E} = \left[\kappa_{ud} \left(f_{ud}^{B}\right)^{-\tilde{\lambda}^B} \left(f_{ud}^{S}\right)^{-\tilde{\lambda}^S} \left(\tau_{ud}^{1-\sigma}\right)^{\tilde{\lambda}^B + \tilde{\lambda}^S}\right]^{\delta_2}, \quad \chi_{ud}^{I} = \left(\tau_{ud}\right)^{1-\sigma},$$

where we define $\tilde{\lambda}^S \equiv \lambda^S/\gamma^S$, $\tilde{\lambda}^B \equiv \lambda^B/\gamma^B$, $\delta_2 \equiv \left[1 - \tilde{\lambda}^S - \tilde{\lambda}^B\right]^{-1}$, and $\{\varrho^E, \varrho^I\}$ are constants invariant across locations. The origin and destination shifters $\{\zeta_u^E, \xi_d^E, \zeta_u^I, \xi_d^I\}$ are functions of $\{w_i, C_{ui}, D_d, D_i^F, e_i, L_i, N_i\}$ as explicitly given by equations (A.20), (A.21), (A.22), (A.23) in Appendix A.2.

Equation (16) implies that geography and spatial frictions affect differently the extensive and intensive margins of trade. The intensive margin is only affected by the iceberg trade cost, $(\tau_{ud})^{1-\sigma}$, as the search costs and matching technology do not affect trade flows once a link is formed. The extensive margin is, in addition, affected by the matching technology efficiency, κ_{ud}, and the bilateral search cost shifters, f_{ud}^B and f_{ud}^S. This is an intuitive rationalization of Fact 3, as different frictions may depend differently on geographic attributes. Notice also that there is an amplification effect compared to a model without the endogenous network margin as long as $1 > \tilde{\lambda}^B + \tilde{\lambda}^S > 0$: the overall trade elasticity (i.e., the elasticity of aggregate trade flows with respect to iceberg trade costs) is larger the greater the search and matching externalities, as governed by the term $\left(\tilde{\lambda}^B + \tilde{\lambda}^S\right)\delta_2$.

Alternative Approaches for Gravity Equations of Production Networks So far, we have developed an analytically tractable framework that can rationalize the data patterns of Facts 1-3 of Section 2.2. We now discuss alternative approaches of modeling endogenous production networks and their predictions regarding these data patterns.

One alternative approach to model endogenous production networks is introduced by Eaton, Kortum, and Kramarz (2022). They model an environment where suppliers produce homogenous products and buyers select the least-cost supplier among the matched ones for each input. Such a framework is particularly well apt to explain the granularity of supplier-buyer relationships –e.g., some firms have no suppliers from certain markets– something that our model with a continuum of suppliers is not designed to capture. However, combined with the assumption of a power law distribution of firm productivity introduced for analytical tractability, their model predicts that the intensive margin of trade flows does not vary across origin locations conditional on the destination. In other words, their framework does not explain the decay of the intensive margin with geographic distance, as we document in Fact 3.

An alternative microfoundation of the gravity equations is through relationship-specific fixed costs (Bernard, Moxnes, and Ulltveit-Moe 2018, Lim 2018, Huneeus 2018, and Dhyne, Kikkawa, Kong, Mogstad, and Tintelnot 2022). In particular, Bernard, Moxnes, and Ulltveit-Moe (2018) show that their model with a power law distribution of productivity predicts aggregate gravity equations in trade flows. This approach has the appeal that it can rationalize additional data patterns, such as the negative degree assortativity. However, to rationalize Fact 3, such theories require fixed costs to decrease in distance, contradictory to our intuition that the costs of forming relationships are likely to increase in distance.

3.3 General Equilibrium

The next step is to embed the production network formation in a general equilibrium framework and discuss how we endogenize the advertisement cost, e_i, firm entry, N_i, wages, w_i, the intermediate goods cost shifter, C_i^*, and the demand shifters, D_i^* and D_i. We present the key equations in this section and delegate the mathematical derivations to the Appendix A.3.

We first discuss the advertisement cost, e_i. Advertisement service is provided by perfectly competitive providers using labor and intermediate goods with Cobb-Douglas production technology with labor share μ. We assume that advertisement firms in location i can source intermediate goods from all firms in region i without search and matching frictions and aggregate these intermediate inputs with the elasticity of substitution σ.[14] Therefore, the cost for advertisement services is given by

$$e_i = A_i \left(w_i\right)^{\mu} \left(C_i^*\right)^{1-\mu}, \tag{17}$$

where A_i is the inverse of the productivity of the advertisement sector. Note that we allow the labor share for the advertisement sector, μ, to be potentially different from the labor share for production, β.

Second, we assume that the measure of firms in location i, N_i, is determined by a free-entry condition. There is a pool of potential entrants in each location. Each of them pays a fixed cost, F_i, in units of local labor and stochastically draws productivity z from distribution $G_i(\cdot)$. The zero-profit condition implies that the aggregate fixed cost payment, $w_i F_i N_i$, is equal to the aggregate post-entry profit, Π_i, where the latter is proportional to aggregate labor compensation, $w_i L_i$. By equating these two objects, we obtain the measure of entrants:

$$N_i = \left(\delta_1 \tilde{\sigma} \tilde{\beta}\right)^{-1} \frac{L_i}{F_i}, \qquad \tilde{\beta} \equiv \beta / \tilde{\sigma} + \sigma^{-1}, \tag{18}$$

where $\tilde{\beta}$ is the labor share in aggregate revenue (including the fixed cost payment for entry).

Third, we assume that labor market clears for each location i. As we show in Appendix A.3.2, the labor market clearing condition is:

$$w_i = \frac{\tilde{\beta}}{1-\tilde{\beta}} \frac{\sum_d X_{id}}{L_i}, \tag{19}$$

where $X_{id} \equiv M_{id} \overline{r}_{id}$ is the aggregate intermediate goods trade flows from i to d. This equation resembles a standard buyer access equation in trade and spatial models (e.g., Anderson and Van Wincoop 2003, Redding and Venables 2004, Donaldson and Hornbeck 2016): the wage in a location depends on the potential revenue of the location from selling to various other locations. However, unlike these standard models, search and matching endogenously changes the production networks, M_{id}.

Fourth, we assume that trade is balanced, i.e., total expenditure in intermediate goods sold in location i is equal to the total sales of intermediate goods sold by firms producing in location i such

[14] For simplicity, we assume that firms supply intermediate goods to advertisement sector at their marginal cost and do not raise profit.

that

$$\sum_u X_{ui} = \sum_d X_{id}. \tag{20}$$

Fifth, the demand shifters $\{D_i^*,\ D_{id},\ D_i^F\}$ can be solved using the accounting relationships (see Appendix A.3.3 for details).

Lastly, the consumer price index, P_i^F, is derived from firm's optimal consumer search decision, such that

$$\left(P_i^F\right)^{1-\sigma} = \tilde{\sigma}^{1-\sigma} N_i a_i^F \left(C_i^*\right)^{1-\sigma} \mathbb{M}_i\left(\delta_1\right). \tag{21}$$

Given these conditions, the general equilibrium is defined by the production cost shifters $\{C_i^*\}$, trade flows $\{X_{ud}, M_{ud}, \overline{r}_{ud}\}$, advertisement costs $\{e_i\}$, the measure of producers $\{N_i\}$, wages $\{w_i\}$, and consumer price indices $\{P_i^F\}$ that satisfy equations (10), (16), (17), (18), (19), (20), and (21).

4 Theoretical Analysis

In this section, we establish the theoretical properties of the endogenous spatial production network model. In Section 4.1, we establish a number of positive properties of the general equilibrium. In Section 4.2, we provide a sufficient statistics expression for evaluating welfare changes from exogenous shocks for each region. Furthermore, in Section 4.3, we characterize the first-order and second-order approximations of the aggregate effects of exogenous shocks.

4.1 Equilibrium Characterization

We start our analysis by showing that the general equilibrium is summarized by two sets of fixed-point equations expressed solely in terms of the endogenous wages $\{w_i\}$ and intermediate goods cost shifters $\{C_i^*\}$, a subset of the global structural parameters $\{\sigma,\ \beta,\ \mu,\ \tilde{\lambda}^B \equiv \lambda^B/\gamma^B,\ \tilde{\lambda}^S \equiv \lambda^S/\gamma^S\}$, and the bilateral connectivity shifters $\{K_{id}\}$.

Theorem 1. *Equilibrium wages $\{w_i\}$ and cost shifters $\{C_i^*\}$ are characterized by the following system of equations:*

$$\left(w_i\right)^{1+\tilde{\lambda}^B\delta_2\mu}\left(C_i^*\right)^{(\sigma-1)\delta_2+\tilde{\lambda}^B\delta_2(1-\mu)} = \frac{1}{L_i}\sum_d K_{id}\left(w_d\right)^{\delta_G}\left(C_d^*\right)^{\frac{(\sigma-1)\delta_2}{1-\beta}-\tilde{\lambda}^S\delta_2(1-\mu)}, \tag{22}$$

$$\left(w_i\right)^{1-\delta_G}\left(C_i^*\right)^{-\frac{(\sigma-1)\delta_2}{1-\beta}+\tilde{\lambda}^S\delta_2(1-\mu)} = \frac{1}{L_i}\sum_u K_{ui}\left(w_u\right)^{-\tilde{\lambda}^B\delta_2\mu}\left(C_u^*\right)^{-(\sigma-1)\delta_2-\tilde{\lambda}^B\delta_2(1-\mu)}, \tag{23}$$

where we define $\delta_G \equiv \left(-\tilde{\lambda}^S\mu + \frac{1-\beta\sigma}{1-\beta}\right)\delta_2$ and the bilateral connectivity shifters $\{K_{id}\}$ as a function of *exogenous variables* $\{\tau_{ud}, \kappa_{ud}, f_{ud}^S, f_{ud}^B, L_i, G_i(\cdot)\}$ *as explicitly defined in Appendix A.4.*

The equilibrium equations are reminiscent of the buyer and supplier access in canonical gravity trade models, notwithstanding the fact that our model accommodates endogenous search and matching in firm-to-firm trade. Note that this system follows the same mathematical architecture as the ones that commonly appear in trade and spatial equilibrium models with fixed networks (Costinot and Rodríguez-Clare 2014, Allen, Arkolakis, and Takahashi 2020). Consequently, we can establish a number of equilibrium properties. First, using the results from Allen, Arkolakis, and Li (2020), we can provide sufficient conditions for the existence and the uniqueness of the equilibrium, summarized by the following proposition:

Proposition 2. *If* $\frac{\beta(\sigma-1)}{1-\beta} \geq (1-\mu)\left(\tilde{\lambda}^B + \tilde{\lambda}^S\right)$ *and* $\delta_G \leq 1$, *the equilibrium exists and it is unique up-to-scale.*

The required conditions are intuitive. The first condition ensures that the scale effects of matching technology have to be small relative to the search cost elasticities, that is, $\tilde{\lambda}^B + \tilde{\lambda}^S = \lambda^B/\gamma^B + \lambda^S/\gamma^S$ is sufficiently small. To understand the second condition, note that δ_G summarizes the elasticity of aggregate trade flows with respect to wages in destination locations (see equation A.33 in Appendix A.4). Therefore, this second condition ensures that a small perturbation in wages attenuates as it propagates to upstream locations.

Second, the equilibrium conditions provide a simple way to study how exogenous shocks to the economy, such as trade costs, search cost, matching technology, population size, or productivity shocks, affect the equilibrium. In particular, following the approach of Dekle, Eaton, and Kortum (2008), we adopt the conventional hat notation to denote the proportional changes of x such that $\hat{x} = x'/x$, where x' is the value of x in the presence of the shocks, and can express the equilibrium system in Theorem 1 in terms of counterfactual changes of wages $\{\hat{w}_i\}$ and intermediate costs $\{\hat{C}_i^*\}$. We then solve it for any change in exogenous shocks as summarized by the changes in connectivity shifters $\{\hat{K}_{id}\}$ (see Appendix A.6 for details). To do so requires only knowledge of structural parameters $\{\sigma$, β, μ, $\tilde{\lambda}^B$, $\tilde{\lambda}^S\}$ and the observed aggregate trade flows, $\{X_{id}\}$. In particular, detailed firm-to-firm trade data, or even the extensive and intensive margin of trade flows $\{M_{id}, \overline{r}_{id}\}$, are not necessary for these counterfactuals.

Finally, we argue that a special case of our model where we shut down endogenous production networks, i.e., $\tilde{\lambda}^S = \tilde{\lambda}^B = 0$, is isomorphic to a wide class of existing gravity trade models. In Appendix A.7, we show that this special case is isomorphic to the multi-region Ricardian model with roundabout intermediate goods trade as in Eaton and Kortum (2002), Alvarez and Lucas (2007); to the model with firm heterogeneity and selective entry as in Melitz (2003), Chaney (2014), Eaton, Kortum, and Kramarz (2011); and to a broad class of gravity-based trade models as studied in Arkolakis, Costinot, and Rodríguez-Clare (2012), Costinot and Rodríguez-Clare (2014) when $\sigma - 1$ is set as the trade elasticity of each model. In what comes next, we theoretically and quantitatively assess the predictions of our endogenous network formation baseline with the exogenous networks benchmark for equilibrium allocations and welfare.

4.2 Sufficient Statistics for Welfare Changes

How does the welfare of each location respond to exogenous shocks? How do endogenous production networks affect these responses? To answer these questions we analyze a shock arising from any changes in iceberg trade cost, $\{\tau_{ud}\}$, search costs, $\{f_{ud}^B, f_{ud}^S\}$, matching efficiency, $\{\kappa_{ud}\}$, productivity, $\{G_d(\cdot)\}$, fixed cost for entry, $\{F_d\}$, and population size, $\{L_d\}$. We follow Arkolakis, Costinot, and Rodríguez-Clare (2012) in order to provide a minimal set of sufficient statistics that summarize the changes in welfare (real wage) of location i, $\widehat{w_i/P_i^F}$. To simplify the exposition, we assume that there are no shocks within location i, i.e., $\hat{\tau}_{ii} = \hat{f}_{ii}^B = \hat{f}_{ii}^S = \hat{\kappa}_{ii} = 1$, $\hat{L}_i = \hat{F}_i = 1$, and $\hat{G}_i(\cdot) = 1$.[15]

Proposition 3. *For any exogenous shocks satisfying $\hat{\tau}_{ii} = \hat{f}_{ii}^B = \hat{f}_{ii}^S = \hat{\kappa}_{ii} = 1$, $\hat{L}_i = \hat{F}_i = 1$, and $\hat{G}_i(\cdot) = 1$, the change in the welfare of location i is expressed as:*

$$\widehat{\frac{w_i}{P_i^F}} = \left(\hat{\Lambda}_{ii}/\hat{M}_{ii}\right)^{-\frac{1}{\sigma-1}\frac{1-\beta}{\beta}\left(1+\frac{1}{\gamma^B}\frac{1-\mu}{\sigma-1}\right)}, \tag{24}$$

where $\Lambda_{ii} \equiv X_{ii}/\left(\sum_\ell X_{\ell i}\right)$.

Regardless of the sources and magnitudes of the shocks, the changes in each location's real GDP and welfare are summarized by only two endogenous variables: aggregate share of expenditure on intermediate goods in location i that are sourced internally, $\hat{\Lambda}_{ii}$, and the change in the number of supplier-buyer linkages within location i, $\hat{M}_{ii}$. In the exogenous network case (i.e., $\tilde{\lambda}^B = \tilde{\lambda}^S = 0$), $\hat{M}_{ii} = 1$, and the proposition yields the familiar expression of Arkolakis, Costinot, and Rodríguez-Clare (2012) and Blaum, Lelarge, and Peters (2018) in imported inputs environments. The first component of the exponent in equation (24), $-\frac{1}{\sigma-1}\frac{1-\beta}{\beta}$, is the inverse of the trade elasticity of these models taking into account the input-output loop. The second component of the exponent, $1 + \frac{1}{\gamma^B}\frac{1-\mu}{\sigma-1}$, captures the gap between producer price index ($\hat{C}_i^*$) and consumer price index ($\hat{P}_i^F$). In particular, if $\mu < 1$, consumer search intensity a_i^F endogenously responds to trade shocks, creating an additional effect on the consumer price index.

In general, with endogenous production network formation (i.e., $\tilde{\lambda}^B > 0$ or $\tilde{\lambda}^S > 0$), we generically have $\hat{M}_{ii} \neq 1$, and hence our welfare expressions differ from that of Arkolakis, Costinot, and Rodríguez-Clare (2012). This difference arises because the changes in production networks within locations ($\hat{M}_{ii}$) alter the aggregate productivity of firms in location i through a love-of-variety effect. This effect resonates with the empirical findings of Goldberg, Khandelwal, Pavcnik, and Topalova (2010) and

[15] If we were to incorporate within-location shocks, it would simply yield an additional multiplicative shifter in equation (24).

Gopinath and Neiman (2014), who emphasize the productivity gains from access to additional input varieties.[16]

4.3 Aggregate Effects of Exogenous Shocks

In this section, we characterize the effects of exogenous shocks on aggregate welfare, instead of location-specific welfare as seen in the previous section. We follow Hulten (1978) and Baqaee and Farhi (2019b) to focus on the first-order and second-order approximation of the effects on aggregate welfare and discuss how those are affected by the endogenous formation of production networks across locations.

First-Order Effects We adopt the conventional notation to denote the marginal percentage change of an equilibrium variable x by $d\log x \approx x'/x - 1$. For the sake of exposition, we consider a shock in iceberg trade costs $\{d\log\tau_{ij}\}$, where it is straightforward to consider shocks on other exogenous variables, such as those on search costs or firm productivity. Following Baqaee and Farhi (2019a), we define changes in aggregate welfare, or "world welfare," as follows:

$$d\log\mathcal{W} \equiv \sum_i w_i L_i \left(d\log w_i - d\log P_i^F\right). \tag{25}$$

Proposition 4. *(i) The first-order effect of a shock in iceberg trade costs $\{d\log\tau_{ij}\}$ on world welfare is given by:*

$$d\log\mathcal{W} = \underbrace{-\sum_{i,j}\varsigma X_{ij} d\log\tau_{ij}}_{\text{technological effect}} + \underbrace{\frac{1}{\sigma-1}\sum_{i,j}\varsigma X_{ij} d\log M_{ij}}_{\text{endogenous network effect}}, \tag{26}$$

where $\varsigma \equiv \frac{1-\beta}{1-\tilde{\beta}}\frac{\tilde{\beta}}{\beta}\left(1+\frac{1}{\gamma^B}\frac{1-\mu}{\sigma-1}\right) \geq 1.$

(ii) The "endogenous network effect" is proportional to the "technological effect," i.e.,

$$\frac{1}{\sigma-1}\sum_{i,j}\varsigma X_{ij} d\log M_{ij} = \frac{\left(\tilde{\lambda}^S+\tilde{\lambda}^B\right)\frac{1-\beta}{\beta}\frac{1-\mu}{\sigma-1}}{1-\left(\tilde{\lambda}^S+\tilde{\lambda}^B\right)\frac{1-\beta}{\beta}\frac{1-\mu}{\sigma-1}}\left(-\sum_{i,j}\varsigma X_{ij} d\log\tau_{ij}\right). \tag{27}$$

Proposition 4 demonstrates that the endogenous formation of production networks has the potential to amplify the first-order effects of the shock. Part (i) of the proposition shows that the first-order effects of iceberg trade costs can be broken down into two terms. The first term, labeled "technological effect," captures the effects of the change in prices propagated through trade and production networks to downstream firms . The elasticity of the "technological effect" with respect to the iceberg cost is ςX_{ij}, which is proportional to the nominal trade flow, X_{ij}. This result resonates Hulten (1978), showing

[16] Note that equation (18) implies that the measure of firms is unchanged by the shock ($\hat{N}_i = 1$) given population size ($\hat{L}_i = 1$) as is standard in single sector trade models. Therefore, $\hat{M}_{ii}$ coincides with the change in the measure of within-location supplier linkages *per firm*.

that in an efficient economy, nominal trade flows summarize the first-order effects of the shock. At the same time, the additional constant ς arises because of the equilibrium inefficiency due to firm entry and imperfect competition.[17] The second term, labeled "endogenous network effect," captures the aforementioned love-of-variety effects of intermediate inputs induced by the changes in supplier linkages. In the special case where production networks are fixed (i.e., $\tilde{\lambda}^B = \tilde{\lambda}^S = 0$), this term is not present (i.e., $d\log M_{ij} = 0$).

Part (ii) addresses a natural next question: whether the "endogenous network effect" amplifies or dampens the "technological effect." The proposition establishes that the "endogenous network effect" always amplifies the "technological effect" when the production network is endogenous (i.e., $\tilde{\lambda}^B > 0$ or $\tilde{\lambda}^S > 0$), production requires intermediate goods inputs ($\beta < 1$), and the advertisement sector requires intermediate goods inputs ($\mu < 1$). In particular, amplification occurs only if $\mu < 1$. In this case, search costs e_i are directly affected by the trade cost shocks through the producer price index C_i^* (equation 17). The presence of this amplification effect is reminiscent of similar forces in growth models when intermediate goods are used for innovation or entry (i.e., Atkeson and Burstein 2010 and Buera, Hopenhayn, Shin, and Trachter 2021).

Second-Order Effects For ease of exposition, we focus on shocks to iceberg trade costs between a particular location pair (i and j), and assume there are no shocks to other location pairs, i.e., $d\log\tau_{ud} = 0$ if $(i,j) \neq (u,d)$. The second-order effects can be derived by totally differentiating the expression for $d\log\mathcal{W}/d\log\tau_{ij}$ in Proposition 4:

Proposition 5. *Consider a shock to iceberg trade costs from location i to j, $d\log\tau_{ij}$. The second-order effect on world welfare is given by:*

$$\frac{d^2\log\mathcal{W}}{d\log\tau_{ij}^2} = -\underbrace{\frac{\varsigma}{1-\left(\tilde{\lambda}^S+\tilde{\lambda}^B\right)\frac{1-\beta}{\beta}\frac{1-\mu}{\sigma-1}}}_{\geq 1}\frac{d\log X_{ij}}{d\log\tau_{ij}}. \tag{28}$$

The proposition demonstrates that the second-order effects of the shock are determined by two terms. The first term is the amplification effect that originates from the amplification of the first-order effects of Proposition 4. The second term captures how trade flows respond to the shock. As it is usually the case that an increase in the trade cost decreases the nominal trade flows, i.e., $d\log X_{ij}/d\log\tau_{ij} < 0$,

[17] Under CES utility and monopolistic competition, firm entry does not generate equilibrium inefficiency if there are no intermediate inputs ($\beta = 1$ and $\mu = 1$) and we have $\varsigma = 1$. This is in agreement with the results of Atkeson and Burstein (2010). Baqaee and Farhi (2020a) analyze a single-location economy with firm entry and imperfect competition with intermediate inputs given exogenous production networks. They show that "forward Domar weights," instead of standard "Domar weights" (Hulten 1978), summarize the first order effects of a shock. In our case, because of the CES demand and Cobb-Douglas production technology, the forward Domar weights are proportional to nominal trade flows X_{ij} (the standard Domar weights). See more discussion in the proof (Appendix A.9).

the proposition implies that second-order effects tend to be positive regardless of the sign of $d\log\tau_{ij}$.[18] From Proposition 4, the first-order effects are positive for a decrease in trade costs, $d\log\tau_{ij}<0$, and negative for an increase in trade costs, $d\log\tau_{ij}>0$. Therefore, the second-order effects tend to amplify the former and dampen the latter. Furthermore, since the trade elasticity is larger if we allow for endogenous responses of production networks (equation 16), the magnitudes of the second-order effects tend to be larger in the presence of endogenous formation of production networks.

5 Quantitative Analysis

The final step of our analysis is to quantify the importance of the endogenous formation of spatial production networks by taking our model to firm-to-firm trade data from Chile and international input-output trade data. In order to provide a more accurate description of the sectoral heterogeneity of spatial production networks, we first extend our model to multiple sectors in Section 5.1. We then calibrate our multiple sector model in Section 5.2. Lastly, we estimate trade costs and search and matching frictions across space in Section 5.3.

5.1 Multiple Sector Model

We consider multiple sectors connected through input-output linkages following the specification of Caliendo and Parro (2015). Below we present some key equations summarizing this extension, and delegate the complete mathematical exposition to Appendix B.

Firms belong to distinct sectors denoted by $k,h\in K$. The unit cost of production for each firm ω in sector k and location i is given by

$$c_{i,k}(\omega)=\frac{1}{z_{i,k}(\omega)}w_i^{\beta_{k,L}}\prod_{h\in K}\left(\int_{\upsilon\in\mathcal{S}_{hk}(\omega)}p(\upsilon,\omega)^{1-\sigma_h}\,d\upsilon\right)^{\frac{\beta_{hk}}{1-\sigma_h}}, \tag{29}$$

where $z_{i,k}(\omega)$ is firm ω's productivity; w_i is the wage at firm ω's production location; $\mathcal{S}_{hk}(\omega)$ is the set of intermediate goods producers in sector h that firm ω in sector k has access to; $p(\upsilon,\omega)$ is the intermediate goods price that supplier υ charges to firm ω (net of iceberg trade cost); $\beta_{k,L}$ is the share of labor input for sector k; β_{hk} is the input share of sector h inputs for sector k production; and σ_h is the elasticity of substitution within input sector h ($\sigma_h>1$). We assume that the production technology is constant returns to scale such that $\beta_{s,L}+\sum_{k\in K}\beta_{ks}=1$.

[18] In principle, it is possible that trade flows respond positively to an increase of iceberg trade cost τ_{ij} because of large terms-of-trade effects. In our counterfactuals in Section 6, we find that the overwhelming majority of cases respond negatively, as discussed in the main text.

Final consumers have Cobb-Douglas preference over sectors k such that their utility is given by:

$$U_i = \prod_{k \in K} \left(\int_{\upsilon \in \mathcal{S}_{i,k}^F} q_k(\upsilon)^{1-\sigma_k} \, d\upsilon \right)^{\frac{\alpha_k}{1-\sigma_k}}, \tag{30}$$

where $\mathcal{S}_{i,k}^F$ is the set of firms in location i and sector k that consumers have access to (determined through firms' consumer search), α_k is the final consumption share for sector k, and $q_k(\upsilon)$ is the consumption of the variety that is produced by firm υ.

Firms' search problem succeeds the basic structure of the single sector model in our main paper, except that firms determine their optimal search intensity for each supplier and buyer *sector* on top of supplier and buyer *location*. Firms' search decisions for buyers, $\{n_{ui,hk}^S\}_{u \in \mathcal{N}, h \in K}$, suppliers, $\{n_{id,kl}^B\}_{d \in \mathcal{N}, l \in K}$, and local final consumers, $n_{i,k}^F$, is given by

$$\begin{aligned} \max_{\{n_{ui,hk}^S\}_{u,h}, \{n_{id,kl}^B\}_{d,l}, n_{i,k}^F} & \frac{1}{\sigma_k} n_{i,k}^F r_{i,k}^F(c) + \frac{1}{\sigma_k} \sum_{l \in K} \sum_{d \in \mathcal{N}} m_{id,kl}^B n_{id,kl}^B r_{d,kl}(c \tau_{id,kl}) \\ & - e_{i,k} \left\{ f_{i,k}^F \frac{\left(n_{i,k}^F\right)^{\gamma_k^F}}{\gamma_k^F} + \sum_{l \in K} \sum_{d \in \mathcal{N}} f_{id,kl}^B \frac{\left(n_{id,kl}^B\right)^{\gamma_k^B}}{\gamma_k^B} + \sum_{h \in K} \sum_{u \in \mathcal{N}} f_{ui,hk}^S \frac{\left(n_{ui,hk}^S\right)^{\gamma_k^S}}{\gamma_k^S} \right\}, \end{aligned} \tag{31}$$

subject to cost function (29). This problem, under the same restriction of the single-sector model in Proposition 1, $\gamma_k^F = \gamma_k^B$ and $1 - \frac{1}{\gamma_k^B} - \frac{1}{\gamma_k^S} \sum_h \beta_{hk} \frac{1-\sigma_k}{1-\sigma_h} > 0$, yields a closed-form solution.

Given the search decisions, the matching market clears every pair of locations *and sectors*. In particular, the total number of matches for each location and sector pair is then given by:

$$M_{ud,kl} = \kappa_{ud,kl} \left(\overline{M}_{ud,kl}^S \right)^{\lambda_{kl}^S} \left(\overline{M}_{ud,kl}^B \right)^{\lambda_{kl}^B}, \tag{32}$$

where $\overline{M}_{ud,kl}^S$ and $\overline{M}_{ud,kl}^B$ are aggregate supplier and buyer search postings, $\kappa_{ud,kl}$ is the matching technology efficiency, and λ_{kl}^S and λ_{kl}^B are the elasticities of matching technology. We can then derive the bilateral gravity equations for each location and sector pair:

$$M_{ud,kl} = \varrho_{kl}^E \chi_{ud,kl}^E \zeta_{u,kl}^E \xi_{d,kl}^E, \quad \overline{r}_{ud,kl} = \varrho_{kl}^I \chi_{ud,kl}^I \zeta_{u,kl}^I \xi_{d,kl}^I, \tag{33}$$

with bilateral resistance shifters given by

$$\chi_{ud,kl}^E = \left[\kappa_{ud,kl} \left(f_{ud,kl}^B \right)^{-\tilde{\lambda}_{kl}^B} \left(f_{ud,kl}^S \right)^{-\tilde{\lambda}_{kl}^S} \left(\tau_{ud,kl}^{1-\sigma} \right)^{\tilde{\lambda}_{kl}^B + \tilde{\lambda}_{kl}^S} \right]^{\delta_{2,kl}}, \quad \chi_{ud,kl}^I = \left(\tau_{ud,kl} \right)^{1-\sigma_k},$$

where we define $\tilde{\lambda}_{kl}^S \equiv \lambda_{kl}^S / \gamma_l^S$, $\tilde{\lambda}_{kl}^B \equiv \lambda_{kl}^B / \gamma_k^B$, and $\delta_{2,kl} \equiv \left[1 - \tilde{\lambda}_{kl}^S - \tilde{\lambda}_{kl}^B \right]^{-1}$; ϱ_{kl}^E and ϱ_{kl}^I are constants that are invariant across locations; and the origin and destination shifters $\{\zeta_{u,kl}^E, \xi_{d,kl}^E, \zeta_{u,kl}^I, \xi_{d,kl}^I\}$ are functions of equilibrium variables summarized by equations (B.21) and (B.22).

Note that the special case of this model with exogenous production networks such that $\lambda_{kl}^S = \lambda_{kl}^B = 0$ for all $k, l \in K$ corresponds to the model of Caliendo and Parro (2015). We can then characterize the counterfactual equilibrium given two sets of information: (i) the regional input-output tables, including the total trade flows across locations and sectors $\{X_{ud,hk}\}$, labor compensation $\{X_{i,k}^L\}$, and final consumption $\{Y_{i,k}^F\}$, and (ii) a subset of structural parameters $\{\alpha_k, \beta_{k,L}, \beta_{hk}, \mu, \gamma_k^B, \gamma_k^S, \lambda_{kl}^B, \lambda_{kl}^S, \sigma_k\}$. Below we discuss how we calibrate these variables and structural parameters in turn.

5.2 Calibration

To map the model to the data we assume that the set of locations consists of a combination of 345 municipalities within Chile and three international locations: United States, China, and the Rest of the World. We focus on the United States and China since they are the two major trading partners of Chile.

We construct regional input-output tables $\{X_{i,k}^L, X_{ud,hk}, Y_{i,k}^F\}$ using various data sources described in Section 2.1. For bilateral trade flows across locations and sectors, $X_{ud,kl}$, we aggregate Chilean firm-to-firm data across municipalities and sectors (when both u and d are municipalities in Chile), customs import and export data (when either of u or d is the international country), and the World Input-Output Tables (when both u and d are international countries). To construct the labor expenditure by location i and sector k, $X_{i,k}^L$, we use the Chilean balance sheet data (when i is the municipality in Chile) and the World Input-Output Tables (when i is an international country). To compute the final sales for location i and sector k, $Y_{i,k}^F$, we aggregate firms' final sales from balance sheet data and firm-to-firm trade data (when i is a municipality in Chile) and use the World Input-Output Tables (when i is an international country).

We next discuss the calibration of structural parameters $\{\alpha_k, \beta_{k,L}, \beta_{hk}, \mu, \gamma_k^B, \gamma_k^S, \lambda_{kl}^B, \lambda_{kl}^S, \sigma_k\}$ summarized in Table 2. For final consumption shares $\{\alpha_k\}$ and sectoral input shares $\{\beta_{k,L}, \beta_{hk}\}$ we use the constructed regional input-output tables. In particular, α_k is the final consumption share of sector k in Chile across all sectors, $\beta_{k,L}$ is the share of expenditure for labor out of all inputs in sector k, and β_{hk} is the corresponding share of expenditure for intermediate inputs that are produced by sector h and sold to sector k. We finally calibrate the labor share of advertisement services, μ, using the labor share of the advertisement sector.

For the search cost elasticities for suppliers and buyers, γ_k^B and γ_k^S, we use our domestic firm-to-firm trade data. From a multi-sector version of Lemma 1 (see Appendix B.1), the measure of suppliers per firm (aggregated across all locations within Chile) is proportional to firm revenue raised to a coefficient $1/\gamma_k^S$ and an aggregate term that depends on region and sector. We thus calibrate $1/\gamma_k^S$ from the regression coefficient of the log number of suppliers on log firm revenue controlling for location-and-sector fixed effects. Similarly, we calibrate $1/\gamma_k^B$ from the regression of the log number of buyers on the log firm revenue controlling for location-and-sector fixed effects.

Matching function elasticities with respect to suppliers and buyers are set to $\lambda_{kl}^S = \lambda_{kl}^B = 0.5$.

Table 2: Calibration: Multi-Sector Model

Parameters	Value	Description	Source
α_k	Figure F.1	Final consumption share	Observed Final Consumption Share in Each Sector
$\{\beta_{k,L}, \beta_{kl}\}$	Figure F.2	Sectoral input share in production	Observed Input Share in Each Sector
μ	0.58	Labor share in advertisement service sector	Observed Labor Share in Advertisement Sector
λ^S	0.5	Matching function elasticity w.r.t. suppliers	Krolikowski and McCallum (2021)
λ^B	0.5	Matching function elasticity w.r.t. buyers	Krolikowski and McCallum (2021)
γ_k^S	Figure F.3	Search cost curvature w.r.t. suppliers	Elasticity of Sales to Number of Suppliers (Cond. on Location FE)
γ_k^B	Figure F.3	Search cost curvature w.r.t. buyers	Elasticity of Sales to Number of Buyers (Cond. on Location FE)
σ_k	Figure F.4	Elasticity of substitution	Fontagne et al (2022) and Gervais and Jensen (2019)

Notes: This table summarizes our calibrated parameters and the sources for our multiple sector model described in Section B.

Although there is limited previous work estimating matching technology in the context of inter- and intra-national trade, these values are in line with some existing work (Krolikowski and McCallum 2021). They are also similar to the elasticities of matching functions in the context of matching between workers and jobs in the labor market (Petrongolo and Pissarides 2001). At the same time, some existing work estimates larger values for the matching function elasticities in production network formation, suggesting the possibility of increasing returns to scale ($\lambda_{kl}^S + \lambda_{kl}^B > 1$; Eaton, Kortum, and Kramarz 2022, Miyauchi 2021). We provide sensitivity analysis to these alternative parameter values when we present the counterfactual simulation results.

Finally, we calibrate σ_k for each sector using existing estimates of trade elasticity. For agriculture and fishing, mining and quarrying, and manufacturing, we use the product-level import elasticity estimated using variation in tariff changes from Fontagné, Guimbard, and Orefice (2022). For services, we use the estimates of Gervais and Jensen (2019). In our model, the trade elasticity (elasticity of total trade flows to the iceberg trade cost) is given by $\sigma_k \left(1 - \tilde{\lambda}_{kl}^B - \tilde{\lambda}_{kl}^S\right)^{-1}$, where we define $\tilde{\lambda}_{kl}^B = \lambda_{kl}^B / \gamma_k^B$ and $\tilde{\lambda}_{kl}^S = \lambda_{kl}^S / \gamma_l^S$ (equation 16 for single sector and equation B.21 and B.22 for multiple sectors). We use these equations and our baseline calibration of $\lambda_{kl}^B = \lambda_{kl}^S = 0.5$ to choose the value of σ_k that replicates the estimates of trade elasticity in the two papers cited above. In other words, we take into account the role of the extensive margin of trade when calibrating σ_k.[19]

There is substantial sectoral heterogeneity in most dimensions of the calibration that follows an intuitive pattern. Labor share $\beta_{k,L}$ of services is around 70%, whereas it is 10% for manufacturing. Consumption share α_k is around 20% for the retail and wholesale sector, whereas it is around 10% for the mining sector. There is significant sectoral heterogeneity of the curvature of the supplier search γ_k^S, while that of the buyer search γ_k^B is more homogenous, consistent with the results documented in Fact 1. Finally, the elasticity of substitution σ_k is lowest in agriculture and highest in services.

[19]Notice that the trade elasticity in our model, $\sigma_k \left(1 - \tilde{\lambda}_{kl}^B - \tilde{\lambda}_{kl}^S\right)^{-1}$, depends both on the supplier sector k and buyer sector l. Given that Fontagné, Guimbard, and Orefice (2022) and Gervais and Jensen (2019) estimate the trade elasticity by seller sector but not separately by buyer sector, we replace $\tilde{\lambda}_{kl}^B$ and $\tilde{\lambda}_{kl}^S$ with their simple averages across buyer sector l to obtain the value of σ_k.

5.3 Estimating Trade Costs and Search-and-Matching Frictions

We finally estimate iceberg trade costs and search and matching frictions across space. We show that both of these frictions are strongly related to geographic proximity of the locations. We also use the elasticities of these frictions to travel time for our counterfactual analysis in Section 6.2.

To estimate iceberg trade costs and search and matching frictions, we use gravity equations (33). The bilateral resistance term of aggregate trade flows, $\chi_{ud,kl}$, is now expressed in two alternative terms, $\chi_{ud,kl} = \chi_{ud,kl}^{\text{matching}} \chi_{ud,kl}^{\text{iceberg}}$, where $\chi_{ud,kl}^{\text{matching}}$ summarizes the influence of search and matching frictions and $\chi_{ud,kl}^{\text{iceberg}}$ summarizes that of iceberg trade costs. They are defined by:

$$\chi_{ud,kl}^{\text{matching}} \equiv \left[\kappa_{ud,kl} \left(f_{ud,kl}^{B}\right)^{-\tilde{\lambda}_{kl}^{B}} \left(f_{ud,kl}^{S}\right)^{-\tilde{\lambda}_{kl}^{S}}\right]^{\delta_{2,kl}}, \quad \chi_{ud,kl}^{\text{iceberg}} \equiv \left(\tau_{ud,kl}^{1-\sigma_k}\right)^{\left(\tilde{\lambda}_{kl}^{B}+\tilde{\lambda}_{kl}^{S}\right)\delta_{2,kl}+1}. \tag{34}$$

Next, we show how $\chi_{ud,kl}^{\text{matching}}$ and $\chi_{ud,kl}^{\text{iceberg}}$ can be estimated for each pair of location-sectors –up to normalization– using the intensive and extensive margin of trade flows. As is standard in gravity-based trade models, we cannot separately identify the bilateral resistance terms, $\chi_{ud,kl}$, from origin and destination shifters. Therefore, we follow Head and Ries (2001) to construct the proxies for bilateral spatial frictions relative to those within location, $\tilde{\chi}_{ud,kl}^{\text{matching}}$ and $\tilde{\chi}_{ud,kl}^{\text{iceberg}}$. Using gravity equations (33), we obtain:

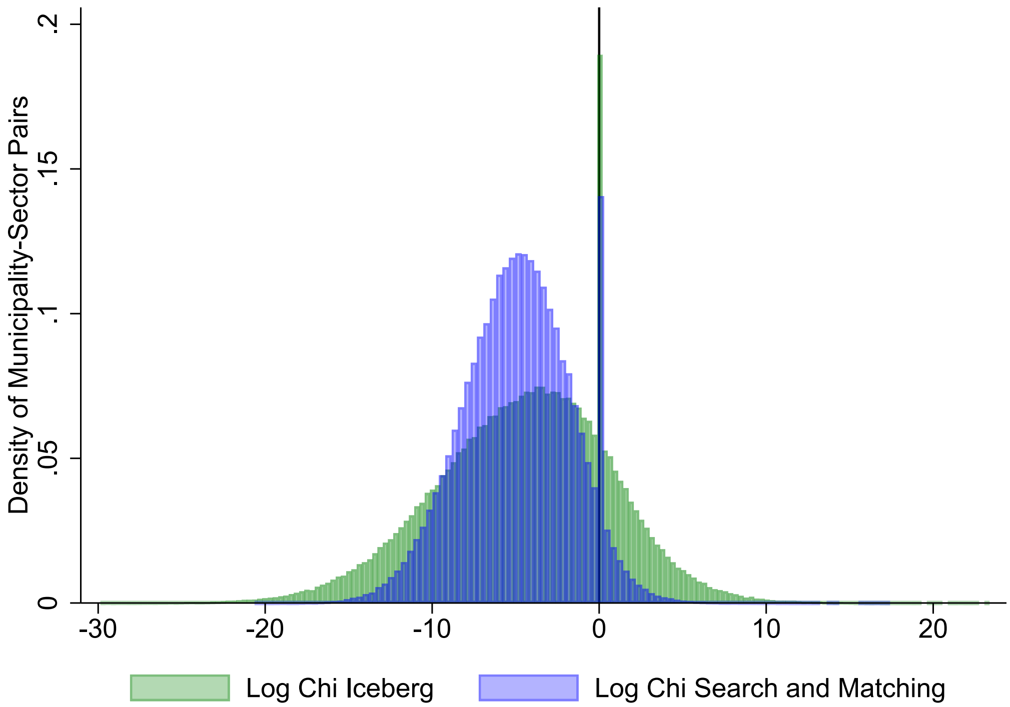
$$\begin{aligned}
\tilde{\chi}_{ud,kl}^{\text{iceberg}} &\equiv \frac{\chi_{ud,kl}^{\text{iceberg}}}{\chi_{uu,kl}^{\text{iceberg}}} \frac{\chi_{du,kl}^{\text{iceberg}}}{\chi_{dd,kl}^{\text{iceberg}}} = \left(\frac{\overline{r}_{ud,kl}}{\overline{r}_{uu,kl}} \frac{\overline{r}_{du,kl}}{\overline{r}_{dd,kl}}\right)^{\left(\tilde{\lambda}_{kl}^{B}+\tilde{\lambda}_{kl}^{S}\right)\delta_{2,kl}+1} \\
\tilde{\chi}_{ud,kl}^{\text{matching}} &\equiv \frac{\chi_{ud,kl}^{\text{matching}}}{\chi_{uu,kl}^{\text{matching}}} \frac{\chi_{du,kl}^{\text{matching}}}{\chi_{dd,kl}^{\text{matching}}} = \left(\frac{M_{ud,kl}}{M_{uu,kl}} \frac{M_{du,kl}}{M_{dd,kl}}\right) \left(\frac{\overline{r}_{ud,kl}}{\overline{r}_{uu,kl}} \frac{\overline{r}_{du,kl}}{\overline{r}_{dd,kl}}\right)^{-\left(\tilde{\lambda}_{kl}^{B}+\tilde{\lambda}_{kl}^{S}\right)\delta_{2,kl}}.
\end{aligned} \tag{35}$$

These expressions are intuitive. Spatial frictions due to iceberg trade costs, $\tilde{\chi}_{ud,kl}^{\text{iceberg}}$, are directly inferred from the intensive margin of trade flows, $\overline{r}_{ud,kl}$, given their one-to-one relationship. Search and matching frictions, $\tilde{\chi}_{ud,kl}^{\text{matching}}$, are inferred from both extensive and intensive margins, since extensive margins of trade flows, $M_{ud,kl}$, are affected both by the search and matching frictions and iceberg trade costs.

Figure 3 presents the distributions of the estimated $\log(\tilde{\chi}_{ud,kl}^{\text{iceberg}})$ and $\log(\tilde{\chi}_{ud,kl}^{\text{matching}})$ across pairs of municipalities and sectors in Chile. We find that both $\log(\tilde{\chi}_{ud,kl}^{\text{matching}})$ and $\log(\tilde{\chi}_{ud,kl}^{\text{iceberg}})$ are on average negative with a similar mean, while $\log(\tilde{\chi}_{ud,kl}^{\text{matching}})$ is less dispersed than $\log(\tilde{\chi}_{ud,kl}^{\text{iceberg}})$. This pattern indicates that both matching frictions and iceberg trade costs contribute to the spatial frictions in trade flows across municipalities.

To further understand the spatial patterns of matching frictions and iceberg trade costs, in Panel (a) of Table 3, we present the regression coefficients of $\log(\tilde{\chi}_{ud,kl}^{\text{matching}})$ and $\log(\tilde{\chi}_{ud,kl}^{\text{iceberg}})$ on proxies for geographic proximity between u and d. We find that both $\log(\tilde{\chi}_{ud,kl}^{\text{iceberg}})$ (in Columns 1 and 2) and $\log(\tilde{\chi}_{ud,kl}^{\text{matching}})$ (in Columns 3 and 4) are negatively related to travel distance (Columns 1 and 3) and travel time (Columns 2 and 4), indicating that both of these frictions tend to increase in geographic

Figure 3: Distribution of $\log(\tilde{\chi}^{\text{iceberg}}_{ud,kl})$ and $\log(\tilde{\chi}^{\text{matching}}_{ud,kl})$



Notes: Probability distribution functions of the log of the Head and Ries (2001)-proxy for the iceberg trade costs, $\log(\tilde{\chi}^{\text{iceberg}}_{ud,kl})$, and search and matching frictions, $\log(\tilde{\chi}^{\text{matching}}_{ud,kl})$, estimated using Equation (35).

distance. The regression coefficients for $\log(\tilde{\chi}^{\text{matching}}_{ud,kl})$ are similar to those for $\log(\tilde{\chi}^{\text{iceberg}}_{ud,kl})$, indicating that the spatial patterns of matching frictions are as important as those of the iceberg trade costs. In Panel (b), we report the same regression coefficients on travel distance for each supplier sector k. [20] While the degree may vary, we find a robust pattern that both search and matching frictions and the iceberg trade costs are important for bilateral trade frictions. These results suggest that solely focusing on the iceberg trade costs, as typically done in the gravity trade and spatial models, may yield a biased picture about the regions' spatial linkages and economic activity.[21]

[20] In Appendix Figure F.5 we document the regression coefficients on travel time for each supplier sector k. In Appendix Figure F.6 we show that the relationships between iceberg and search and matching frictions and log travel distance are relatively well approximated by a log-linear relationship as the one from Table 3.

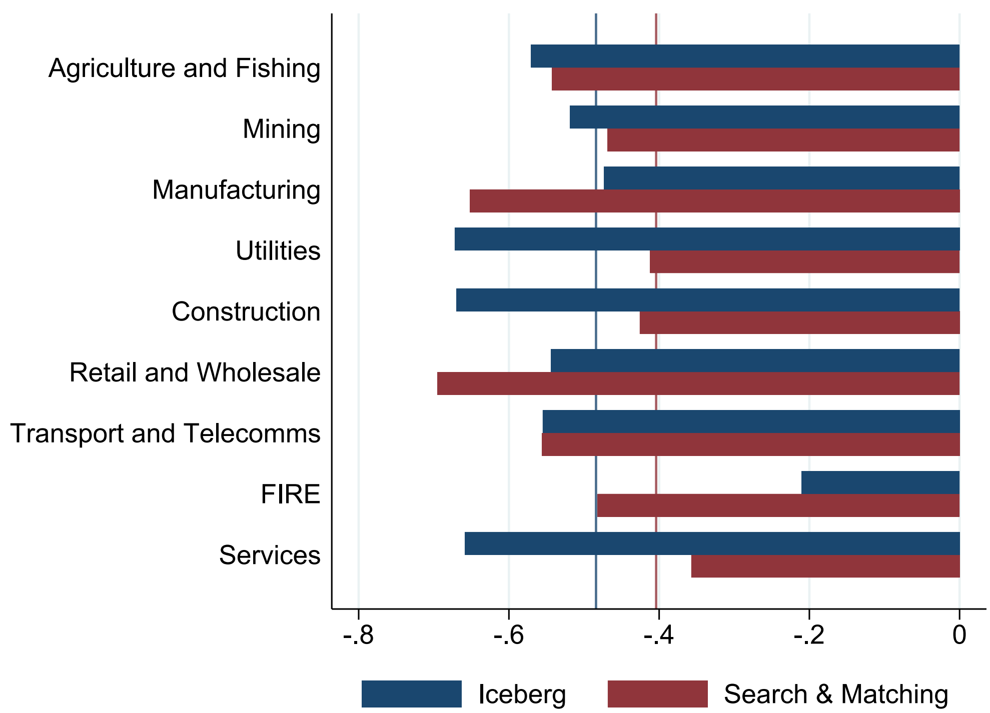
[21] This finding resonates with recent literature emphasizing the importance of search and matching frictions in inter- and intra-national trade relationships, e.g., Chaney (2014), Allen (2014), Brancaccio, Kalouptsidi, and Papageorgiou (2020), Dasgupta and Mondria (2018), Eaton, Jinkins, Tybout, and Xu (2016), Lenoir, Martin, and Mejean (2020), Krolikowski and McCallum (2021), Startz (2021), Miyauchi (2021). In particular, Eaton, Kortum, and Kramarz (2022) provide a similar decomposition of trade frictions into iceberg cost and search frictions using a different theoretical framework, and they reach a similar conclusion about the relative importance of search and matching frictions.

Table 3: Relationships between $\log(\tilde{\chi}^{\text{iceberg}}_{ud,kl})$ and $\log(\tilde{\chi}^{\text{matching}}_{ud,kl})$ and Geographic Proximity

(a) Aggregate across Sectors

	Iceberg		Search and Matching	
	(1)	(2)	(3)	(4)
Log Distance	-0.484***		-0.404***	
	(0.002)		(0.001)	
Log Time Travel		-0.584***		-0.480***
		(0.003)		(0.001)
R^2	0.213	0.213	0.572	0.570
Origin Municipality-Sector-Year FE	✓	✓	✓	✓
Destination Municipality-Sector-Year FE	✓	✓	✓	✓
Same Municipality-Year FE	✓	✓	✓	✓
N	767971	767971	767971	767971

(b) Coefficient on Travel Distance by Supplier Sector



Notes: Panel (a) presents the regression results of iceberg trade cost, $\log\left(\tilde{\chi}^{\text{iceberg}}_{ud,kl}\right)$, and search-matching frictions, $\log\left(\tilde{\chi}^{\text{matching}}_{ud,kl}\right)$, on log travel time and log travel distance between municipalities. Panel (b) presents the regression coefficients on log distance (Columns 1 and 3) from Table 3 for each supplier sector k.

6 Counterfactuals: Inter- and Intra-national Trade Shocks

We proceed to quantify the importance of the endogenous formation of spatial production networks through two sets of counterfactual simulations: international trade tariff changes with major trading partners and a planned domestic transportation infrastructure improvement within Chile.

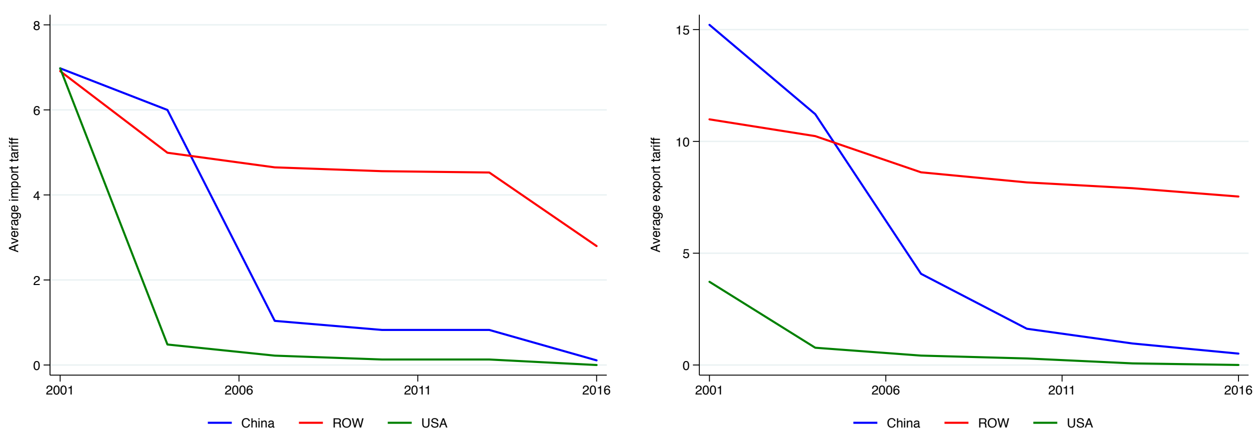
6.1 International Trade Tariff Changes

Chile has signed significant trade treaties with multiple countries in the last two decades (Linarello 2018 and Fontagné, Guimbard, and Orefice 2022). We use our calibrated model to study the effects of the reduction in tariffs of Chile with the United States (U.S.) and China, the two largest trading partners for Chile both in terms of imports and exports.[22]

The U.S. and Chile implemented a Preferential Trade Agreement (PTA) in 2004. This PTA reduced Chile's (average) preferential import tariff toward US products by 93% (from an average applied tariff of 6.9 percentage points to 0.5 percentage points (Fontagné, Guimbard, and Orefice 2022)), with a peak of a 100% tariff cut (i.e. the complete removal of import tariffs) for many organic and inorganic chemical products and many plastic and rubber products (Fontagné, Guimbard, and Orefice 2022). It had similar effects on export tariffs of Chile to the U.S. In addition, Chile has implemented a trade liberalization agenda that in particular reduced tariffs from and to China. Average import tariffs with China were reduced from 6.9 percentage points in 2001 to 0.1 percentage points in 2016. Figure 4 shows a significant tariff decline from and to China and the US, while there is only a moderate decline to the Rest of the World (ROW). These tariff cuts were particularly relevant for intermediate imports. Major imported products from China include engines, and those from the US include gas and also chemical products. Table 4 summarizes the tariff changes between 2001 and 2016 from and to the US and China for three main sectors where the majority of trade liberalization occurred: Agricultural and Fishing, Mining, and Manufacturing. While import tariff reductions are relatively homogenous across sectors, export tariff reductions are heterogeneous across sectors.

Figure 4: Import and Export Tariffs of Chile during 2001-2016

(a) Average Import Tariffs (b) Average Export Tariffs



Notes: These figures present the average import and export tariffs of Chile (averaged across sectors) with China, the US, and the rest of the world (ROW), computed using the dataset built by Fontagné, Guimbard, and Orefice (2022). Panel (A) presents average import tariffs and Panel (B) presents average export tariffs (imposed by the counterpart countries).

[22]In 2018, imports from China and the US constituted about 24% and 19% of overall imports, which corresponds to 6% and 4% of Chile's GDP, respectively. Exports to China and the US were about 33% and 14% of overall exports, which amounts to 8% and 4% of Chile's GDP, respectively.

We use our calibrated model to simulate how these tariff changes from and to the US and China affect the international and domestic production networks and welfare across Chilean municipalities. In particular, we implement a backward-looking counterfactual evaluating the return to the tariffs Chile had with the US and China in 2001. To study the role of endogenous production networks, we compare our results with a special case of our model where we shut down endogenous formation of production networks ($\tilde{\lambda}^S = \tilde{\lambda}^B = 0$).

Table 4: Import and Export Tariff Change of Chile with Main Trade Partners: Across Sectors (%)

	Imports		Exports	
	China	US	China	US
a) Agriculture and Fishing	-6.54	-6.54	-12.84	-1.86
b) Mining	-6.45	-6.45	-2.63	-0.20
c) Manufacturing	-6.45	-6.45	-13.06	-3.85

Notes: This table presents the average percentage point changes in tariffs from and to China and the US, between 2001 and 2016, across different sectors and for import and export tariffs, computed using the dataset built by Fontagné, Guimbard, and Orefice (2022)

Aggregate Effects Table 5 presents how these tariff changes from and to the US and China affect the aggregate welfare, trade patterns, and spatial production networks in overall Chile. In Row (a), we report the simulation results from our baseline specification. In Rows (b)-(d), we report the results from the alternative specification with exogenous production networks ($\tilde{\lambda}^S = \tilde{\lambda}^B = 0$). To ensure that the difference from the endogenous network specification is not driven by the implied trade elasticities, we report the results under three different values for the elasticities of substitution: our Baseline σ_k (in Row c), a Low Sigma case, $\sigma_k - 1$ (Row b), and a High Sigma case, $\sigma_k + 1$ (Row d). We apply this uniform shift of the sectoral elasticities to retain the heterogeneity of elasticities across sectors.

We find that the tariff increase reduces the aggregate welfare of the Chilean economy, measured as the income-weighted changes in real wages, by 0.67 percent in our baseline specification (Column 1, Row a). In contrast, these welfare losses are substantially smaller (in absolute value) when we shut down endogenous production networks (Column 1, Rows b, c, d), ranging from 0.32 to 0.40 percent. Thus, these alternative models predict only 47 to 60 percent of welfare losses compared to our baseline model (Column 2).

In the remaining columns of Table 5, we report the effects on trade flows and production networks. In our baseline model (Row a), we find that intermediate imports from China and the US decrease by 5.95% (Column 3). This reduction in imports is accompanied by an increase in expenditure sourced within Chile by 0.23% (Column 4), indicating the presence of import substitution. A significant part of these responses is driven by the reorganization of production networks: the measure of supplier linkages to China and the US decrease by 2.69% (Column 5), while the measure of supplier linkages within Chile decreases by only 0.25% (Column 6). In our exogenous network specifications in Row (b)-(d), we find that the responses of trade flows vary significantly across the values of the elasticity of substitution, σ_k. If we use the same value as our baseline model (Row c), the responses of trade flows

Table 5: Aggregate Effects from Tariff Changes from China and the US (%)

	1) $\widehat{\text{Welfare}}$	2) Rel. to Baseline	3) $\hat{X}_{ui,u\in\{\text{US,China}\}}$	4) $\hat{X}_{ui,u\in\text{Chile}}$	5) $\hat{M}_{ui,u\in\{\text{US,China}\}}$	6) $\hat{M}_{ui,u\in\text{Chile}}$
a) Baseline	-0.67	100	-5.95	0.23	-2.69	-0.25
b) Exogenous Network: Low Sigma	-0.40	60	-2.35	0.10	0	0
c) Exogenous Network: Baseline Sigma	-0.32	48	-4.22	0.16	0	0
d) Exogenous Network: High Sigma	-0.32	47	-5.98	0.21	0	0

Notes: This table presents the results of the counterfactual simulation of reverting the tariff changes from and to the US and China as observed between 2001 and 2016. We implement this counterfactual as a tariff increase, that is, increasing tariffs from the levels in 2016 to the ones in 2001 (the inverse of the numbers in Table 4). Column 1 shows the welfare changes as measured by the percent change in the income-weighted average of welfare changes across municipalities. Column 2 presents the welfare changes of the exogenous production networks ($\tilde{\lambda}_{kl}^S = \tilde{\lambda}_{kl}^B = 0$) relative to the baseline model, in percentages. Column 3 presents the import changes from China and the US. Column 4 presents the changes in domestic expenditure in intermediate inputs. Column 5 presents the changes in the number of linkages with China and the US. Column 6 presents the changes in the number of supplier linkages with domestic firms. As with Column 1, all numbers are income-weighted averages across municipalities in Chile. Row (a) presents the results from the baseline model. Rows (b)-(d) present the results from the model with exogenous production networks ($\tilde{\lambda}_{kl}^S = \tilde{\lambda}_{kl}^B = 0$) under three different values for the elasticities of substitution: our Baseline σ_k (in Row c), a Low Sigma case, $\sigma_k - 1$ (Row b), and a High Sigma case, $\sigma_k + 1$ (Row d).

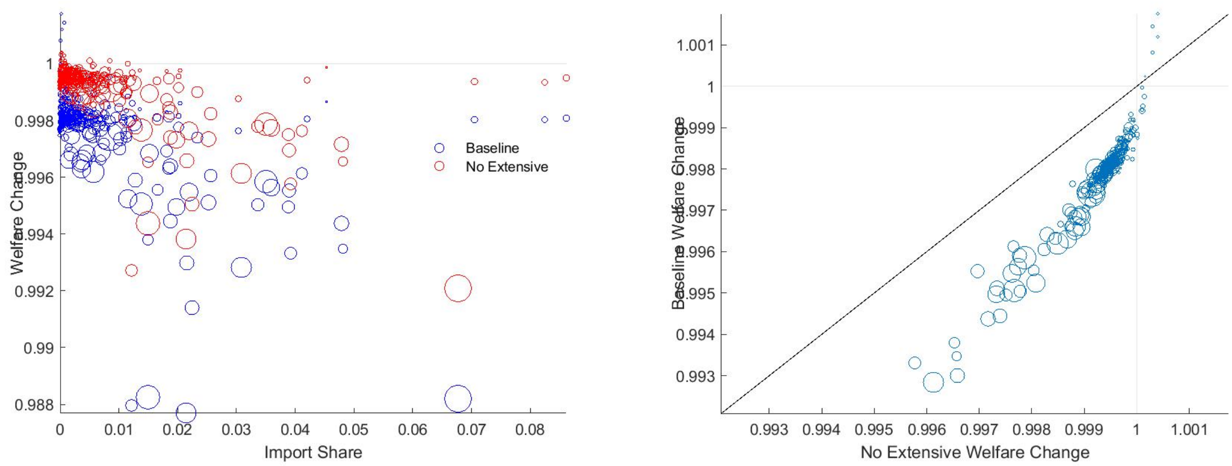
are smaller than our baseline model (Columns 3 and 4). This finding is consistent with the observation that endogenous production networks increase trade elasticity given the value of σ_k (Equation 16). If we use a larger value for the elasticity of substitution, σ_k (Row d), the specification with exogenous production networks predicts similar responses of trade flows to our baseline specification. However, even if the two specifications predict similar responses in trade flows, our baseline model with endogenous production networks predicts a substantially larger welfare change as already emphasized above. These results indicate that endogenous network formation implies a larger aggregate welfare effect (in absolute value) than the exogenous network model even if one recalibrates trade elasticities.

Heterogeneous Effects across Municipalities The aggregate welfare changes presented so far mask significant heterogeneity across municipalities in Chile. Figure 5 presents the spatial heterogeneity of the predicted welfare changes from the same counterfactual simulation. In Panel (a), we plot the welfare changes against the import share from China and the US for those 3 sectors where the majority of trade liberalization occurred (agriculture, mining, and manufacturing) at the municipality level. We show both the prediction from our baseline model (in blue) and the prediction from our alternative specification of exogenous production networks (in red). In both cases, we find a large variation of welfare changes across municipalities. These welfare changes (in absolute value) are positively correlated with the direct import share. At the same time, there is a significant variation in welfare changes conditional on the direct import share. The patterns are consistent with the interpretation that international trade shocks affect regions not only directly through imports and exports but also indirectly through domestic production networks.

We also report the differences in these heterogeneous effects between our baseline specification and the exogenous network specification. In Panel (b), we plot the predicted changes using our baseline model against the exogenous network specification for each municipality. If the observation lies below the 45-degree line, it means that our baseline model predicts a larger welfare change for the municipality than the model with exogenous production networks and vice versa. We find that our baseline model predicts larger welfare changes for all municipalities. At the same time, the relationship is steeper than

Figure 5: Heterogeneous Welfare Changes from Tariff Changes from China

(a) Welfare Changes by Import Share (b) Welfare Changes: Baseline versus Exogenous Networks



Notes: This figure presents the welfare changes across Chilean municipalities from the counterfactual simulation of the tariff changes from and to the US and China as observed between 2016 and 2001 (inverse of the numbers shown in Table 4). Panel (a) plots the welfare changes against the import share from China and the US for those 3 sectors where the majority of trade liberalization occurred (agriculture, mining, and manufacturing) at the municipality level, using our baseline model (in blue) and using an alternative specification of exogenous production networks with the same σ_k (in red). Panel (b) plots the welfare changes in the baseline model against those in the exogenous network model, with a 45-degree line in black. In both panels, the size of each circle indicates the aggregate labor income of the municipality.

45 degrees, indicating that there is a larger dispersion of welfare changes in our baseline model than those in the exogenous network specification.

Sensitivity Analysis and Additional Results In Appendix Table G.1, we report the sensitivity of these results to alternative parameter values. We find that amplification is weaker when β is larger, μ is larger, and $\tilde{\lambda}^S + \tilde{\lambda}^B$ is smaller, as anticipated from Proposition 4. Interestingly, conditional on the value of $\tilde{\lambda}^S + \tilde{\lambda}^B$, whether the elasticity is loaded on suppliers ($\tilde{\lambda}^S$) or buyers ($\tilde{\lambda}^B$) has small effects on aggregate welfare.[23] We also find that, even if we set $\mu = 1$, there is still significant amplification from endogenous network formation. In that calibration shutting down endogenous network formation reduces aggregate welfare changes by 37%. This result arises partly because of the presence of second-order effects (Proposition 5) and partly because of the sectoral reallocation that is not captured in Proposition 4. Overall, while the magnitudes vary, incorporating endogenous production networks amplifies the aggregate effects on welfare in all of these sensitivity exercises.[24]

In Appendix Table G.3, instead of changing the import and export tariffs simultaneously, we change each of them one by one. In our baseline model, increasing only import tariffs reduces aggregate welfare

[23]This robustness exercise is partially motivated by the fact that in general our model is inefficient due to search and matching externalities (Hosios 1990). In Appendix D, we show that a necessary condition for the optimality of equilibrium search in a single-sector and location model is $\lambda^S/\lambda^B = \sigma/(\sigma-1)$, equating the relative search externality of supplier and buyer search with the relative share of resources used for supplier and buyer search in the equilibrium.

[24]We also implement these exercises evaluating an increase of tariffs in the same magnitude as presented in Table 4, that is, the inverse of tariff changes of our baseline counterfactual. The results commented in this paragraph are qualitatively robust to this alternative counterfactual (Table G.2).

by 0.59 percent, and increasing only export tariffs reduces aggregate welfare by 0.08 percent. We also find that incorporating endogenous network formation amplifies the welfare changes in both exercises, and the amplification is larger for the export tariff shocks than for the import tariff shocks. This result is consistent with the interpretation that, for import tariff shocks, the additional changes are somewhat muted because of the import substitution.

In Appendix Table G.4, besides using the observed tariff changes, we simulate a decrease of tariffs in the same magnitude as those shown in Table 4. To show how our results compare to small changes in tariffs, we also implement an increase and decrease of 10% of the observed tariff changes. We find that, when a shock is small (10% of the observed changes in tariffs), the absolute magnitude of aggregate welfare changes are similar (around 0.06%) regardless of the signs of tariff changes. On the other hand, when the shock is large (the observed tariff changes), aggregate welfare changes are asymmetric; 0.99% increase in welfare from a decrease of tariffs and 0.67% decrease in welfare from an increase of tariffs (our baseline counterfactual). We also find that endogenous network formation leads to a larger amplification for a decrease of tariffs than its increase. These results are consistent with the prediction in Proposition 5 that the endogenous network formation tends to amplify the aggregate effect of a decrease of trade costs and dampen that of an increase of trade costs.

6.2 Transportation Infrastructure

In our second counterfactual simulation, we study the impact of intra-national trade shocks. In particular, we simulate the impact of a large-scale transportation infrastructure development, a new bridge between the mainland of Chile and Chiloé Island planned to open in 2025.[25] Chiloé is the largest island in Chile, and it is populated by approximately 1% of Chile's population. As of 2023, the only available transportation mode to access Chiloé island from the mainland is through a ferry crossing the Chacao Channel, which takes about 35 minutes (including waiting time) over around 2 kilometers of sea travel. To promote the economic development of the island, the Chilean government plans to open a new suspension bridge that is estimated to reduce the time of crossing the Chacao channel to virtually zero.

To simulate the effects of this new bridge we start by calibrating how much the reduction of expected travel time reduces the iceberg trade costs and search and matching frictions. We assume that the two types of frictions decrease iso-elastically to the observed relationships between these frictions and bilateral travel time. More concretely, denoting the change in travel time between location u and d from the new bridge by $\hat{T}_{ud}$, we assume that $\hat{\chi}^{\text{iceberg}}_{ud,kl} = \hat{T}_{ud}^{\nu_k^{\text{iceberg}}}$ and $\hat{\chi}^{\text{matching}}_{ud,kl} = \hat{T}_{ud}^{\nu_k^{\text{matching}}}$, where $\chi^{\text{iceberg}}_{ud,kl}$ and $\chi^{\text{matching}}_{ud,kl}$ are bilateral frictions defined in Section 5.3 that summarize the iceberg trade costs and search and matching frictions, and ν_k^{iceberg} and ν_k^{matching} are the elasticities of the two types of spatial

[25] See Figure E.1 for the planned location of this bridge.

frictions with respect to travel time.[26] We calibrate ν_k^{iceberg} and ν_k^{matching} from the observed relationships between the spatial frictions and travel time as documented in Figure F.5 (ν_k^{iceberg} from the blue bars and ν_k^{matching} from the red bars).

Table 6 presents the results of the counterfactual simulation. Panel (a) presents the aggregate effects. Similarly to Table 5, in Row (a) we report the simulation results from our baseline specification, and in Rows (b)-(d) we report the results from the alternative specification with exogenous production networks under different values of σ_k.

Table 6: Predicted Welfare Gains from the Bridge to Chiloé Island

(a) Aggregate Effects

	1) $\widehat{\text{Welfare}}$	2) Rel. to Baseline	3) $\hat{X}_{ui,u\in\text{Chiloe}}$	4) $\hat{X}_{ui,u\notin\text{Chiloe}}$	5) $\hat{M}_{ui,u\in\text{Chiloe}}$	6) $\hat{M}_{ui,u\notin\text{Chiloe}}$
a) Baseline	0.25	100	1.13	-0.01	2.29	-0.08
b) Exogenous Network: Low Sigma	0.17	68	0.37	-0.00	0	0
c) Exogenous Network: Baseline Sigma	0.16	62	0.61	-0.00	0	0
d) Exogenous Network: High Sigma	0.14	58	0.77	-0.00	0	0

(b) Heterogeneous Effects

	1) All Municipalities	2) High Exposure Municipalities	3) Low Exposure Municipalities
(A) $\widehat{\text{Welfare}(\%)}$			
a) Baseline	0.25	2.15	0.22
b) Exogenous Network: Baseline Sigma	0.16	1.19	0.14
(B) $\hat{M}_{ui,u\in\text{Chiloe}}$			
c) Baseline	2.29	8.48	2.19
d) Exogenous Network: Baseline Sigma	0	0	0

Notes: This table presents the results of the counterfactual simulation of the expected travel time reduction from the Chiloé bridge. Panel (a) presents the aggregate effects. Column 1 shows the gains as measured by the percent increase in the income-weighted average of welfare gains across municipalities. Column 2 presents the welfare gains of the exogenous production networks ($\tilde{\lambda}^S = \tilde{\lambda}^B = 0$) relative to the baseline model, in percentages. Column 3 presents the changes in trade volume from Chiloé island and Column 4 presents those from locations outside Chiloé island. Column 5 presents the changes in the number of supplier linkages from Chiloé island and Column 6 presents those from locations outside Chiloé island. As with Column 1, all numbers are income-weighted averages across municipalities in Chile. Row (a) presents the results from the baseline model. Rows (b)-(d) presents the results from the model with exogenous production networks ($\tilde{\lambda}^S_{kl} = \tilde{\lambda}^B_{kl} = 0$) under three different values for the elasticities of substitution: our Baseline σ_k (in Row c), a Low Sigma case, $\sigma_k - 1$ (Row b), and a High Sigma case, $\sigma_k + 1$ (Row d). In Panel (b), we report the welfare gains and changes in supplier linkages to Chiloé island for all municipalities (Column 1), for municipalities with high exposure to Chiloé island (top 5% municipalities in terms of the share of intermediate inputs from Chiloé island, Column 2), and for municipalities with low exposure to Chiloé island (the rest of the municipalities, Column 3).

Our baseline model predicts an aggregate welfare gain of 0.25 percent (Column 1, Row a). In contrast, these welfare gains are substantially smaller when we shut down endogenous production networks (Column 1, Rows b, c, d), ranging from 0.14 to 0.17 percent. This implies that these alternative models predict only 58 to 68 percent of welfare gains compared to our baseline model (Column 2). We also find an increase of trade flows and supplier linkages from Chiloé island (Columns 3-6), consistent with the interpretation that production networks reorganize as a response to the opening of the bridge.

Panel (b) of Table 6 presents the heterogeneous effects across municipalities. We report the welfare gains and changes in supplier linkages to Chiloé island for all municipalities (Column 1), for municipalities with high exposure to Chiloé island (top 5% municipalities in terms of the share of intermediate

[26] We set $\hat{T}_{ud} = (T_{ud} - 35)/T_{ud}$ if either u or d is in Chiloé island and the other is outside the island and $\hat{T}_{ud} = 1$ otherwise, where T_{ud} is the travel minutes given the existing land or water transportation method. 35 minutes corresponds to the expected travel time reduction induced by this bridge.

inputs from Chiloé island, Column 2), and for municipalities with low exposure to Chiloé island (the rest of the municipalities, Column 3). We find that high-exposure municipalities increase welfare by 2.15 percent whereas the other municipalities increase welfare only by 0.22 percent, suggesting a large heterogeneity in welfare gains across municipalities. When we shut down endogenous network formation, we find that the welfare gains are substantially smaller in percentage points for high exposure municipality (1.19 percent relative to 2.15 percent) than the rest of the municipalities (0.14 percent relative to 0.22 percent). Consistent with our findings in the tariff counterfactual, these results suggest that endogenous network formation leads to larger and more dispersed welfare gains from domestic trade cost shocks.

7 Conclusion

In this paper, we study how production networks are organized in space and how their endogenous formation shapes the spatial distribution of economic activity. Using rich administrative firm-to-firm transaction-level data from Chile, we document that production networks are related to firms' size and geography. Guided by these pieces of evidence, we build a microfounded model of spatial production network formation where firms form supplier and buyer relationships across space facing iceberg trade costs and matching frictions. We characterize how spatial frictions shape the production networks and, in turn, how endogenous production network formation determines the spatial distribution of economic activity and aggregate welfare in general equilibrium. Using a quantitative version of our theory with multiple sectors we demonstrate –in accordance with our theoretical findings– that the endogenous formation of production networks leads to larger and more dispersed effects of both international and intra-national trade cost changes.

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Appendix for "Spatial Production Networks"

A Proofs and Mathematical Derivations

A.1 Proof of Lemma 1

We first note that firms' search problem (8) is a strictly convex optimization problem when $\gamma^B > 1$ and $\gamma^S > 1$. Therefore, there is a unique solution to the problem, and the first order conditions are necessary and sufficient for the solution. Imposing $\gamma^B = \gamma^F$, the first-order conditions of (8) with respect to n_i^F, n_{id}^B , and n_{ui}^S are given by:

$$e_i f_i^F \left(n_i^F\right)^{\gamma^B - 1} = \frac{\tilde{\sigma}^{1-\sigma}}{\sigma} D_i^F \frac{w_i^{\beta(1-\sigma)} \left(\sum_{u\in\mathcal{N}} n_{ui}^S m_{ui}^S \left(C_{ui}\right)^{1-\sigma}\right)^{1-\beta}}{z^{1-\sigma}} \tag{A.1}$$

$$e_i f_{id}^B \left(n_{id}^B\right)^{\gamma^B - 1} = \frac{\tilde{\sigma}^{1-\sigma}}{\sigma} m_{id}^B D_d \left(\tau_{id}\right)^{1-\sigma} \frac{w_i^{\beta(1-\sigma)} \left(\sum_{u\in\mathcal{N}} n_{ui}^S m_{ui}^S \left(C_{ui}\right)^{1-\sigma}\right)^{1-\beta}}{z^{1-\sigma}} \tag{A.2}$$

$$\begin{aligned} e_i f_{ui}^S \left(n_{ui}^S\right)^{\gamma^S - 1} = & \frac{\tilde{\sigma}^{1-\sigma}}{\sigma} \left\{ n_i^F D_i^F + \sum_{d\in\mathcal{N}} n_{id}^B m_{id}^B D_d \left(\tau_{id}\right)^{1-\sigma} \right\} (1-\beta) \\ & \times \frac{w_i^{\beta(1-\sigma)} \left(\sum_{u'\in\mathcal{N}} n_{u'i}^S m_{u'i}^S \left(C_{u'i}\right)^{1-\sigma}\right)^{-\beta}}{z^{1-\sigma}} m_{ui}^S \left(C_{ui}\right)^{1-\sigma} \end{aligned} \tag{A.3}$$

Now, we conjecture that the solutions take the form of (9), replicated here:

$$n_i^F(z) = a_i^F z^{\frac{\delta_1}{\gamma^B}};\ n_{id}^B(z) = a_{id}^B z^{\frac{\delta_1}{\gamma^B}};\ n_{ui}^S(z) = a_{ui}^S z^{\frac{\delta_1}{\gamma^S}}, \tag{A.4}$$

where we define $\delta_1 \equiv (\sigma - 1) / \left\{1 - \frac{1}{\gamma^B} - \frac{1-\beta}{\gamma^S}\right\} > 0$ and $\{a_i^F, a_{id}^B, a_{ui}^S\}$ are unknown constants. Plugging these equations into (A.1), (A.2), and (A.3), we obtain the expressions for $\{a_i^F, a_{id}^B, a_{ui}^S\}$ given by

$$a_i^F = \left(\frac{\tilde{\sigma}^{1-\sigma}}{\sigma} \frac{D_i^F}{e_i f_i^F} \left(C_i^*\right)^{1-\sigma}\right)^{\frac{1}{\gamma^B - 1}}, \tag{A.5}$$

$$a_{id}^B = \left(\frac{\tilde{\sigma}^{1-\sigma}}{\sigma} m_{id}^B \frac{D_d}{e_i f_{id}^B} \left(\tau_{id}\right)^{1-\sigma} \left(C_i^*\right)^{1-\sigma}\right)^{\frac{1}{\gamma^B - 1}}, \tag{A.6}$$

$$a_{ui}^S = \left(\frac{\tilde{\sigma}^{1-\sigma}}{\sigma} \frac{(1-\beta) D_i^*}{e_i w_i^{-\frac{\beta}{1-\beta}(1-\sigma)} f_{ui}^S} m_{ui}^S \left(C_i^*\right)^{\beta\frac{\sigma-1}{1-\beta}} \left(C_{ui}\right)^{1-\sigma}\right)^{\frac{1}{\gamma^S - 1}}, \tag{A.7}$$

where we further define the demand shifter from buyers in all locations by

$$D_i^* = a_i^F D_i^F + \sum_d m_{id}^B a_{id}^B D_d \left(\tau_{id}\right)^{1-\sigma}, \tag{A.8}$$

and the production cost shifter for firms in location i by

$$\left(C_i^*\right)^{1-\sigma} \equiv w_i^{\beta(1-\sigma)} \left(\sum_{u\in\mathcal{N}} a_{ui}^S m_{ui}^S C_{ui}^{1-\sigma}\right)^{1-\beta}. \tag{A.9}$$

Since the solution is unique, this is the only possible solution.

By plugging these equations into the cost function (equation 7), the unit cost of a firm with productivity z is given by

$$c_i\left(z\right) = \frac{w_i^\beta \left(\sum_{u\in\mathcal{N}} a_{ui}^S z^{\frac{\delta_1}{\gamma^S}} m_{ui}^S \left(C_{ui}\right)^{1-\sigma}\right)^{\frac{1-\beta}{1-\sigma}}}{z} = \left(C_i^*\right) z^{\frac{\delta_1}{\gamma^S}\frac{1-\beta}{1-\sigma}-1}. \tag{A.10}$$

The revenue of a firm with productivity z is given by

$$r_i\left(z\right) = \left\{n_i^F D_i^F + \sum_{d\in\mathcal{N}} n_{id}^B m_{id}^B D_d \left(\tau_{id}\right)^{1-\sigma}\right\} \left(\tilde{\sigma} c_i\left(z\right)\right)^{1-\sigma} = \left(\tilde{\sigma}\right)^{1-\sigma} D_i^* \left(C_i^*\right)^{1-\sigma} \left(z\right)^{\delta_1}, \tag{A.11}$$

from equation (A.10).

□

For later aggregation purposes, we also derive the expression for the revenue, profit, and demand for advertisement for each firm in the following lemma.

Lemma 1. *The profit of the firm, $\pi_i\left(z\right)$, the payment for advertisement, $h_i\left(z\right)$, and labor compensation, $l_i\left(z\right)$, are all proportional to firm revenue such that:*

$$\pi_i\left(z\right) = \vartheta^P r_i\left(z\right); \quad h_i\left(z\right) = \vartheta^A r_i\left(z\right); \quad l_i\left(z\right) = \vartheta^L r_i\left(z\right) \tag{A.12}$$

where $\vartheta^P \equiv \frac{1}{\delta_1 \tilde{\sigma}}$, $\vartheta^A \equiv \frac{1}{\sigma}\left\{\frac{1}{\gamma^B} + \frac{1-\beta}{\gamma^S}\right\}$, and $\vartheta^L \equiv \beta\frac{\sigma-1}{\sigma}$.

Proof. First, the profit of the firm is obtained by plugging equations (A.1), (A.2), and (A.3) into the optimal firm profit (8) such that:

$$
\begin{aligned}
\pi_i(z) =& \frac{\tilde{\sigma}^{1-\sigma}}{\sigma} D_i^* \left(c_i(z)\right)^{1-\sigma} - e_i \left\{ f_i^B \frac{\left(n_i^F\right)^{\gamma^B}}{\gamma^B} + \sum_{d\in\mathcal{N}} f_{id}^B \frac{\left(n_{id}^B\right)^{\gamma^B}}{\gamma^B} + \sum_{u\in\mathcal{N}} f_{ui}^S \frac{\left(n_{ui}^S\right)^{\gamma^S}}{\gamma^S} \right\} \\
=& \frac{\tilde{\sigma}^{1-\sigma}}{\sigma} D_i^* \left(c_i(z)\right)^{1-\sigma} \\
&- \frac{\tilde{\sigma}^{1-\sigma}}{\sigma} \left(c_i(z)\right)^{1-\sigma} \left\{ \frac{n_i^F D_i^F + \sum_{d\in\mathcal{N}} n_{id}^B m_{id}^B D_d \left(\tau_{id}\right)^{1-\sigma}}{\gamma^B} + \frac{1-\beta}{\gamma^S} \left\{ n_i^F D_i^F + \sum_d m_{id}^B n_{id}^B D_d \left(\tau_{id}\right)^{1-\sigma} \right\} \right\} \\
=& \frac{1}{\sigma} \left\{ 1 - \frac{1}{\gamma^B} - \frac{1-\beta}{\gamma^S} \right\} \left(\tilde{\sigma}\right)^{1-\sigma} D_i^* \left(C_i^*\right)^{1-\sigma} (z)^{\delta_1} \\
=& \frac{1}{\delta_1 \tilde{\sigma}} \left(\tilde{\sigma}\right)^{1-\sigma} D_i^* \left(C_i^*\right)^{1-\sigma} (z)^{\delta_1} \\
=& \vartheta^P r_i(z),
\end{aligned}
$$

where we used that $\delta_1 \equiv (\sigma-1) / \left(1 - \frac{1}{\gamma^B} - \frac{1-\beta}{\gamma^S}\right)$. From the above expression, the demand for advertisement services is $\vartheta^A = \frac{1}{\sigma}\left\{\frac{1}{\gamma^B} + \frac{1-\beta}{\gamma^S}\right\}$ of the revenue. Lastly, labor compensation for production is a fraction β of the total factor payment, $\frac{\sigma-1}{\sigma} r_i(z)$. Therefore the statement is proved. $\square$

It is also useful to derive the expression for the average cost of intermediate goods that are produced in location u and sold to location i, C_{ui}:

Lemma 2. *C_{ui} is given by*

$$
C_{ui} = \overline{C}_u \tau_{ui}, \quad \overline{C}_u^{1-\sigma} = \left(\tilde{\sigma}\right)^{1-\sigma} \left(C_u^*\right)^{1-\sigma} \frac{\mathbb{M}_u\left(\delta_1\right)}{\mathbb{M}_u\left(\frac{\delta_1}{\gamma^B}\right)}, \tag{A.13}
$$

where we define $\mathbb{M}_d(\chi) \equiv \int z^\chi dG_d(z)$.

Proof. From the definition of C_{ui} in equation (7), we have

$$
C_{ui}^{1-\sigma} \equiv \int \left(\tilde{\sigma} c_u(z) \tau_{ui}\right)^{1-\sigma} dG_{ui}^B(z)
$$

where

$$
dG_{ui}^B(z) = \frac{n_{ui}^B(z)\, dG_u(z)}{\int n_{ui}^B(z')\, dG_u(z')} = \frac{z^{\frac{\delta_1}{\gamma^B}} dG_u(z)}{\int z'^{\frac{\delta_1}{\gamma^B}} dG_u(z')} = \frac{z^{\frac{\delta_1}{\gamma^B}} dG_u(z)}{\mathbb{M}_u\left(\frac{\delta_1}{\gamma^B}\right)}.
$$

Therefore we have

$$
\begin{aligned}
C_{ui}^{1-\sigma} &= \int \left(\tilde{\sigma} C_u^* z^{-\frac{\delta_1}{\gamma^S}\frac{1-\beta}{\sigma-1}-1} \tau_{ui} \right)^{1-\sigma} dG_{ui}^B(z) \\
&= \left(\tilde{\sigma} C_u^* \tau_{ui}\right)^{1-\sigma} \int z^{\left(-\frac{\delta_1}{\gamma^S}\frac{1-\beta}{\sigma-1}-1\right)(1-\sigma)} \frac{z^{\frac{\delta_1}{\gamma^B}} dG_u(z)}{\mathbb{M}_u\left(\frac{\delta_1}{\gamma^B}\right)} \\
&= \left(\tilde{\sigma} C_u^* \tau_{ui}\right)^{1-\sigma} \frac{\mathbb{M}_u(\delta_1)}{\mathbb{M}_u\left(\frac{\delta_1}{\gamma^B}\right)},
\end{aligned}
\tag{A.14}
$$

where we used the fact that $\delta_1 = \left(-\frac{\delta_1}{\gamma^S}\frac{1-\beta}{\sigma-1} - 1\right)(1-\sigma) + \frac{\delta_1}{\gamma^B}$. □

A.2 Derivations for Gravity Equations

We illustrate the derivation of the gravity equations (16).

We derive the gravity equation of the extensive margin by solving equations (9), (12), (13), (14), and (15). Combine equations, (12), (15), and (A.7) to obtain:

$$
M_{ud} = m_{ud}^S N_d a_{ud}^S \mathbb{M}_d \left(\frac{\delta_1}{\gamma^S}\right) = \left(m_{ud}^S\right)^{\frac{\gamma^S}{\gamma^S-1}} \tilde{a}_{ud}^S,
\tag{A.15}
$$

where

$$
\begin{aligned}
\tilde{a}_{ud}^S &\equiv N_d \mathbb{M}_d \left(\frac{\delta_1}{\gamma^S}\right) a_{ud}^S / \left(m_{ud}^S\right)^{\frac{1}{\gamma^S-1}} \\
&= N_d \mathbb{M}_d \left(\frac{\delta_1}{\gamma^S}\right) \left(\frac{\tilde{\sigma}^{1-\sigma}}{\sigma} \frac{1}{e_d f_{ud}^S} D_d^* (1-\beta) w_d^{\frac{\beta(1-\sigma)}{1-\beta}} \left(C_d^*\right)^{-\frac{\beta(1-\sigma)}{1-\beta}} \left(\overline{C}_u \tau_{ud}\right)^{1-\sigma} \right)^{\frac{1}{\gamma^S-1}},
\end{aligned}
\tag{A.16}
$$

where we used $C_{ui} = \overline{C}_u \tau_{ui}$ from Lemma 2. Similarly, combine equation (13), (15), and (A.6) to obtain:

$$
M_{ud} = m_{ud}^B N_u a_{ud}^B \mathbb{M}_u \left(\frac{\delta_1}{\gamma^B}\right) = \left(m_{ud}^B\right)^{\frac{\gamma^B}{\gamma^B-1}} \tilde{a}_{ud}^B,
\tag{A.17}
$$

where

$$
\tilde{a}_{ud}^B = N_u \mathbb{M}_u \left(\frac{\delta_1}{\gamma^B}\right) \left(\frac{\tilde{\sigma}^{1-\sigma}}{\sigma} \frac{1}{e_u f_{ud}^B} D_d \left(\tau_{ud}\right)^{1-\sigma} \left(C_u^*\right)^{1-\sigma} \right)^{\frac{1}{\gamma^B-1}}.
\tag{A.18}
$$

Combining equations (A.15) and (A.17) yields:

$$
\left(m_{ud}^S\right)^{\frac{\gamma^S}{\gamma^S-1}} = \left(m_{ud}^B\right)^{\frac{\gamma^B}{\gamma^B-1}} \left(\frac{\tilde{a}_{ud}^B}{\tilde{a}_{ud}^S}\right).
\tag{A.19}
$$

Now, by plugging (15) into equation (14), we have

$$
\begin{aligned}
& \left(m_{ud}^{S}\right)^{\lambda^{S}}\left(m_{ud}^{B}\right)^{\lambda^{B}}=\kappa_{ud} M_{ud}^{\lambda^{S}+\lambda^{B}-1} \\
\Longleftrightarrow\ & \left(\left(m_{ud}^{B}\right)^{\frac{\gamma^{B}}{\gamma^{B}-1}}\left(\frac{\tilde{a}_{ud}^{B}}{\tilde{a}_{ud}^{S}}\right)\right)^{\lambda^{S} \frac{\gamma^{S}-1}{\gamma^{S}}}\left(m_{ud}^{B}\right)^{\lambda^{B}}=\kappa_{ud}\left(\left(m_{ud}^{B}\right)^{\frac{\gamma^{B}}{\gamma^{B}-1}} \tilde{a}_{ud}^{B}\right)^{\lambda^{S}+\lambda^{B}-1} \\
\Longleftrightarrow\ & \left(m_{ud}^{B}\right)^{\frac{\gamma^{B}}{\gamma^{B}-1}\left\{\frac{\lambda^{S}}{\gamma^{S}}+\frac{\lambda^{B}}{\gamma^{B}}-1\right\}}=\kappa_{ud}^{-1}\left(\tilde{a}_{ud}^{B}\right)^{1-\frac{\lambda^{S}}{\gamma^{S}}-\lambda^{B}}\left(\tilde{a}_{ud}^{S}\right)^{-\lambda^{S} \frac{\gamma^{S}-1}{\gamma^{S}}}
\end{aligned}
$$

where we used equations (A.15) and (A.19). By plugging this equation into equation (A.17), we have

$$
M_{ud}=\left[\kappa_{ud}\left(\tilde{a}_{ud}^{B}\right)^{\lambda^{B} \frac{\gamma^{B}-1}{\gamma^{B}}}\left(\tilde{a}_{ud}^{S}\right)^{\lambda^{S} \frac{\gamma^{S}-1}{\gamma^{S}}}\right]^{\delta_{2}},
$$

where $\delta_2=\left[1-\tilde{\lambda}^S-\tilde{\lambda}^B\right]^{-1}$, $\tilde{\lambda}^S=\lambda^S/\gamma^S$, and $\tilde{\lambda}^B=\lambda^B/\gamma^B$ as defined in our main paper.

Plugging $\tilde{a}_{ud}^B$ and $\tilde{a}_{ud}^S$ from equations (A.16) and (A.18) in the equation above yields the gravity equation:

$$
M_{ud}=\varrho^{E} \chi_{ud}^{E} \zeta_{u}^{E} \xi_{d}^{E}
$$

where $\varrho^E=\left(\tilde{\sigma}^{1-\sigma}/\sigma\right)^{\left(\tilde{\lambda}^B+\tilde{\lambda}^S\right)\delta_2}(1-\beta)^{\tilde{\lambda}^S\delta_2}$ is a constant term and the bilateral resistance term is given by $\chi_{ud}^E=\left[\kappa_{ud}\left(f_{ud}^B\right)^{-\tilde{\lambda}^B}\left(f_{ud}^S\right)^{-\tilde{\lambda}^S}\left(\tau_{ud}^{1-\sigma}\right)^{\tilde{\lambda}^B+\tilde{\lambda}^S}\right]^{\delta_2}$. The origin-specific shifter ζ_u^E is given by:

$$
\zeta_{u}^{E}=\left[\left(N_{u} \mathbb{M}_{u}\left(\frac{\delta_{1}}{\gamma^{B}}\right)\right)^{\lambda^{B} \frac{\gamma^{B}-1}{\gamma^{B}}}\left\{e_{u}^{-1}\left(C_{u}^{*}\right)^{1-\sigma}\right\}^{\tilde{\lambda}^{B}}\left(\overline{C}_{u}\right)^{(1-\sigma) \tilde{\lambda}^{S}}\right]^{\delta_{2}}, \tag{A.20}
$$

which summarizes the capability of location u to generate buyer relationships. The destination-specific shifter ξ_d^E is given by:

$$
\xi_{d}^{E}=\left[\left(N_{d} \mathbb{M}_{d}\left(\frac{\delta_{1}}{\gamma^{S}}\right)\right)^{\lambda^{S} \frac{\gamma^{S}-1}{\gamma^{S}}}\left(D_{d}\right)^{\tilde{\lambda}^{B}}\left\{D_{d}^{*} e_{d}^{-1} w_{d}^{\frac{\beta(1-\sigma)}{1-\beta}}\left(C_{d}^{*}\right)^{\frac{\beta(\sigma-1)}{1-\beta}}\right\}^{\tilde{\lambda}^{S}}\right]^{\delta_{2}}, \tag{A.21}
$$

which summarizes the capability of location d to generate supplier relationships.

Using similar steps, we derive the intensive margin gravity equation. From equation (6), the average volume of transactions between suppliers in location u and buyers in location d, $\overline{r}_{ud}$, is expressed as

$$
\overline{r}_{ud}=\frac{N_{u} \int D_{d}\left(\tilde{\sigma} c_{u}(z) \tau_{ud}\right)^{1-\sigma} n_{ud}^{B} m_{ud}^{B} d G_{u}(z)}{M_{ud}},
$$

where the numerator is the total transaction volume from u to d, and the denominator is the number of realized matches from u to d.

The numerator is rewritten as

$$\begin{aligned}
&N_u \int D_d \left(\tilde{\sigma} c_u(z) \tau_{ud}\right)^{1-\sigma} n_{ud}^B m_{ud}^B dG_u(z) \\
&= N_u a_{ud}^B m_{ud}^B D_d \left(\tilde{\sigma} \tau_{ud}\right)^{1-\sigma} \int \left(C_u^*\right)^{1-\sigma} z^{\frac{\delta_1}{\gamma^S}(1-\beta)+(\sigma-1)} z^{\frac{\delta_1}{\gamma^B}} dG_u(z) \\
&= N_u a_{ud}^B m_{ud}^B D_d \left(\tilde{\sigma} \tau_{ud}\right)^{1-\sigma} \left(C_u^*\right)^{1-\sigma} \mathbb{M}_u \left(\delta_1\right),
\end{aligned}$$

where the last transformation used the fact that $\delta_1 = \frac{\delta_1}{\gamma^S}(1-\beta) + (\sigma-1) + \frac{\delta_1}{\gamma^B}$. The denominator is rewritten as

$$M_{ud} = N_u \int a_{ud}^B z^{\frac{\delta_1}{\gamma^B}} m_{ud}^B dG_u(z) = N_u a_{ud}^B m_{ud}^B \mathbb{M}_u \left(\frac{\delta_1}{\gamma^B}\right).$$

Putting together, we have the gravity equation for intensive margin trade flows:

$$\overline{r}_{ud} = \varrho^I \chi_{ud}^I \zeta_u^I \xi_d^I,$$

where $\varrho^I = (\tilde{\sigma})^{1-\sigma}$ is a constant term that only depends on parameters; the bilateral resistance term is given by $\chi_{ud}^I = (\tau_{ud})^{1-\sigma}$; the origin-specific shifter ζ_u^I is given by:

$$\zeta_u^I = \frac{\mathbb{M}_u \left(\delta_1\right)}{\mathbb{M}_u \left(\frac{\delta_1}{\gamma^B}\right)} \left(C_u^*\right)^{1-\sigma}, \tag{A.22}$$

and the destination-specific shifter ξ_d^I is given by

$$\xi_d^I = D_d. \tag{A.23}$$

A.3 General Equilibrium

This section provides detailed mathematical derivation for the general equilibrium in Section 3.3.

A.3.1 Firm Entry

The zero-profit condition implies that the aggregate fixed cost payment $w_i F_i N_i$ equals to the aggregate post-entry profit Π_i. We first characterize Π_i. From Lemma 1, aggregate post-entry profit is a fraction ϑ^P of aggregate revenue, i.e., $\Pi_i = \vartheta^P R_i$. Furthermore, aggregate labor compensation $w_i L_i$ is given by:

$$w_i L_i = \tilde{\beta} R_i, \tag{A.24}$$

where $\tilde{\beta} \equiv \vartheta^L + \vartheta^A + \vartheta^P = \frac{\sigma-1}{\sigma}\beta + \frac{1}{\sigma}$ is the labor share in aggregate revenue, and $\vartheta^L R_i$, $\vartheta^A R_i$, and $\vartheta^P R_i$ are labor compensation for production, search costs, and fixed costs, respectively.[*]

[*]See Lemma 1 for the values of ϑ^L, ϑ^A, and ϑ^P. Note that, although the labor share of the advertisement sector is μ, which may be less than one, the aggregate labor compensation in advertisement sector is $\vartheta^A R_i$, not $\mu\vartheta^A R_i$. This is because we define R_i as aggregate revenue excluding the intermediate goods used by advertisement sector. Therefore, aggregating the labor input used in a infinite sequence of input-output loops to produce the intermediate goods used by the advertisement sector, we have the result above.

Combining these results, we obtain $\Pi_i = \frac{\vartheta^P}{\tilde{\beta}} w_i L_i = \frac{1}{\delta_1 \tilde{\sigma} \tilde{\beta}} w_i L_i$. Equating this with $w_i F_i N_i$, we derive the expression for entry (18).

A.3.2 Labor Market Clearing

We assume that labor markets clear for each location. Labor demand is given by equation (A.24), where aggregate firm revenue, R_i, is given by the sum of final goods consumption and intermediate goods sales such that:

$$R_i = w_i L_i + \sum_d M_{id} \overline{r}_{id}.$$

By combining the two equations, we derive the expression (19).

A.3.3 Deriving Demand Shifters (D_i^*, D_i)

We first characterize D_i^*. Using equation (A.11), aggregate firm revenue R_i is expressed using D_i^* as

$$R_i = N_i \int r_i(z) G_i(z) dz = (\tilde{\sigma})^{1-\sigma} N_i D_i^* \left(C_i^*\right)^{1-\sigma} \mathbb{M}_i\left(\delta_1\right). \tag{A.25}$$

By equating this expression with $R_i = \frac{1}{\tilde{\beta}} w_i L_i$ from equation (A.24), we have:

$$D_i^* = \frac{(\tilde{\sigma})^{1-\sigma}}{\tilde{\beta}} \frac{w_i L_i}{\left(C_i^*\right)^{1-\sigma} N_i} \frac{1}{\mathbb{M}_i\left(\delta_1\right)}. \tag{A.26}$$

Next, we characterize D_d using intermediate goods market clearing condition. To do so, the aggregate intermediate goods revenue in location d is given by:

$$\begin{aligned}
&\sum_u N_u \int \left(\tau_{ud}\right)^{1-\sigma} D_d \left(\tilde{\sigma} c_u(z)\right)^{1-\sigma} n_{ud}^B(z) m_{ud}^B dG_u(z) \\
&= D_d \sum_u N_u \left(\tau_{ud}\right)^{1-\sigma} (\tilde{\sigma})^{1-\sigma} \int \left(C_u^*\right)^{1-\sigma} a_{ud}^B m_{ud}^B z^{\frac{\delta_1}{\gamma^B} + \frac{\delta_1}{\gamma^S}(1-\beta) + (\sigma-1)} dG_u(z) \\
&= D_d \sum_u \left(\tau_{ud}\right)^{1-\sigma} (\tilde{\sigma})^{1-\sigma} \left(C_u^*\right)^{1-\sigma} N_u a_{ud}^B m_{ud}^B \mathbb{M}_u\left(\delta_1\right) \\
&= D_d \sum_u \left(\tau_{ud}\right)^{1-\sigma} (\tilde{\sigma})^{1-\sigma} \left(C_u^*\right)^{1-\sigma} N_d a_{ud}^S m_{ud}^S \frac{\mathbb{M}_d\left(\frac{\delta_1}{\gamma^S}\right)}{\mathbb{M}_u\left(\frac{\delta_1}{\gamma^B}\right)} \mathbb{M}_u\left(\delta_1\right) \quad \text{(from equations 12, 13, 15)} \\
&= D_d N_d \mathbb{M}_d\left(\frac{\delta_1}{\gamma^S}\right) \sum_u \left(\tau_{ud}\right)^{1-\sigma} \overline{C}_u^{1-\sigma} a_{ud}^S m_{ud}^S \quad \text{(from equation A.13)} \\
&= D_d N_d \mathbb{M}_d\left(\frac{\delta_1}{\gamma^S}\right) \left[w_d^{-\beta} C_d^*\right]^{\frac{1-\sigma}{1-\beta}} \quad \text{(from equation A.9)}
\end{aligned}$$

At the same time, the aggregate intermediate goods demand is given by $\left(1-\tilde{\beta}\right) R_d = \frac{1-\tilde{\beta}}{\tilde{\beta}} w_d L_d$.

Equation of these two expressions gives

$$D_d = \frac{1-\tilde{\beta}}{\tilde{\beta}} \frac{L_d}{N_d \mathbb{M}_d\left(\frac{\delta_1}{\gamma^S}\right)} \left(w_d\right)^{\frac{1-\beta\sigma}{1-\beta}} \left(C_d^*\right)^{\frac{\sigma-1}{1-\beta}}. \tag{A.27}$$

A.3.4 Consumer Price Index

The final consumer price, P_i^F, is defined using standard CES preferences in equation (5) as

$$\begin{aligned}
\left(P_i^F\right)^{1-\sigma} &= N_i \int \left\{\tilde{\sigma} c_i(z)\right\}^{1-\sigma} n_i^F(z)\, dz \\
&= \tilde{\sigma}^{1-\sigma} N_i a_i^F \int \left(C_i^*\right)^{1-\sigma} z^{\frac{\delta_1}{\gamma^B}+\frac{\delta_1}{\gamma^S}(1-\beta)+(\sigma-1)} dz \\
&= \tilde{\sigma}^{1-\sigma} N_i a_i^F \left(C_i^*\right)^{1-\sigma} \mathbb{M}_i\left(\delta_1\right),
\end{aligned}$$

which corresponds to equation (21) and a_i^F is given by equation (A.5). Furthermore, from the definition of D_i^F in equation (5), we have

$$D_i^F = \left(\frac{1}{P_i^F}\right)^{1-\sigma} w_i L_i. \tag{A.28}$$

Combining, we have

$$\begin{aligned}
\left(P_i^F\right)^{1-\sigma} &= \tilde{\sigma}^{1-\sigma} N_i a_i^F \left(C_i^*\right)^{1-\sigma} \mathbb{M}_i\left(\delta_1\right) \\
&= \tilde{\sigma}^{1-\sigma} N_i \left(\frac{\tilde{\sigma}^{1-\sigma}}{\sigma} \frac{D_i^F}{e_i f_i^F} \left(C_i^*\right)^{1-\sigma}\right)^{\frac{1}{\gamma^B-1}} \left(C_i^*\right)^{1-\sigma} \mathbb{M}_i\left(\delta_1\right) \\
&= \tilde{\sigma}^{1-\sigma} N_i \left(\frac{\tilde{\sigma}^{1-\sigma}}{\sigma} \frac{1}{e_i f_i^F} \left(\frac{1}{P_i^F}\right)^{1-\sigma} w_i L_i \left(C_i^*\right)^{1-\sigma}\right)^{\frac{1}{\gamma^B-1}} \left(C_i^*\right)^{1-\sigma} \mathbb{M}_i\left(\delta_1\right).
\end{aligned}$$

Further manipulations finally imply

$$\left(P_i^F\right)^{1-\sigma} = \left(\tilde{\sigma}^{1-\sigma} N_i \mathbb{M}_i\left(\delta_1\right)\right)^{\frac{\gamma^B-1}{\gamma^B}} \left(\frac{\tilde{\sigma}^{1-\sigma}}{\sigma} \frac{1}{e_i f_i^F} w_i L_i\right)^{\frac{1}{\gamma^B}} \left(C_i^*\right)^{1-\sigma}. \tag{A.29}$$

A.4 Proof of Theorem 1

We begin by expressing the measure of supplier-to-buyer relationships, $M_{ud} = \varrho^E \chi_{ud}^E \zeta_u^E \xi_d^E$, and the average transaction volume per relationship, $\overline{r}_{ud} = \varrho^I \chi_{ud}^I \zeta_u^I \xi_d^I$, only in terms of $\{w_n, C_n^*\}_{n\in\mathcal{N}}$ and exogenous variables. The origin-specific shifter of the extensive margin gravity equation, ζ_i^E, is obtained by plugging advertisement cost e_i (equation 17), entry N_i (equation 18), and the expression for $\overline{C}_u$ (equation A.13) into equation (A.20) as:

$$\zeta_i^E = \left[\left(\frac{1}{\delta_1 \tilde{\sigma} \tilde{\beta}} \frac{L_i}{F_i} \mathbb{M}_i \left(\frac{\delta_1}{\gamma^B} \right) \right)^{\lambda^B \frac{\gamma^B - 1}{\gamma^B}} \left\{ A_i^{-1} w_i^{-\mu} C_i^{*-(1-\mu)} C_i^{*1-\sigma} \right\}^{\tilde{\lambda}^B} \left(\tilde{\sigma}^{1-\sigma} C_i^{*1-\sigma} \frac{\mathbb{M}_i(\delta_1)}{\mathbb{M}_i\left(\frac{\delta_1}{\gamma^B}\right)} \right)^{\tilde{\lambda}^S} \right]^{\delta_2}$$

$$= K_i^{\zeta,E} L_i^{\lambda^B \frac{\gamma^B - 1}{\gamma^B} \delta_2} w_i^{-\tilde{\lambda}^B \delta_2 \mu} \left(C_i^* \right)^{-(\sigma-1)\left[\tilde{\lambda}^B + \tilde{\lambda}^S\right]\delta_2 - \tilde{\lambda}^B \delta_2 (1-\mu)}, \tag{A.30}$$

where $K_i^{\zeta,E}$ is a combination of exogenous parameters. Together with the intensive margin gravity equation, the origin-specific shifter of the total margin gravity equation, $\zeta_i^E \zeta_i^I$, is derived from equations (A.30) and (A.22) as

$$\zeta_i^E \zeta_i^I = K_i^{\zeta,E} L_i^{\lambda^B \frac{\gamma^B - 1}{\gamma^B} \delta_2} w_i^{-\tilde{\lambda}^B \delta_2 \mu} \left(C_i^* \right)^{-(\sigma-1)\left[\tilde{\lambda}^B + \tilde{\lambda}^S\right]\delta_2 - \tilde{\lambda}^B \delta_2 (1-\mu)} C_i^{*1-\sigma} \frac{\mathbb{M}_i(\delta_1)}{\mathbb{M}_i\left(\frac{\delta_1}{\gamma^B}\right)}$$

$$= K_i^{\zeta} L_i^{\lambda^B \frac{\gamma^B - 1}{\gamma^B} \delta_2} w_i^{-\tilde{\lambda}^B \delta_2 \mu} \left(C_i^* \right)^{-(\sigma-1)\delta_2 - \tilde{\lambda}^B \delta_2 (1-\mu)}, \tag{A.31}$$

where K_i^{ζ} is a combination of exogenous parameters and we used that $\left(\tilde{\lambda}^B + \tilde{\lambda}^S\right)\delta_2 + 1 = \delta_2$.

The origin-specific shifter of the extensive margin gravity equation, ξ_d^E, is similarly obtained by plugging equations (17), (18), the expression for D_d^* (equation A.26) and D_d (equation A.27) into equation (A.21) as

$$\xi_d^E = \left[\left(N_d \mathbb{M}_d \left(\frac{\delta_1}{\gamma^S} \right) \right)^{\lambda^S \frac{\gamma^S - 1}{\gamma^S}} \left(D_d \right)^{\tilde{\lambda}^B} \left\{ D_d^* \left(A_d \left(w_d \right)^{\mu} \left(C_d^* \right)^{1-\mu} \right)^{-1} w_d^{\frac{\beta(1-\sigma)}{1-\beta}} \left(C_d^* \right)^{-\frac{\beta(1-\sigma)}{1-\beta}} \right\}^{\tilde{\lambda}^S} \right]^{\delta_2}$$

$$= \left(N_d \mathbb{M}_d \left(\frac{\delta_1}{\gamma^S} \right) \right)^{\lambda^S \frac{\gamma^S - 1}{\gamma^S} \delta_2} A_d^{-\tilde{\lambda}^S \delta_2} w_d^{\tilde{\lambda}^S \delta_2 \left(\frac{\beta(1-\sigma)}{1-\beta} - \mu \right)} \left(D_d \right)^{\tilde{\lambda}^B \delta_2} \left\{ D_d^* \right\}^{\tilde{\lambda}^S \delta_2} \left(C_d^* \right)^{\tilde{\lambda}^S \delta_2 \left(-\frac{\beta(1-\sigma)}{1-\beta} - (1-\mu) \right)}$$

$$= \left(N_d \mathbb{M}_d \left(\frac{\delta_1}{\gamma^S} \right) \right)^{\lambda^S \frac{\gamma^S - 1}{\gamma^S} \delta_2} A_d^{-\tilde{\lambda}^S \delta_2} w_d^{\tilde{\lambda}^S \delta_2 \left(\frac{\beta(1-\sigma)}{1-\beta} - \mu \right)} \left(\frac{1 - \tilde{\beta}}{\tilde{\beta}} \frac{L_d}{N_d \mathbb{M}_d \left(\frac{\delta_1}{\gamma^S} \right)} \left(w_d \right)^{\frac{1-\beta\sigma}{1-\beta}} \left(C_d^* \right)^{\frac{\sigma-1}{1-\beta}} \right)^{\tilde{\lambda}^B \delta_2}$$

$$\times \left\{ \frac{(\tilde{\sigma})^{1-\sigma}}{\tilde{\beta}} \frac{w_d L_d}{\left(C_d^* \right)^{1-\sigma} N_d} \frac{1}{\mathbb{M}_d(\delta_1)} \right\}^{\tilde{\lambda}^S \delta_2} \left(C_d^* \right)^{\tilde{\lambda}^S \delta_2 \left(-\frac{\beta(1-\sigma)}{1-\beta} - (1-\mu) \right)}$$

$$= K_d^{\xi,E} \left(L_d \right)^{\lambda^S \frac{\gamma^S - 1}{\gamma^S} \delta_2} \left(C_d^* \right)^{\frac{\sigma-1}{1-\beta}\left(\tilde{\lambda}^B + \tilde{\lambda}^S \right)\delta_2 - (1-\mu)\tilde{\lambda}^S \delta_2} \left(w_d \right)^{\tilde{\lambda}^S \delta_2 \left(\frac{\beta(1-\sigma)}{1-\beta} - \mu \right) + \frac{1-\beta\sigma}{1-\beta} \tilde{\lambda}^B \delta_2 + \tilde{\lambda}^S \delta_2}, \tag{A.32}$$

where $K_d^{\xi,E}$ is a combination of exogenous parameters. Similarly, the destination-specific shifter of the total margin gravity equation, $\xi_d^E \xi_d^I$, is derived from equations (A.32) and (A.23) as

$$\xi_d^E \xi_d^I = K_d^{\xi,E} (L_d)^{\lambda^S \frac{\gamma^S - 1}{\gamma^S} \delta_2} (C_d^*)^{\frac{\sigma-1}{1-\beta}(\tilde{\lambda}^B + \tilde{\lambda}^S)\delta_2 - (1-\mu)\tilde{\lambda}^S \delta_2} (w_d)^{\tilde{\lambda}^S \delta_2 \left(\frac{\beta(1-\sigma)}{1-\beta} - \mu\right) + \frac{1-\beta\sigma}{1-\beta}\tilde{\lambda}^B \delta_2 + \tilde{\lambda}^S \delta_2}$$
$$\times \frac{1-\tilde{\beta}}{\tilde{\beta}} \frac{L_d}{N_d \mathbb{M}_d \left(\frac{\delta_1}{\gamma^S}\right)} (w_d)^{\frac{1-\beta\sigma}{1-\beta}} (C_d^*)^{\frac{\sigma-1}{1-\beta}}$$
$$= K_d^{\xi} (L_d)^{\lambda^S \frac{\gamma^S - 1}{\gamma^S} \delta_2} (C_d^*)^{\frac{\sigma-1}{1-\beta}\delta_2 - (1-\mu)\tilde{\lambda}^S \delta_2} (w_d)^{\delta_G}, \tag{A.33}$$

where we define $\delta_G = \tilde{\lambda}^S \delta_2 \left(-\mu + \frac{1-\beta\sigma}{1-\beta}\right) + \left(\tilde{\lambda}^B \delta_2 + 1\right) \frac{1-\beta\sigma}{1-\beta} = -\tilde{\lambda}^S \delta_2 \mu + \frac{1-\beta\sigma}{1-\beta}\delta_2$ as defined in Theorem 1 (note again that $\left(\tilde{\lambda}^B + \tilde{\lambda}^S\right)\delta_2 + 1 = \delta_2$) and K_d^{ζ} is a combination of exogenous parameters.

By plugging equations (A.31) and (A.33) into wage equation (19),

$$w_i = \frac{\tilde{\beta}}{1-\tilde{\beta}} \frac{1}{L_i} \zeta_i^E \zeta_i^I \sum_d \chi_{id}^E \chi_{id}^I \xi_d^E \xi_d^I$$
$$= \frac{1}{L_i} w_i^{-\tilde{\lambda}^B \delta_2 \mu} (C_i^*)^{-(\sigma-1)\delta_2 - \tilde{\lambda}^B \delta_2 (1-\mu)} \sum_d K_{id} (C_d^*)^{\frac{(\sigma-1)\delta_2}{1-\beta} - (1-\mu)\tilde{\lambda}^S \delta_2} (w_d)^{\delta_G},$$

which correspond to equation (22) of Theorem 1, where we define

$$K_{id} \equiv \frac{\tilde{\beta}}{1-\tilde{\beta}} \chi_{id}^E \chi_{id}^I K_i^{\zeta} K_d^{\xi} L_i^{\lambda^B \frac{\gamma^B - 1}{\gamma^B} \delta_2} L_d^{\lambda^S \frac{\gamma^S - 1}{\gamma^S} \delta_2}. \tag{A.34}$$

Similarly, using wage equation (19) and the trade balancing condition (20), we have

$$w_i = \frac{\tilde{\beta}}{1-\tilde{\beta}} \frac{1}{L_i} \xi_i^E \xi_i^I \sum_u \zeta_u^E \zeta_u^I \chi_{ui}^E \chi_{ui}^I$$
$$= \frac{1}{L_i} (C_i^*)^{\frac{(\sigma-1)\delta_2}{1-\beta} - (1-\mu)\tilde{\lambda}^S \delta_2} (w_i)^{\delta_G} \sum_u K_{ui} w_u^{-\tilde{\lambda}^B \delta_2 \mu} (C_u^*)^{-(\sigma-1)\delta_2 - \tilde{\lambda}^B \delta_2 (1-\mu)},$$

which corresponds to equation (22) of Theorem 1.

A.5 Proof of Proposition 2

We take advantage of Allen, Arkolakis, and Li (2020) and express the system in terms of their notation. Notice that the matrices $K_{id}^D, K_{id}^U > 0$. Define the matrices

$$\Gamma = \begin{bmatrix} 1 + \tilde{\lambda}^B \delta_2 \mu & (\sigma - 1)\delta_2 + \tilde{\lambda}^B \delta_2 (1-\mu) \\ 1 - \delta_G & -\frac{(\sigma-1)\delta_2}{1-\beta} + \tilde{\lambda}^S \delta_2 (1-\mu) \end{bmatrix} = \begin{bmatrix} 1 + c_1 & c_2 \\ 1 - \delta_G & -c_3 \end{bmatrix}$$

and

$$B=\begin{bmatrix}\delta_G & \frac{(\sigma-1)\delta_2}{1-\beta}-\tilde{\lambda}^S\delta_2(1-\mu)\\ -\tilde{\lambda}^B\delta_2\mu & -(\sigma-1)\delta_2-\tilde{\lambda}^B\delta_2(1-\mu)\end{bmatrix}=\begin{bmatrix}\delta_G & c_3\\ -c_1 & -c_2\end{bmatrix}$$

where

$$c_1=\tilde{\lambda}^B\delta_2\mu$$
$$c_2=(\sigma-1)\delta_2+\tilde{\lambda}^B\delta_2(1-\mu)$$
$$c_3=\frac{(\sigma-1)\delta_2}{1-\beta}-\tilde{\lambda}^S\delta_2(1-\mu)$$

where $c_1>0$ and $c_2>0$ under our model parameter assumptions. A sufficient condition for the equilibrium uniqueness is that the spectral radius of $A=|B\Gamma^{-1}|$ is equal to 1, where

$$\begin{aligned}B\Gamma^{-1}&=\frac{1}{-c_3(1+c_1)-c_2(1-\delta_G)}\begin{bmatrix}\delta_G & c_3\\ -c_1 & -c_2\end{bmatrix}\begin{bmatrix}-c_3 & -c_2\\ -(1-\delta_G) & 1+c_1\end{bmatrix}\\ &=\frac{1}{-c_3(1+c_1)-c_2(1-\delta_G)}\begin{bmatrix}-c_3 & -\delta_G c_2+c_3(1+c_1)\\ c_1c_3+(1-\delta_G)c_2 & -c_2\end{bmatrix}\end{aligned}$$

We now show that, when $\delta_G\leq 1$ and $\frac{\beta(\sigma-1)}{1-\beta}>(1-\mu)\left(\tilde{\lambda}^B+\tilde{\lambda}^S\right)$ as assumed in Proposition 2, the largest eigenvalue of $|B\Gamma^{-1}|$ is indeed less than one. From the second condition, we have $c_3>0$ and $c_3>c_2$. Furthermore, $-\delta_G c_2+c_3(1+c_1)\geq c_1c_3+(1-\delta_G)c_2>0$. Therefore,

$$|B\Gamma^{-1}|=\frac{1}{c_3(1+c_1)+c_2(1-\delta_G)}\begin{bmatrix}c_3 & -\delta_G c_2+c_3(1+c_1)\\ c_1c_3+(1-\delta_G)c_2 & c_2\end{bmatrix}$$

Note that the sum of the rows for the first column and second column are both one. Therefore, from Collatz–Wielandt Formula (see Allen, Arkolakis, and Li (2020)), the largest eigenvalue of $|B\Gamma^{-1}|$ is one under this condition. Therefore, when $\delta_G<1$ and $\frac{\beta(\sigma-1)}{1-\beta}>(1-\mu)\left(\tilde{\lambda}^B+\tilde{\lambda}^S\right)$, the equilibrium exists and it is unique up to scale.

A.6 Exact-Hat Algebra for Counterfactuals

We prove the following statement:

Proposition 6. *Given the set of structural parameters $\{\sigma,\beta,\mu,\tilde{\lambda}^B,\tilde{\lambda}^S\}$ and the observed aggregate trade flows, $\{X_{id}\}$, the counterfactual changes of wages $\{\hat{w}_i\}$ and intermediate costs $\{\hat{C}_i^*\}$ induced by exogenous shocks $\{\hat{K}_{id}\}$ are obtained by solving the following system of equations:*

$$(\hat{w}_i)^{1+\tilde{\lambda}^B\delta_2\mu}\left(\hat{C}_i^*\right)^{(\sigma-1)\delta_2+\tilde{\lambda}^B\delta_2(1-\mu)}=\sum_d\hat{K}_{id}(\hat{w}_d)^{\delta_G}\left(\hat{C}_d^*\right)^{\frac{(\sigma-1)\delta_2}{1-\beta}-\tilde{\lambda}^S\delta_2(1-\mu)}\Psi_{id},\tag{A.35}$$

$$(\hat{w}_i)^{1-\delta_G}\left(\hat{C}_i^*\right)^{-\frac{(\sigma-1)\delta_2}{1-\beta}+\tilde{\lambda}^S\delta_2(1-\mu)}=\sum_u\hat{K}_{ui}(\hat{w}_u)^{-\tilde{\lambda}^B\delta_2\mu}\left(\hat{C}_u^*\right)^{-(\sigma-1)\delta_2-\tilde{\lambda}^B\delta_2(1-\mu)}\Lambda_{ui},\tag{A.36}$$

where we define $\Psi_{id} \equiv X_{id} / \left(\sum_\ell X_{i\ell}\right)$ *and* $\Lambda_{ui} \equiv X_{ui} / \left(\sum_\ell X_{\ell i}\right)$.

Proof. Following similar manipulations as in Appendix A.4, we have

$$\Psi_{id} = \frac{X_{id}}{\sum_\ell X_{i\ell}} = \frac{\chi^E_{id}\chi^I_{id}\xi^E_d\xi^I_d}{\sum_\ell \chi^E_{i\ell}\chi^I_{i\ell}\xi^E_\ell\xi^I_\ell} = \frac{K_{id}\left(w_d\right)^{\delta_G}\left(C^*_d\right)^{\frac{(\sigma-1)\delta_2}{1-\beta}-\tilde{\lambda}^S\delta_2(1-\mu)}}{\sum_\ell K_{i\ell}\left(w_\ell\right)^{\delta_G}\left(C^*_\ell\right)^{\frac{(\sigma-1)\delta_2}{1-\beta}-\tilde{\lambda}^S\delta_2(1-\mu)}},$$

$$\Lambda_{ui} = \frac{X_{ui}}{\sum_\ell X_{\ell i}} = \frac{\zeta^E_u\zeta^I_u\chi^E_{ui}\chi^I_{ui}}{\sum_\ell \zeta^E_\ell\zeta^I_\ell\chi^E_{\ell i}\chi^I_{\ell i}} = \frac{K_{ui}\left(w_u\right)^{-\tilde{\lambda}^B\delta_2\mu}\left(C^*_u\right)^{-(\sigma-1)\delta_2-\tilde{\lambda}^B\delta_2(1-\mu)}}{\sum_\ell K_{\ell i}\left(w_\ell\right)^{-\tilde{\lambda}^B\delta_2\mu}\left(C^*_\ell\right)^{-(\sigma-1)\delta_2-\tilde{\lambda}^B\delta_2(1-\mu)}}.$$

Now, by denoting the variable x in the new equilibrium by x' (with a prime) and the ratio change of x as $\hat{x} = x/x'$, we can rearrange equation (22) as

$$\begin{aligned}
\left(\hat{w}_i\right)^{1+\tilde{\lambda}^B\delta_2}\left(\hat{C}^*_i\right)^{(\sigma-1)\delta_2+\tilde{\lambda}^B\delta_2(1-\mu)} &= \frac{\sum_d K'_{id}\left(w'_d\right)^{\delta_G}\left(C^{*\prime}_d\right)^{\frac{(\sigma-1)\delta_2}{1-\beta}-\tilde{\lambda}^S\delta_2(1-\mu)}}{\sum_d K_{id}\left(w_d\right)^{\delta_G}\left(C^*_d\right)^{\frac{(\sigma-1)\delta_2}{1-\beta}-\tilde{\lambda}^S\delta_2(1-\mu)}} \\
&= \sum_d K'_{id}\left(w'_d\right)^{\delta_G}\left(C^{*\prime}_d\right)^{\frac{(\sigma-1)\delta_2}{1-\beta}-\tilde{\lambda}^S\delta_2(1-\mu)} \frac{\Psi_{id}}{K_{id}\left(w_d\right)^{\delta_G}\left(C^*_d\right)^{\frac{(\sigma-1)\delta_2}{1-\beta}-\tilde{\lambda}^S\delta_2(1-\mu)}} \\
&= \sum_d \hat{K}_{id}\left(\hat{w}_d\right)^{\delta_G}\left(\hat{C}^*_d\right)^{\frac{(\sigma-1)\delta_2}{1-\beta}-\tilde{\lambda}^S\delta_2(1-\mu)}\Psi_{id},
\end{aligned}$$

which corresponds to equation (A.35) of Proposition 6. Similarly, we have

$$\left(\hat{w}_i\right)^{1-\delta_G}\left(\hat{C}^*_i\right)^{-\frac{(\sigma-1)\delta_2}{1-\beta}+\tilde{\lambda}^S\delta_2(1-\mu)} = \sum_u \hat{K}_{ui}\left(\hat{w}_u\right)^{-\tilde{\lambda}^B\delta_2\mu}\left(\hat{C}^*_u\right)^{-(\sigma-1)\delta_2-\tilde{\lambda}^B\delta_2(1-\mu)}\Lambda_{ui},$$

which corresponds to equation (A.36) of Proposition 6. □

A.7 Isomorphism to Gravity Trade Models when $\tilde{\lambda}^S = \tilde{\lambda}^B = 0$

In this section, we discuss that our model becomes isomorphic to canonical gravity trade models in the literature when we set $\tilde{\lambda}^S = \tilde{\lambda}^B = 0$. Under these parameter values, we have $\delta_2 = 1$ and $\delta_G = \frac{1-\beta\sigma}{1-\beta}$, and the equilibrium conditions (22) and (23) come down to the following set of equations:

$$\left(w_i\right)\left(C^*_i\right)^{(\sigma-1)} = \sum_d K^D_{id}\left(w_d\right)^{-\frac{\beta\sigma-1}{1-\beta}}\left(C^*_d\right)^{\frac{\sigma-1}{1-\beta}}, \tag{A.37}$$

$$\left(w_i\right)^{1+\frac{\beta\sigma-1}{1-\beta}}\left(C^*_i\right)^{-\frac{\sigma-1}{1-\beta}} = \sum_u K^U_{ui}\left(C^*_u\right)^{-(\sigma-1)}, \tag{A.38}$$

where $K^D_{id} = \frac{1}{L_i}K_{id}$ and $K^D_{ui} = \frac{1}{L_i}K_{ui}$.

To see the isomorphism to canonical gravity trade models more closely, we redefine the cost shifter $\tilde{C}_i$ such that

$$\tilde{C}_i = \left(C_i^*\right)^{\frac{1}{1-\beta}} w_i^{-\frac{\beta}{1-\beta}}.$$

Using the newly defined $\tilde{C}_i$, the first equation (A.37) is rewritten as

$$\left(w_i\right)^{1+\beta(\sigma-1)} \tilde{C}_i^{(1-\beta)(\sigma-1)} = \sum_d K_{id}^D \left(w_d\right)^{-\frac{\beta\sigma-1}{1-\beta}+\frac{\beta(\sigma-1)}{1-\beta}} \left(\tilde{C}_d\right)^{\sigma-1} \iff$$

$$\left(w_i\right)^{1+\beta(\sigma-1)} \tilde{C}_i^{(1-\beta)(\sigma-1)} = \sum_d K_{id}^D w_d \left(\tilde{C}_d\right)^{\sigma-1},$$

and the second equation (A.38) is rewritten as

$$\left(w_i\right)^{1+\frac{\beta\sigma-1}{1-\beta}-\frac{\sigma-1}{1-\beta}\beta} \left(\tilde{C}_i\right)^{-(\sigma-1)} = \sum_u K_{ui}^U w_i^{-\beta(\sigma-1)} \tilde{C}_i^{-(1-\beta)(\sigma-1)} \iff$$

$$\left(\tilde{C}_i\right)^{-(\sigma-1)} = \sum_u K_{ui}^U w_i^{-\beta(\sigma-1)} \tilde{C}_i^{-(1-\beta)(\sigma-1)}$$

The first and second equations correspond to equation (3.10, 3.14) and (3.8) in Alvarez and Lucas (2007) with $\theta = 1/(\sigma-1)$ without taxes, respectively. Furthermore, the first and second equations correspond to (45) and (41-45) in Eaton, Kortum, and Kramarz (2011) with $\theta = \sigma - 1$ without taxes, respectively.

A.8 Proof of Proposition 3

We first characterize the changes of cost shifter $\hat{C}_i^*$. The share of intermediate goods expenditure on suppliers in location u, Λ_{ui}, is given by:

$$\Lambda_{ui} = \frac{\int n_{ui}(z)\, m_{ui}\overline{r}_{ui} dG_i(z)}{\sum_{\ell\in\mathcal{N}} \int n_{\ell i}(z)\, m_{\ell i}\overline{r}_{\ell i} dG_i(z)} = \frac{a_{ui}^S m_{ui}^S \left(\overline{C}_u \tau_{ui}\right)^{1-\sigma}}{\sum_{\ell\in\mathcal{N}} a_{\ell i}^S m_{\ell i}^S \left(\overline{C}_\ell \tau_{\ell i}\right)^{1-\sigma}}. \tag{A.39}$$

By combining this expression with the definition of C_i^* in equation (A.9), we have

$$\begin{aligned}
\left(\hat{C}_u^*\right)^{1-\sigma} &= \hat{w}_u^{\beta(1-\sigma)} \left(\hat{a}_{uu}^S \hat{m}_{uu}^S \left(\hat{\overline{C}}_u\right)^{1-\sigma} \hat{\Lambda}_{uu}^{-1}\right)^{1-\beta} \\
&= \hat{w}_u^{\beta(1-\sigma)} \left(\hat{a}_{uu}^S \hat{m}_{uu}^S \left(\hat{C}_u^*\right)^{1-\sigma} \hat{\Lambda}_{uu}^{-1}\right)^{1-\beta} \quad \text{(from Lemma 2)} \\
&= \hat{w}_u^{\beta(1-\sigma)} \left(\hat{M}_{uu} \hat{N}_u^{-1} \left(\hat{C}_u^*\right)^{1-\sigma} \hat{\Lambda}_{uu}^{-1}\right)^{1-\beta} \quad \text{(from equations 12 and 15)} \\
&= \hat{w}_u^{\beta(1-\sigma)} \left(\hat{M}_{uu} \left(\hat{C}_u^*\right)^{1-\sigma} \hat{\Lambda}_{uu}^{-1}\right)^{1-\beta} \quad \text{(from equations 18)} \\
&= \hat{w}_u^{(1-\sigma)} \left(\hat{M}_{uu}\right)^{\frac{1-\beta}{\beta}} \hat{\Lambda}_{uu}^{-\frac{1-\beta}{\beta}}.
\end{aligned} \tag{A.40}$$

Next, we characterize the change in consumer price index $\hat{P}_i^F$. From equation (A.29),

$$\left(\hat{P}_i^F\right)^{1-\sigma} = \left(\frac{\hat{w}_i}{\hat{e}_i}\right)^{\frac{1}{\gamma^B}} \left(\hat{C}_i^*\right)^{1-\sigma} = \left(\hat{w}_i/\hat{C}_i^*\right)^{\frac{1}{\gamma^B}(1-\mu)} \left(\hat{C}_i^*\right)^{1-\sigma}.$$

Therefore,

$$\widehat{\frac{w_i}{P_i^F}} = \frac{\hat{w}_i}{\left(\hat{w}_i/\hat{C}_i^*\right)^{\frac{1}{\gamma^B}\frac{1-\mu}{1-\sigma}} \left(\hat{C}_i^*\right)} = \left(\frac{\hat{w}_i}{\hat{C}_i^*}\right)^{1+\frac{1}{\gamma^B}\frac{1-\mu}{\sigma-1}}. \tag{A.41}$$

By combining equation (A.40) in this expression, we obtain equation (24) of Proposition 3.

A.9 Proof of Proposition 4

A.9.1 Part (i)

We first define "forward" Domar weights using "forward" Leontief inverse matrix and show that it is proportional to nominal trade flow. This lemma plays a crucial role in the proof of Part (i).

Definition 1. Forward Domar weights are defined by:

$$\psi_{ij} \equiv \left[\mathbf{Y^F}'\Phi\right]_j (1-\beta)\,\Lambda_{ij}, \tag{A.42}$$

where $\Lambda_{ij} = X_{ij}/\left(\sum_\ell X_{\ell j}\right)$, $\mathbf{Y^F}$ is the vector of nominal GDP of size $|\mathcal{N}|$ whose j-th component is $Y_j^F = w_j L_j$, $[\mathbf{x}]_j$ denotes the j-th element of vector $\mathbf{x}$, and Φ is the forward Leontief inverse matrix of size $|\mathcal{N}| \times |\mathcal{N}|$ defined by

$$\Phi = \left(\mathbf{I} - (1-\beta)\,\mathbf{\Lambda}'\right)^{-1}, \tag{A.43}$$

where $\mathbf{I}$ is the $|\mathcal{N}| \times |\mathcal{N}|$ identity matrix, and $\mathbf{\Lambda}$ is the $|\mathcal{N}| \times |\mathcal{N}|$ matrix whose (i,j)-th element corresponds to Λ_{ij}.

We now show that the forward Domar weights are proportional to nominal trade flows.

Lemma 3. *The forward Domar weights for location pairs i and j are defined by:*

$$\psi_{ij} = \tilde{\varsigma} X_{ij}, \tag{A.44}$$

where $\tilde{\varsigma} \equiv \frac{1-\beta}{1-\tilde{\beta}}\frac{\tilde{\beta}}{\beta} \geq 1$.

Proof. From the intermediate goods clearing condition, aggregate firm revenue in location i,

$$R_i = Y_i^F + \sum_d \Lambda_{id}\left(1-\tilde{\beta}\right)R_d$$
$$= Y_i^F + \sum_d \Lambda_{id}\left(\beta-\tilde{\beta}\right)R_d + \sum_d \Lambda_{id}\left(1-\beta\right)R_d$$
$$= Y_i^F\frac{\beta}{\tilde{\beta}} + \sum_d \Lambda_{id}\left(1-\beta\right)R_d, \tag{A.45}$$

where the last transformation uses $Y_i^F = w_iL_i = \tilde{\beta}R_i = \tilde{\beta}\sum_d \Lambda_{id}R_d$ (last equation is from trade balancing condition). Rewriting equation (A.45) in a vector form,

$$\mathbf{R} = \mathbf{Y^F}\frac{\beta}{\tilde{\beta}} + (1-\beta)\,\mathbf{\Lambda R} \iff \mathbf{R} = \mathbf{Y^F}'\Phi\frac{\beta}{\tilde{\beta}}, \tag{A.46}$$

where Φ is the forward Leontief inverse matrix defined by equation (A.43). Using this relationship,

$$\psi_{ij} \equiv \left[\mathbf{Y^F}'\Phi\right]_j (1-\beta)\,\Lambda_{ij} = (1-\beta)\,\Lambda_{ij}R_j\frac{\tilde{\beta}}{\beta} = \frac{1-\beta}{1-\tilde{\beta}}\frac{\tilde{\beta}}{\beta}X_{ij},$$

where we used $X_{ij} = \left(1-\tilde{\beta}\right)\Lambda_{ij}R_j$. $\square$

We are now ready to prove Part (i) of Proposition 4. We first note that equation (A.35) is rewritten as

$$(\hat{w}_i)^{1-\frac{1-\beta\sigma}{1-\beta}}\left(\hat{C}_i^*\right)^{-\frac{\sigma-1}{1-\beta}} = \sum_u \hat{\chi}_{ui}^I\hat{M}_{ui}\left(\hat{C}_u^*\right)^{-(\sigma-1)}\Lambda_{ui}.$$

By log-linearizing this equation, we have

$$\left(1-\frac{1-\beta\sigma}{1-\beta}\right)d\log w_i - \frac{\sigma-1}{1-\beta}d\log C_i^* = \sum_u \Lambda_{ui}\left(d\log\chi_{ui}^I + d\log M_{ui} - (\sigma-1)\,d\log C_u^*\right). \tag{A.47}$$

By rewriting this equation in vector notation,

$$-\left(\frac{\sigma-1}{1-\beta}\mathbf{I}-(\sigma-1)\,\mathbf{\Lambda}'\right)d\log\boldsymbol{C}^* = -\left(1-\frac{1-\beta\sigma}{1-\beta}\right)d\log\boldsymbol{w} + \left(\mathbf{\Lambda}\cdot\left(d\log\chi^{\mathbf{I}} + d\log\mathbf{M}\right)\right)\mathbf{1}$$

$$\iff -d\log\boldsymbol{C}^* = \left(\frac{1-\beta}{\sigma-1}\right)\left\{-\left(1-\frac{1-\beta\sigma}{1-\beta}\right)\Phi d\log\boldsymbol{w} + \Phi\left[\left(\mathbf{\Lambda}\cdot\left(d\log\chi^{\mathbf{I}} + d\log\mathbf{M}\right)\right)\mathbf{1}\right]\right\},$$

where $\mathbf{1}$ is a vector of one with $|\mathcal{N}|$ elements.

We now characterize $d\log\mathcal{W} \equiv \sum_i w_iL_i\left(d\log w_i - d\log P_i^F\right)$. The first term of $d\log\mathcal{W}$ is zero, because $\sum_i Y_i^F d\log w_i = \sum_i w_iL_i d\log w_i = 0$ (recall our normalization of $\sum_i w_iL_i = 1$).

Therefore,

$$\begin{aligned}
d\log\mathcal{W} &= -\mathbf{Y^{F}}'d\log\boldsymbol{P^{F*}} \\
&= -\left(1+\frac{1}{\gamma^B}\frac{1-\mu}{\sigma-1}\right)\mathbf{Y^{F}}'d\log\mathbf{C}^* \quad \text{(from equation A.41)} \\
&= \left(1+\frac{1}{\gamma^B}\frac{1-\mu}{\sigma-1}\right)\left(\frac{1-\beta}{\sigma-1}\right) \\
&\quad\times\left\{-\left(1-\frac{1-\beta\sigma}{1-\beta}\right)\mathbf{Y^{F}}'\Phi d\log\boldsymbol{w}+\mathbf{Y^{F}}'\Phi\left[\left(\boldsymbol{\Lambda}\cdot\left(d\log\chi^{\mathbf{I}}+d\log\mathbf{M}\right)\right)\mathbf{1}\right]\right\}.
\end{aligned}$$

The first term is zero because $\mathbf{Y^{F}}'\Phi=\frac{\tilde{\beta}}{\beta}\mathbf{R}'$ and $\mathbf{R}'d\log\boldsymbol{w}=\frac{1}{\tilde{\beta}}\sum_i Y_i^F d\log w_i=0$. Rewriting the second term using Definition 1, we have

$$d\log\mathcal{W}=\sum_{ij}\left(1+\frac{1}{\gamma^B}\frac{1-\mu}{\sigma-1}\right)\left(\frac{1}{\sigma-1}\right)\psi_{ij}\left(d\log\chi_{ij}^I+d\log M_{ij}\right).$$

Lastly, using the fact that that $d\log\chi_{ij}^I=(1-\sigma)\,d\log\tau_{ij}$ (from equation 16) and using Lemma 3 yields equation (26) of Proposition 4.

A.9.2 Part (ii)

We first define "endogenous-network" Domar weights and show that they are proportional to forward Domar weights. This lemma plays a crucial role in the proof of Part (ii).

Definition 2. "Endogenous-network" Domar weights are defined by

$$\psi_{ij}^N\equiv\left[\mathbf{Y^{F}}'\Phi^N\right]_j(1-\beta)\,\Lambda_{ij},$$

where Φ^N is the "endogenous-network" Leontief inverse matrix of size $|\mathcal{N}|\times|\mathcal{N}|$ defined by

$$\Phi^N=\frac{1}{c^S}\left(\mathbf{I}-\frac{c^B}{c^S}(1-\beta)\,\boldsymbol{\Lambda}'\right)^{-1},$$

where

$$\begin{aligned}
c^S &= 1-\tilde{\lambda}^S\frac{1-\mu}{\sigma-1}(1-\beta) \\
c^B &= 1+\tilde{\lambda}^B\frac{1-\mu}{\sigma-1}.
\end{aligned}$$

The following lemma shows that endogenous-network Domar weights are proportional to forward Domar weights.

Lemma 4. *"Endogenous network" Domar weights are proportional to "forward" Domar weights, i.e.,*

$$\psi_{ij}^{N} = \frac{1}{1 - \left(\tilde{\lambda}^S + \tilde{\lambda}^B\right) \frac{1-\beta}{\beta} \frac{1-\mu}{\sigma-1}} \psi_{ij}.$$

Proof. By rewriting the intermediate goods clearing condition (A.45) similarly in the Proof of Lemma 3,

$$\begin{aligned}
R_i &= Y_i^F + \sum_d \Lambda_{id} \left(1 - \tilde{\beta}\right) R_d \\
&= Y_i^F + \sum_d \Lambda_{id} \left(\left(1 - \tilde{\beta}\right) - \frac{c^B}{c^S} \left(1 - \beta\right)\right) R_d + \sum_d \Lambda_{id} \frac{c^B}{c^S} \left(1 - \beta\right) R_d \\
&= Y_i^F \left(1 + \frac{\left(1 - \tilde{\beta}\right) - \frac{c^B}{c^S} \left(1 - \beta\right)}{\tilde{\beta}}\right) + \sum_d \Lambda_{id} \frac{c^B}{c^S} \left(1 - \beta\right) R_d \\
&= Y_i^F \left(\frac{1 - \frac{c^B}{c^S} \left(1 - \beta\right)}{\tilde{\beta}}\right) + \sum_d \Lambda_{id} \frac{c^B}{c^S} \left(1 - \beta\right) R_d.
\end{aligned}$$

Writing this equation in a vector form,

$$\mathbf{R} = \mathbf{Y^{F'}} \Phi^N \left(\frac{c^S - c^B \left(1 - \beta\right)}{\tilde{\beta}}\right) = \mathbf{Y^{F'}} \Phi^N \left(1 - \left(\tilde{\lambda}^S + \tilde{\lambda}^B\right) \frac{1 - \mu}{\sigma - 1} \left(\frac{1 - \beta}{\beta}\right)\right) \frac{\beta}{\tilde{\beta}}. \tag{A.48}$$

Then, endogenous network Domar weights are given by:

$$\begin{aligned}
\psi_{ij}^N &\equiv \left[\mathbf{Y^{F'}} \Phi^N\right]_j \left(1 - \beta\right) \Lambda_{ij} \qquad \text{(A.49)} \\
&= \left(\frac{1}{1 - \left(\tilde{\lambda}^S + \tilde{\lambda}^B\right) \frac{1-\mu}{\sigma-1} \left(\frac{1-\beta}{\beta}\right)}\right) \frac{\tilde{\beta}}{\beta} \left(1 - \beta\right) R_j \Lambda_{ij} \\
&= \frac{1}{1 - \left(\tilde{\lambda}^S + \tilde{\lambda}^B\right) \frac{1-\mu}{\sigma-1} \left(\frac{1-\beta}{\beta}\right)} \psi_{ij},
\end{aligned}$$

where the last transformation uses Lemma A.45 and that $X_{ij} = \left(1 - \tilde{\beta}\right) R_j \Lambda_{ij}$. □

We are now ready to prove Part (ii) of Proposition 4. We first reproduce equation (A.36) as

$$\left(\hat{w}_i\right)^{1-\delta_G} \left(\hat{C}_i^*\right)^{-\frac{(\sigma-1)\delta_2}{1-\beta} + \tilde{\lambda}^S \delta_2 (1-\mu)} = \sum_u \hat{K}_{ui} \left(\hat{w}_u\right)^{-\tilde{\lambda}^B \delta_2 \mu} \left(\hat{C}_u^*\right)^{-(\sigma-1)\delta_2 - \tilde{\lambda}^B \delta_2 (1-\mu)} \Lambda_{ui},$$

where $\hat{K}_{ui} = \hat{\tau}_{ui}^{(1-\sigma)\left(1+\delta_2\left(\tilde{\lambda}^S + \tilde{\lambda}^B\right)\right)} = \hat{\tau}_{ui}^{(1-\sigma)\delta_2}$. By log-linearizing this equation and dividing both hand sides by δ_2, we have

$$\frac{1-\delta_G}{\delta_2} d\log w_i - c^S \frac{\sigma-1}{1-\beta} d\log C_i^* = \sum_u \Lambda_{ui} \left((1-\sigma)\, d\log\tau_{ui} - \tilde{\lambda}^B \mu d\log w_u - c^B (\sigma-1)\, d\log C_u^* \right), \tag{A.50}$$

By rewriting this equation in matrix form,

$$\left(c^S \frac{\sigma-1}{1-\beta} \mathbf{I} - c^B (\sigma-1)\, \mathbf{\Lambda} \right) d\log \boldsymbol{C}^* = \left(\frac{1-\delta_G}{\delta_2} \mathbf{I} + \tilde{\lambda}^B \mu \mathbf{\Lambda} \right) d\log \boldsymbol{w} + (\mathbf{\Lambda} \cdot (\sigma-1)\,(d\log\tau))\, \mathbf{1}$$

$$\iff d\log \boldsymbol{C}^* = \left(\frac{1-\beta}{\sigma-1} \right) \left\{ \Phi^N \left(\frac{1-\delta_G}{\delta_2} \mathbf{I} + \tilde{\lambda}^B \mu \mathbf{\Lambda} \right) d\log \boldsymbol{w} + \Phi^N (\mathbf{\Lambda} \cdot (\sigma-1)\,(d\log\tau))\, \mathbf{1} \right\} \tag{A.51}$$

and $d\log \boldsymbol{P}^{\boldsymbol{F}} = \left(1 + \frac{1}{\gamma^B} \frac{1-\mu}{\sigma-1}\right) d\log \boldsymbol{C}^*$.

The final step of the proof is to apply these formulas to derive changes in aggregate welfare,

$$d\log \mathcal{W} = -\mathbf{Y}^{\mathbf{F}'} d\log \boldsymbol{P}^{\boldsymbol{F}}.$$

Notice that multiplying $\mathbf{Y}^{\mathbf{F}'}$ with the first term of the RHS of equation $d\log \boldsymbol{C}^*$ implies

$$\begin{aligned} \mathbf{Y}^{\mathbf{F}'} \Phi^N \left(\frac{1-\delta_G}{\delta_2} \mathbf{I} + \tilde{\lambda}^B \mu \mathbf{\Lambda} \right) d\log \boldsymbol{w} &= \frac{\tilde{\beta}}{\beta} \varsigma^N \mathbf{R}' \left(\frac{1-\delta_G}{\delta_2} \mathbf{I} + \tilde{\lambda}^B \mu \mathbf{\Lambda} \right) d\log \boldsymbol{w} \\ &= \frac{\tilde{\beta}}{\beta} \varsigma^N \left(\frac{1-\delta_G}{\delta_2} \mathbf{I} + \tilde{\lambda}^B \mu \mathbf{\Lambda} \right)' \mathbf{R} d\log \boldsymbol{w} \\ &= 0, \end{aligned}$$

where $\varsigma^N \equiv 1/\left(1 - \left(\tilde{\lambda}^S + \tilde{\lambda}^B\right) \frac{1-\mu}{\sigma-1} \left(\frac{1-\beta}{\beta}\right)\right)$. Therefore, we can ignore that term and express welfare as

$$\begin{aligned} d\log \mathcal{W} &= -\left(1 + \frac{1}{\gamma^B} \frac{1-\mu}{\sigma-1}\right) \left(\frac{1-\beta}{\sigma-1}\right) \mathbf{Y}^{\mathbf{F}'} \Phi^N (\mathbf{\Lambda} \cdot (\sigma-1)\,(d\log\tau))\, \mathbf{1} \\ &= -\left(1 + \frac{1}{\gamma^B} \frac{1-\mu}{\sigma-1}\right) \varsigma^N \sum_{ij} \psi_{ij} d\log\tau_{ij}, \end{aligned}$$

where the last transformation uses Lemma 4. We complete the proof of the statement by combining this expression with Part (i) of Proposition 4.

A.10 Proof of Proposition 5

From Proposition 4, the first-order effects of the shock to iceberg trade costs between a particular location pair (i and j) is given by

$$\frac{d\log \mathcal{W}}{d\log \tau_{ij}} = -\frac{\varsigma}{1 - \left(\tilde{\lambda}^S + \tilde{\lambda}^B\right) \frac{1-\beta}{\beta} \frac{1-\mu}{\sigma-1}} X_{ij}.$$

Totally differentiating this expression by $\log \tau_{ij}$ yields the desired expression.

B Multiple Sector Model

In this appendix, we extend our model to incorporate multiple sectors.

B.1 Model Set-up

We assume that intermediate goods belong to distinct sectors denoted by $k, h, l \in K$. Each firm produces a distinct variety within a sector. The production function takes the Cobb-Douglas form using labor and the set of intermediate goods. The unit cost of production for each firm ω in sector k and location i is given by

$$c_{i,k}(\omega) = \frac{1}{z_{i,k}(\omega)} w_i^{\beta_{k,L}} \prod_{h \in K} \left(\int_{\upsilon \in \mathcal{S}_{hk}(\omega)} p(\upsilon, \omega)^{1-\sigma_h} d\upsilon \right)^{\frac{\beta_{hk}}{1-\sigma_h}}, \tag{B.1}$$

where $z_{i,k}(\omega)$ is firm ω's productivity; w_i is the wage at firm ω's production location; $\mathcal{S}_{hk}(\omega)$ is the set of intermediate goods producers in sector h that firm ω in sector k has access to; $p(\upsilon, \omega)$ is the intermediate goods price that supplier υ charges to firm ω (net of iceberg trade cost); $\beta_{k,L}$ is the share of labor input; β_{hk} is the input share of sector h inputs for sector k production; and σ_h is the elasticity of substitution across different intermediate goods ($\sigma_h > 1$). We assume that the production technology is constant returns to scale such that $\beta_{k,L} + \sum_{h \in K} \beta_{hk} = 1$. The set of intermediate goods producers $S_{hk}(\omega)$ is endogenously determined in the equilibrium through search and matching as described below.

Firms' search problem succeeds the basic structure of the single sector model in our main paper, except that firms determine their optimal search intensity for each supplier and buyer *sector* on top of supplier and buyer *location*. More specifically, we denote the expected revenue from a matched buyer in location d and sector l by a firm in location i and sector s with marginal production cost c as follows:

$$r_{d,kl}(c\tau_{id,kl}) = (\tilde{\sigma}_k c \tau_{id,kl})^{1-\sigma_k} D_{d,kl}, \tag{B.2}$$

where $D_{d,kl}$ is the demand shifter for intermediate goods for firms in sector l and location d to firms in sector k, $\tau_{id,kl}$ is the iceberg trade cost from location i, sector k to location d, sector l, and $\tilde{\sigma}_k = \sigma_k/(\sigma_k - 1)$.

Given this notation, firms' search decision for buyers, $\{n^S_{ui,hk}\}_{u \in \mathcal{N}, h \in K}$, and suppliers, $\{n^B_{id,kl}\}_{d \in \mathcal{N}, l \in K}$, is represented by:

$$\begin{aligned}
\pi_{i,k}\left(z\right) = & \max_{\{n^S_{ui,hk}\}_{u\in\mathcal{N},h\in K},\{n^B_{id,kl}\}_{d\in\mathcal{N},l\in K},n^F_{i,k}} \\
& \frac{\tilde{\sigma}_k^{1-\sigma_k}}{\sigma_k} n^F_{i,k} D^F_{i,k}\left(c\right)^{1-\sigma_k} + \frac{\tilde{\sigma}_k^{1-\sigma_k}}{\sigma_k}\sum_{l\in K}\sum_{d\in\mathcal{N}} m^B_{id,kl} n^B_{id,kl} D_{d,kl}\left(c\tau_{id,kl}\right)^{1-\sigma_k} \\
& - e_{i,k}\left\{ f^F_{i,k}\frac{\left(n^F_{i,k}\right)^{\gamma^B_k}}{\gamma^B_k} + \sum_{l\in K}\sum_{d\in\mathcal{N}} f^B_{id,kl}\frac{\left(n^B_{id,kl}\right)^{\gamma^B_k}}{\gamma^B_k} + \sum_{h\in K}\sum_{u\in\mathcal{N}} f^S_{ui,hk}\frac{\left(n^S_{ui,hk}\right)^{\gamma^S_k}}{\gamma^S_k}\right\} \\
\text{subject to}\quad c = & \frac{w_i^{\beta_{k,L}}\prod_h\left(\sum_{u\in\mathcal{N}} n^S_{ui,hk} m^S_{ui,hk}\left(C_{ui,hk}\right)^{1-\sigma_h}\right)^{\frac{\beta_{hk}}{1-\sigma_h}}}{z}
\end{aligned} \tag{B.3}$$

This problem extends equation (8) allowing for firms' search for each supplier and buyer sectors.

We impose a parameter restriction that $1-\frac{1}{\gamma^B_k}-\frac{1}{\gamma^S_k}\sum_h\beta_{hk}\frac{1-\sigma_k}{1-\sigma_h}>0$, which guarantees that firms make positive sales and profit. The first-order conditions of these equations are:

$$\begin{aligned}
e_{i,k} f^B_{id,kl}\left(n^B_{id,kl}\right)^{\gamma^B_k-1} = & \frac{\tilde{\sigma}_k^{1-\sigma_k}}{\sigma_k} m^B_{id,kl} D_{d,kl}\left(\tau_{id,kl}\right)^{1-\sigma_k} \\
& \times\frac{w_i^{\beta_{k,L}(1-\sigma_k)}\prod_h\left(\sum_{u\in\mathcal{N}} n^S_{ui,hk} m^S_{ui,hk}\left(C_{ui,hk}\right)^{1-\sigma_h}\right)^{\frac{\beta_{hk}}{1-\sigma_h}(1-\sigma_k)}}{z^{1-\sigma_k}}
\end{aligned} \tag{B.4}$$

$$e_{i,k} f^F_{i,k}\left(n^F_{i,k}\right)^{\gamma^B_k-1} = \frac{\tilde{\sigma}_k^{1-\sigma_k}}{\sigma_k} D^F_{i,k}\frac{w_i^{\beta_{k,L}(1-\sigma_k)}\prod_h\left(\sum_{u\in\mathcal{N}} n^S_{ui,hk} m^S_{ui,hk}\left(C_{ui,hk}\right)^{1-\sigma_h}\right)^{\frac{\beta_{hk}}{1-\sigma_h}(1-\sigma_k)}}{z^{1-\sigma_k}} \tag{B.5}$$

$$\begin{aligned}
e_{i,k} f^S_{ui,hk}\left(n^S_{ui,hk}\right)^{\gamma^S_k-1} = & \frac{\tilde{\sigma}_k^{1-\sigma_k}}{\sigma_k}\left\{ n^F_{i,k} D^F_{i,k}\left(c\right)^{1-\sigma_k} + \sum_{l\in K}\sum_{d\in\mathcal{N}} n^B_{id,kl} m^B_{id,kl} D_{d,kl}\left(\tau_{id,k}\right)^{1-\sigma}\right\}\frac{1-\sigma_k}{1-\sigma_h}\beta_{hk} \\
& \times\frac{w_i^{\beta_{hk}(1-\sigma_k)}\prod_h\left(\sum_{u\in\mathcal{N}} n^S_{ui,hk} m^S_{ui,hk}\left(C_{ui,hk}\right)^{1-\sigma_h}\right)^{\frac{\beta_{hk}}{1-\sigma_h}(1-\sigma_k)}}{z^{1-\sigma_k}} \\
& \times\frac{m^S_{ui,hk}\left(C_{ui,hk}\right)^{1-\sigma_h}}{\sum_{u\in\mathcal{N}} n^S_{ui,hk} m^S_{ui,hk}\left(C_{ui,hk}\right)^{1-\sigma_h}}
\end{aligned} \tag{B.6}$$

For any given firm of type z the solution of the optimization takes the form:

$$n^S_{ui,hk}\left(z\right) = a^S_{ui,hk} z^{\frac{\delta_{1,k}}{\gamma^S_k}};\; n^B_{id,kl}\left(z\right) = a^B_{id,kl} z^{\frac{\delta_{1,k}}{\gamma^B_k}};\; n^F_{i,k}\left(z\right) = a^F_{i,k} z^{\frac{\delta_{1,k}}{\gamma^B_k}} \tag{B.7}$$

where $\delta_{1,k} \equiv \frac{\sigma_k - 1}{1 - \frac{1}{\gamma_k^B} - \frac{1}{\gamma_k^S} \sum_h \beta_{hk} \frac{1-\sigma_k}{1-\sigma_h}} > \sigma_k - 1$, and $\{a^S_{ui,hk}, a^B_{id,kl}, a^F_{i,k}\}$ can be written as

$$a^B_{id,kl} = \left(\frac{\tilde{\sigma}_k^{1-\sigma_k}}{\sigma_k} m^B_{id,kl} \frac{D_{d,kl}}{e_i f^B_{id,kl}} \left(\tau_{id,kl}\right)^{1-\sigma_k} \left(C^*_{i,k}\right)^{1-\sigma_k} \right)^{\frac{1}{\gamma_k^B - 1}}, \tag{B.8}$$

$$a^F_{i,k} = \left(\frac{\tilde{\sigma}_k^{1-\sigma_k}}{\sigma_k} \frac{D^F_{i,k}}{e_i f^F_{i,k}} \left(C^*_{i,k}\right)^{1-\sigma_k} \right)^{\frac{1}{\gamma_k^B - 1}}, \tag{B.9}$$

$$a^S_{ui,hk} = \left(\frac{\tilde{\sigma}_k^{1-\sigma_k}}{\sigma_k} \frac{1-\sigma_k}{1-\sigma_h} \beta_{hk} \frac{D^*_{i,k}}{e_i f^S_{ui,hk}} \left(C^*_{i,k}\right)^{1-\sigma_k} m^S_{ui,hk} \left(C_{ui,hk}\right)^{1-\sigma_h} \left(\tilde{C}_{i,hk}\right)^{\sigma_h - 1} \right)^{\frac{1}{\gamma_k^S - 1}}, \tag{B.10}$$

where we further define the demand shifter for buyers of sector k in location i by

$$D^*_{i,k} = a^F_{i,k} D^F_{i,k} + \sum_l \sum_d m^B_{id,kl} a^B_{id,kl} D_{d,kl} \left(\tau_{id,kl}\right)^{1-\sigma_k}, \tag{B.11}$$

and the corresponding production cost shifter by

$$C^*_{i,k} \equiv w_i^{\beta_{k,L}} \prod_{h \in K} \tilde{C}_{i,hk}^{\beta_{hk}}, \tag{B.12}$$

$$\tilde{C}_{i,hk} = \left(\sum_{u \in \mathcal{N}} a^S_{ui,hk} m^S_{ui,hk} \left(C_{ui,hk}\right)^{1-\sigma_h} \right)^{\frac{1}{1-\sigma_h}}. \tag{B.13}$$

By plugging these equations into the cost function (constraint of equation B.3), the unit cost of a firm with productivity z is given by

$$c_{i,k}(z) = C^*_{i,k} z^{\frac{\delta_{1,k}}{\gamma_k^S} \sum_h \frac{\beta_{hk}}{1-\sigma_h} - 1}. \tag{B.14}$$

and the revenue of a firm with productivity z is given by

$$r_{i,k}(z) = \left(\tilde{\sigma}_k\right)^{1-\sigma_k} D^*_{i,k} \left(C^*_{i,k}\right)^{1-\sigma} (z)^{\delta_{1,k}}. \tag{B.15}$$

Similarly to Lemma 2, we can express $C_{ui,hk} = \overline{C}_{u,h} \tau_{ui,hk}$, where

$$\overline{C}_{u,h}^{1-\sigma_h} = \left(\tilde{\sigma}_h\right)^{1-\sigma_h} \left(C^*_{u,h}\right)^{1-\sigma_h} \frac{\mathbb{M}_{u,h}\left(\delta_{1,h}\right)}{\mathbb{M}_{u,h}\left(\frac{\delta_{1,h}}{\gamma_h^B}\right)}. \tag{B.16}$$

The matching rates between suppliers and buyers, $m^S_{ud,kl}$ and $m^B_{ud,kl}$, are determined for each pair of location and sectors. To compute those we perform a standard change of variables to consider all the allocations for firms of different efficiencies z. We define $G_{d,l}(z)$ as the distribution of firm productivity for location d and sector l. Then the aggregate number of supplier and buyer postings for each pair of location and sectors are defined similarly as in Section 3.2 as follows:

$$\overline{M}^S_{ud,kl} = N_{d,l} \int n^S_{ud,kl}(z) dG_{d,k}(z) = N_{d,l} a^S_{ud,kl} \mathbb{M}_{d,l} \left(\frac{\delta_{1,l}}{\gamma^S_l} \right), \tag{B.17}$$

$$\overline{M}^B_{ud,kl} = N_{u,k} \int n^B_{ud,kl}(z) dG_{u,k}(z) = N_{u,k} a^B_{ud,kl} \mathbb{M}_{u,k} \left(\frac{\delta_{1,k}}{\gamma^B_k} \right), \tag{B.18}$$

with

$$\mathbb{M}_{d,l}(\chi) \equiv \int z^{\chi} dG_{d,l}(z).$$

The total number of matches for each location and sector pair is then given by:

$$M_{ud,kl} = \kappa_{ud,kl} \left(\overline{M}^S_{ud,kl} \right)^{\lambda^S_{kl}} \left(\overline{M}^B_{ud,kl} \right)^{\lambda^B_{kl}}, \tag{B.19}$$

Last, the number of total supplier-to-buyer matches between bilateral regions, the matching rates $m^S_{ud,kl}$ and $m^B_{ud,kl}$ are now defined by:

$$m^S_{ud,kl} = \frac{M_{ud,kl}}{\overline{M}^S_{ud,kl}}, \quad m^B_{ud,kl} = \frac{M_{ud,kl}}{\overline{M}^B_{ud,kl}}. \tag{B.20}$$

B.2 Gravity Equations

We aggregate trade flows between any given pair of locations and pair of sectors. Following the same algebra as in Section 3.2, the number of supplier-to-buyer relationships $M_{ud,kl}$ from supplier location u and sector k to buyer location d and sector l is given the following gravity equation:

$$M_{ud,kl} = \varrho^E_{kl} \chi^E_{ud,kl} \zeta^E_{u,kl} \xi^E_{d,kl}, \tag{B.21}$$

where the bilateral resistance term $\chi^E_{ud,kl}$ is given by:

$$\chi^E_{ud,kl} = \left[\kappa_{ud,kl} \left(f^B_{ud,kl} \right)^{-\tilde{\lambda}^B_{kl}} \left(f^S_{ud,kl} \right)^{-\tilde{\lambda}^S_{kl}} \left(\tau^{1-\sigma}_{ud,kl} \right)^{\tilde{\lambda}^B_{kl} + \tilde{\lambda}^S_{kl}} \right]^{\delta_{2,kl}},$$

where we define $\tilde{\lambda}^S_{kl} \equiv \lambda^S_{kl} / \gamma^S_l$ and $\tilde{\lambda}^B_{kl} \equiv \lambda^B_{kl} / \gamma^B_k$ as the ratio of matching function elasticities and search cost elasticities, and also $\delta_{2,kl} \equiv \left[1 - \tilde{\lambda}^S_{kl} - \tilde{\lambda}^B_{kl} \right]^{-1}$, and ϱ^E_{kl} is a constant. The origin- and destination-specific shifter takes the form:

$$\zeta^E_{u,kl} = \left[\left(N_{u,k} \mathbb{M}_{u,k} \left(\frac{\delta_{1,k}}{\gamma^B_k} \right) \right)^{\lambda^B_{kl} \frac{\gamma^B_k - 1}{\gamma^B_k}} \left\{ e^{-1}_{u,k} \left(C^*_{u,k} \right)^{1-\sigma_k} \right\}^{\tilde{\lambda}^B_{kl}} \left(\overline{C}_{u,k} \right)^{(1-\sigma_k)\tilde{\lambda}^S_{kl}} \right]^{\delta_{2,kl}},$$

$$\xi_{d,kl}^{E}=\left[\left(N_{d,l}\mathbb{M}_{d,l}\left(\frac{\delta_{1,l}}{\gamma_{kl}^{S}}\right)\right)^{\lambda_{kl}^{S}\frac{\gamma_{l}^{S}-1}{\gamma_{l}^{S}}}\left(D_{d,kl}\right)^{\tilde{\lambda}_{kl}^{B}}\left\{D_{d,l}^{*}e_{d,l}^{-1}\left(C_{d,l}^{*}\right)^{(1-\sigma_{l})}\left(\tilde{C}_{d,kl}\right)^{(\sigma_{k}-1)}\right\}^{\tilde{\lambda}_{kl}^{S}}\right]^{\delta_{2,kl}}.$$

We can also derive the intensive margin of trade flows, namely the average volume of bilateral transactions from suppliers in location u and sector k to buyers in location d and sector l. In particular,

$$\overline{r}_{ud,kl}=\varrho_{kl}^{I}\chi_{ud,kl}^{I}\zeta_{u,kl}^{I}\xi_{d,kl}^{I}, \tag{B.22}$$

where $\varrho_{kl}^{I}=\left(\tilde{\sigma}_{k}\right)^{1-\sigma_{k}}$, and the bilateral component is only a function of iceberg costs,

$$\chi_{ud,kl}^{I}=\left(\tau_{ud,kl}\right)^{1-\sigma_{k}},$$

and the origin- and destination-specific shifters are given by:

$$\zeta_{u,kl}^{I}=\left(C_{u,k}^{*}\right)^{1-\sigma_{k}}\frac{\mathbb{M}_{u,k}\left(\delta_{1,k}\right)}{\mathbb{M}_{u,k}\left(\frac{\delta_{1,k}}{\gamma_{k}^{B}}\right)},\ \xi_{d,kl}^{I}=D_{d,kl}.$$

B.3 General Equilibrium

To embed the the aforementioned search and matching multi-sector framework in general equilibrium we proceed again in a number of steps.

Advertisement Cost First, we assume that advertisement service is provided by perfectly competitive providers that use labor and intermediate goods with Cobb-Douglas production technology. Therefore, the price of advertisement service $e_{i,k}$ is given by

$$e_{i,k}=A_{i,k}w_{i}^{\mu}\left(C_{i,k}^{*}\right)^{1-\mu}, \tag{B.23}$$

where $A_{i,k}$ captures the inverse of productivity of the advertisement sector.

Intermediate Demand Shifters Similarly as in Appendix B.22, the demand shifter of the firms in location d and sector l purchasing from sector k can be solved as the following integral:

$$\begin{aligned}
Y^I_{d,kl} &= (\tilde{\sigma}_k)^{1-\sigma_k} \sum_u N_{u,k} \int (\tau_{ud,kl})^{1-\sigma_k} D_{d,kl} (c_{u,k}(z))^{1-\sigma_k} n^B_{ud,kl}(z)\, m^B_{ud,kl} dG_{u,k}(z) \\
&= (\tilde{\sigma}_k)^{1-\sigma_k} D_{d,kl} \sum_u N_{u,k} (\tau_{ud,kl})^{1-\sigma_k} \int \left(C^*_{u,k}\right)^{1-\sigma_k} a^B_{ud,kl} m^B_{ud,kl} z^{\frac{\delta_{1,k}}{\gamma^B_k} + \frac{\delta_{1,k}}{\gamma^S_k}\sum_h \frac{\beta_{hk}}{\sigma_h-1}(\sigma_k-1)+(\sigma_k-1)} dG_{u,k}(z) \\
&= (\tilde{\sigma}_k)^{1-\sigma_k} D_{d,kl} \sum_u (\tau_{ud,kl})^{1-\sigma_k} \left(C^*_{u,k}\right)^{1-\sigma_k} N_{d,l} a^S_{ud,kl} m^S_{ud,kl} \frac{\mathbb{M}_{d,l}\left(\frac{\delta_{1,l}}{\gamma^S_l}\right)}{\mathbb{M}_{u,k}\left(\frac{\delta_{1,k}}{\gamma^B_k}\right)} \mathbb{M}_{u,k}(\delta_{1,k}) \\
&= D_{d,kl} N_{d,l} \mathbb{M}_{d,l}\left(\frac{\delta_{1,l}}{\gamma^S_l}\right) \sum_u (\tau_{ud,kl})^{1-\sigma_k} \overline{C}^{1-\sigma_k}_{u,k} a^S_{ud,kl} m^S_{ud,kl} \quad \text{(from equation B.16)} \\
&= D_{d,kl} N_{d,l} \mathbb{M}_{d,l}\left(\frac{\delta_{1,l}}{\gamma^S_l}\right) \left(\tilde{C}_{d,kl}\right)^{1-\sigma_k}
\end{aligned}$$

Now, aggregate intermediate goods demand (net of the usage by the advertisement sector) is also derived from the demand side, that is:

$$Y^I_{d,kl} = \tilde{\beta}_{kl} \tilde{\sigma}^{1-\sigma_l}_l D^*_{d,l} \left(C^*_{d,l}\right)^{1-\sigma_l} \mathbb{M}_{d,l}(\delta_{1,l}) N_{d,l},$$

where $\tilde{\beta}_{kl} = \beta_{kl}/\tilde{\sigma}_l + \sigma_l^{-1}$ is the intermediate good share for sector k in aggregate revenue in sector l and the remaining term is the aggregate revenue (from equation B.15). Combining the two expressions for $Y^I_{d,kl}$ we finally obtain,

$$D_{d,kl} = \frac{\tilde{\beta}_{kl} \tilde{\sigma}^{1-\sigma_l}_l D^*_{d,l} \left(C^*_{d,l}\right)^{1-\sigma_l} \mathbb{M}_{d,l}(\delta_{1,l})}{\mathbb{M}_{d,l}\left(\frac{\delta_{1,l}}{\gamma^S_l}\right) \left(\tilde{C}_{d,kl}\right)^{1-\sigma_k}} \tag{B.24}$$

Final Demand Shifters From the goods market clearing condition, we have

$$\begin{aligned}
N_{i,k} \int a^F_{i,k} z^{\frac{\delta_{1,k}}{\gamma^B_k}} D^F_{i,k} \left(C^*_{i,k} z^{\frac{\delta_{1,k}}{\gamma^S_k}\sum_h \frac{\beta_{hk}}{1-\sigma_h} - 1} \right)^{1-\sigma_k} dG_{i,k}(z) = \alpha_k w_i L_i \\
\iff D^F_{i,k} = \frac{\left(C^*_{i,k}\right)^{\sigma_k - 1} w_i}{a^F_{i,k} N_{i,k} \mathbb{M}(\delta_{1,k})} \alpha_k L_i
\end{aligned} \tag{B.25}$$

Firm Entry Lastly, we characterize firm entry $N_{i,k}$. We follow a long tradition in international trade and spatial economics and assume that in each region, there is a pool of potential entrants of intermediate goods producers (firms). Conditional on paying a fixed cost $F_{i,k}$ in units of local labor, firms in region i sector k draw a productivity z from the cumulative distribution function $G_{i,k}(\cdot)$. The zero-profit condition for the potential entrants implies that

$$w_i F_{i,k} = \vartheta^P_k \tilde{\sigma}^{1-\sigma_k}_k \left[D^*_{i,k} \left(C^*_{i,k}\right)^{1-\sigma_k} \mathbb{M}_{i,k}(\delta_{1,k}) \right] \iff$$

$$D^*_{i,k} = \frac{w_i F_{i,k}}{\vartheta^P_k \tilde{\sigma}_k^{1-\sigma_k} \left(C^*_{i,k}\right)^{1-\sigma_k} \mathbb{M}_{i,k}\left(\delta_{1,k}\right)}, \tag{B.26}$$

where $\vartheta^P_k \equiv \frac{1}{\delta_{1,k}\tilde{\sigma}_k}$ is the share of profit to revenue (i.e., Lemma 1). Combining equation (B.11), (B.18), (B.20), (B.26),

$$\frac{w_i F_{i,k}}{\vartheta^P_k \tilde{\sigma}_k^{1-\sigma_k} \left(C^*_{i,k}\right)^{1-\sigma_k} \mathbb{M}_{i,k}\left(\delta_{1,k}\right)} = a^F_{i,k} D^F_{i,k} + \sum_l \sum_d \frac{M_{id,kl}}{N_{i,k}} \left(\mathbb{M}_{i,k}\left(\frac{\delta_{1,k}}{\gamma^S_k}\right)\right)^{-1} D_{d,kl} \left(\tau_{id,kl}\right)^{1-\sigma_k}$$

By multiplying both hand side by $N_{i,k}$, we have

$$N_{i,k} = \frac{\vartheta^P_k \tilde{\sigma}_k^{1-\sigma_k} \mathbb{M}_{i,k}\left(\delta_{1,k}\right)}{F_{i,k}} \frac{\left(C^*_{i,k}\right)^{1-\sigma_k}}{w_i} \left[a^F_{i,k} D^F_{i,k} N_{i,k} + \sum_l \sum_d M_{id,kl} \left(\mathbb{M}_{i,k}\left(\frac{\delta_{1,k}}{\gamma^S_k}\right)\right)^{-1} D_{d,kl} \left(\tau_{id,kl}\right)^{1-\sigma_k}\right] \tag{B.27}$$

Wages The aggregate labor demand is given by

$$Y^L_{i,k} = \tilde{\beta}_{k,L} \tilde{\sigma}_k^{1-\sigma_k} D^*_{i,k} \left(C^*_{i,k}\right)^{1-\sigma_l} \mathbb{M}_{i,k}\left(\delta_{1,k}\right) N_{i,k},$$

where $\tilde{\beta}_{k,L} \equiv 1 - \sum_h \tilde{\beta}_{hk}$ is the labor share in aggregate revenue in sector k (including the advertisement and fixed cost payment). Therefore, labor market clearing requires that:

$$w_i = \frac{1}{L_i} \sum_k \tilde{\beta}_{k,L} \tilde{\sigma}_k^{1-\sigma_k} D^*_{i,k} \left(C^*_{i,k}\right)^{1-\sigma_k} \mathbb{M}_{i,k}\left(\delta_{1,k}\right) N_{i,k}. \tag{B.28}$$

Equilibrium The general equilibrium is defined by $\{C^*_{i,k}, \tilde{C}_{i,hk}, \overline{C}_{u,h}, M_{ud,kl}, \overline{r}_{ud,kl}, D_{i,kl}, D^F_{i,k}, D^*_{i,k}, N_{i,k}, w_i\}$, that satisfy equations (B.12), (B.13), (B.16), (B.21), (B.22), (B.23), (B.24), (B.25), (B.26), and (B.28).

B.4 Counterfactual Equilibrium

We now characterize the counterfactual changes in equilibrium as a response to shocks. Similarly to the approach of Dekle, Eaton, and Kortum (2008) and Caliendo and Parro (2015), we can characterize the counterfactual equilibrium given two sets of information: (i) the regional input-output tables, including the total trade flows across locations and sectors $\{X_{ud,hk}\}$, labor compensation $\{X^L_{i,k}\}$, and final consumption $\{Y^F_{i,k}\}$, and (ii) a subset of structural parameters $\{\alpha_k, \beta_{k,L}, \beta_{hk}, \mu, \gamma^B_k, \gamma^S_k, \lambda^B_{kl}, \lambda^S_{kl}, \sigma_k\}$ as follows:

$$\hat{C}^*_{d,k} \equiv \hat{w}_d^{\beta_{l,L}} \prod_{k \in K} \hat{\tilde{C}}_{d,kl}^{\beta_{kl}}$$

$$\hat{\tilde{C}}_{d,kl} = \left(\sum_{u \in \mathcal{N}} \frac{\hat{M}_{ud,kl}}{\hat{N}_{d,l}} \left(\hat{C}^*_{u,k}\right)^{1-\sigma_h} \left(\hat{\tau}_{ud,k}\right)^{1-\sigma_h} \Lambda_{ud,kl}\right)^{\frac{1}{1-\sigma_k}}$$

$$\hat{M}_{ud,kl} = \hat{\chi}^E_{ud,kl}\hat{\zeta}^E_{u,kl}\hat{\xi}^E_{d,kl}$$

$$\hat{\zeta}^E_{u,kl} = \left[\left(\hat{N}_{u,k}\right)^{\lambda^B_{kl}\frac{\gamma^B_k-1}{\gamma^B_k}} \left\{\hat{e}^{-1}_{u,k}\left(\hat{C}^*_{u,k}\right)^{1-\sigma_k}\right\}^{\tilde{\lambda}^B_{kl}} \left(\hat{C}^*_{u,k}\right)^{(1-\sigma_k)\tilde{\lambda}^S_{kl}}\right]^{\delta_{2,kl}}$$

$$\hat{\xi}^E_{d,kl} = \left[\left(\hat{N}_{d,l}\right)^{\lambda^S_{kl}\frac{\gamma^S_l-1}{\gamma^S_l}} \left(\hat{D}_{d,kl}\right)^{\tilde{\lambda}^B_{kl}} \left\{\hat{D}^*_{d,l}\left(\hat{e}_{d,l}\right)^{-1}\left(\hat{C}^*_{d,l}\right)^{(1-\sigma_l)}\left(\hat{\tilde{C}}_{d,kl}\right)^{\sigma_k-1}\right\}^{\tilde{\lambda}^S_{kl}}\right]^{\delta_{2,kl}}$$

$$\hat{e}_{d,l} = \hat{w}^{\mu}_d\left(\hat{C}^*_{d,k}\right)^{1-\mu}$$

$$\hat{D}_{d,kl} = \frac{\hat{D}^*_{d,l}\left(\hat{C}^*_{d,l}\right)^{1-\sigma_l}}{\left(\hat{\tilde{C}}_{d,kl}\right)^{1-\sigma_k}}$$

$$\hat{D}^*_{i,k} = \frac{\hat{w}_i}{\left(\hat{C}^*_{i,k}\right)^{1-\sigma_k}}$$

$$\hat{N}_{u,k} = \frac{\left(\hat{C}^*_{u,k}\right)^{1-\sigma_k}}{\hat{w}_u}\left[S^F_{u,k}\hat{w}_u\left(\hat{C}^*_{u,k}\right)^{\sigma_k-1} + \left(1-S^F_{u,k}\right)\sum_l\sum_d \hat{M}_{ud,kl}\hat{D}_{d,kl}\left(\hat{\tau}_{ud,kl}\right)^{1-\sigma_k}\Psi_{ud,kl}\right]$$

$$\hat{w}_i = \sum_k \frac{X^L_{i,k}}{\sum_l X^L_{i,l}}\hat{D}^*_{i,k}\left(\hat{C}^*_{i,k}\right)^{1-\sigma_k}\hat{N}_{i,k}$$

where $S^F_{i,k}$ is the share of final goods sales relative to total sales by firms in location i and sector k given by

$$S^F_{i,k} = \frac{\alpha_{i,k}\sum_m X^L_{i,m}}{\sum_{j,m} X_{ij,km} + \alpha_{i,k}\sum_m X^L_{i,m}},$$

$\Lambda_{ud,kl}$ is the share of sales in sector k to sector l by firms in d that comes from location u

$$\Lambda_{ud,kl} = \frac{a^S_{ud,kl}m^S_{ud,kl}\left(C^*_{u,k}\right)^{1-\sigma_k}\left(\tau_{ud,kl}\right)^{1-\sigma_k}}{\sum_i a^S_{id,kl}m^S_{id,kl}\left(C^*_{i,k}\right)^{1-\sigma_k}\left(\tau_{id,kl}\right)^{1-\sigma_k}} = \frac{X_{ud,kl}}{\sum_i X_{id,kl}},$$

and $\Psi_{ud,kl}$ is the share of intermediate goods sales by firms in location u and sector k that goes to location d and sector l

$$\Psi_{ud,kl} = \frac{m^B_{ud,kl}a^B_{ud,kl}D_{d,kl}\left(\tau_{ud,kl}\right)^{1-\sigma_k}}{\sum_h\sum_i m^B_{ui,kh}a^B_{ui,kh}D_{i,kh}\left(\tau_{ui,kh}\right)^{1-\sigma_k}} = \frac{X_{ud,kl}}{\sum_h\sum_i X_{ui,kh}}.$$

Lastly, the changes of final goods price index is given by

$$\hat{P}^F_i = \prod_k\left(\hat{P}^F_{i,k}\right)^{\alpha_k} = \prod_k\left(\hat{N}_{i,k}\right)^{\frac{\gamma^B_k-1}{\gamma^B_k}\frac{\alpha_k}{1-\sigma_k}}\left(\frac{\hat{w}_i}{\hat{e}_{i,k}}\right)^{\frac{1}{\gamma^B_k}\frac{\alpha_k}{1-\sigma_k}}\left(\hat{C}^*_{i,k}\right)^{\alpha_k}$$

Online Appendix for “Spatial Production Networks”

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January, 2023

C Incorporate Labor Mobility

In this subsection, we extend our model to incorporate labor mobility.

We assume that the utility of workers who reside in location i is given by:

$$U_j = B_j \left(\frac{w_j}{P_j^F}\right) L_j^{-1/\upsilon},$$

where B_j is the exogenous residential amenity, w_j is the nominal wage, P_j is the consumer price index, and L_j is the population size j. Parameter υ governs the dispersion force, which includes housing costs, negative residential spillovers, and idiosyncratic preference heterogeneity.

Workers are freely mobile across locations. This implies that the utility is equalized across locations, i.e., $U_j = U$ for all locations j. Therefore, the population size of location j is given by:

$$L_j = \frac{B_j^\upsilon \left(\frac{w_j}{P_j^F}\right)^\upsilon}{\sum_\ell B_\ell^\upsilon \left(\frac{w_\ell}{P_\ell^F}\right)^\upsilon}, \tag{C.1}$$

and the utility of workers in the economy is given by:

$$\mathcal{U} = \left(\sum_\ell B_\ell^\upsilon \left(\frac{w_\ell}{P_\ell^F}\right)^\upsilon\right)^{1/\upsilon}. \tag{C.2}$$

Taking L_j as another endogenous variables, the system of equilibrium equations (22) and (23) become:

$$(w_i)^{1+\tilde{\lambda}^B\delta_2\mu} (C_i^*)^{(\sigma-1)\delta_2+\tilde{\lambda}^B\delta_2(1-\mu)} (L_i)^{-\lambda^B\frac{\gamma^B-1}{\gamma^B}\delta_2-1} = \sum_d \tilde{K}_{id} (w_d)^{\delta_G} (C_d^*)^{\frac{(\sigma-1)\delta_2}{1-\beta}-\tilde{\lambda}^S\delta_2(1-\mu)} (L_d)^{\lambda^S\frac{\gamma^S-1}{\gamma^S}\delta_2}, \tag{C.3}$$

$$(w_i)^{1-\delta_G} (C_i^*)^{-\frac{(\sigma-1)\delta_2}{1-\beta}+\tilde{\lambda}^S\delta_2(1-\mu)} (L_i)^{-\lambda^S\frac{\gamma^S-1}{\gamma^S}\delta_2-1} = \sum_u \tilde{K}_{ui} (w_u)^{-\tilde{\lambda}^B\delta_2\mu} (C_u^*)^{-(\sigma-1)\delta_2-\tilde{\lambda}^B\delta_2(1-\mu)} (L_u)^{\lambda^B\frac{\gamma^B-1}{\gamma^B}\delta_2}, \tag{C.4}$$

where $\tilde{K}_{id}$ is the combination of exogenous variables defined by similar manipulation in Appendix 1.

Furthermore, from equation (A.29), the final price index is $\left(P_i^F\right)^{1-\sigma} \propto L_i \left(C_i^*\right)^{1-\sigma-\frac{1}{\gamma^B}(1-\mu)} w_i^{\frac{1}{\gamma^B}(1-\mu)}$. By combining with equation (C.1), we have:

$$(w_i)^{\upsilon\left(1-\frac{1-\mu}{\gamma^B(1-\sigma)}\right)} \left(C_i^*\right)^{-\upsilon\left(1-\frac{1-\mu}{\gamma^B(1-\sigma)}\right)} (L_i)^{\frac{\upsilon}{\sigma-1}-1} = \sum_\ell \tilde{B}_{i\ell} (w_\ell)^{\upsilon\left(1-\frac{1-\mu}{\gamma^B(1-\sigma)}\right)} \left(C_\ell^*\right)^{-\upsilon\left(1-\frac{1-\mu}{\gamma^B(1-\sigma)}\right)} (L_\ell)^{\frac{\upsilon}{\sigma-1}}, \tag{C.5}$$

where $\tilde{B}_{i\ell}$ is a function of exogenous variables. Equations (C.3), (C.4), and (C.5) jointly characterize the equilibrium in terms of $\{w_i, C_i^*, L_i\}$.

D Optimality of Search Decision

In this section of the appendix, we analyze the optimality of the equilibrium search decision in a single location model ($|\mathcal{N}| = 1$). We derive a necessary condition for the equilibrium search decision to be socially efficient.

To analyze the efficiency of the equilibrium search decisions, we consider a planner who taxes the search costs. Denote the tax rates for search costs for final consumers, for intermediate buyers, and for suppliers by τ^F, τ^B, τ^S, respectively. We assume that the planner also taxes labor income at rate τ^I. Suppressing the subscripts for locations and normalizing wage and population size to one, the equilibrium search intensity $\{a^F, a^B, a^S\}$ reduces to

$$\left(1+\tau^F\right) e \left(a^F\right)^{\gamma^B-1} = \varrho^F \left(1-\tau^I\right) \left(\frac{1}{P_i^F}\right)^{1-\sigma} (C^*)^{1-\sigma}, \tag{D.1}$$

$$\left(1+\tau^B\right) e \left(a^B\right)^{\gamma^B-1} = \varrho^B m^B (C^*)^{\beta\frac{\sigma-1}{1-\beta}}, \tag{D.2}$$

$$\left(1+\tau^S\right) e \left(a^S\right)^{\gamma^S-1} = \varrho^S m^S (C^*)^{\beta\frac{\sigma-1}{1-\beta}}, \tag{D.3}$$

where the unit cost for advertisement and matching rates are given by

$$e = \varrho^E (C^*)^{1-\mu}, \tag{D.4}$$

$$m^S = \varrho^{MS} \left(a^S\right)^{\lambda^S-1} \left(a^B\right)^{\lambda^B}, \tag{D.5}$$

$$m^B = \varrho^{MB} \left(a^S\right)^{\lambda^S} \left(a^B\right)^{\lambda^B-1}, \tag{D.6}$$

where $\{\varrho^F, \varrho^B, \varrho^S, \varrho^E, \varrho^{MS}, \varrho^{MB}\}$ are functions of exogenous parameters. Furthermore, C^* and P^F are given by

$$\left(C_i^*\right)^{\frac{\beta(1-\sigma)}{1-\beta}} \equiv \varrho^C a^S m^S, \tag{D.7}$$

$$\left(P^F\right)^{1-\sigma} = \varrho^P N a^F (C^*)^{1-\sigma}. \tag{D.8}$$

Now we characterize the government budget constraint. From Lemma 1, the aggregate search costs for suppliers is $\vartheta^A \frac{1-\beta}{\gamma^S}$ fraction of aggregate revenue, where the aggregate revenue is in turn given by $\left(1-\tau^I\right) wL/\tilde{\beta} = \left(1-\tau^I\right)/\tilde{\beta}$. Together, the aggregate search cost payment for suppliers is given by

$$\left(1+\tau^{S}\right)\frac{1}{\gamma^{B}}ef^{S}\left(a^{S}\right)^{\gamma^{S}}N=\frac{\vartheta^{A}}{\gamma^{S}}\frac{1-\beta}{\tilde{\beta}}\left(1-\tau^{I}\right). \tag{D.9}$$

Furthermore, from Lemma 1, the aggregate search costs for final consumers and intermediate buyers together are $\frac{\vartheta^{A}}{\gamma^{B}}$ fraction of aggregate revenue $\left(1-\tau^{I}\right)/\tilde{\beta}$. Furthermore, final consumption and intermediate sales are $\tilde{\beta}$ and $1-\tilde{\beta}$ share of the aggregate revenue, respectively. Therefore, we have

$$\left(1+\tau^{F}\right)\frac{1}{\gamma^{B}}ef^{F}\left(a^{F}\right)^{\gamma^{B}}N=\frac{\vartheta^{A}}{\gamma^{B}}\left(1-\tau^{I}\right), \tag{D.10}$$

$$\left(1+\tau^{B}\right)\frac{1}{\gamma^{B}}ef^{B}\left(a^{B}\right)^{\gamma^{B}}N=\frac{\vartheta^{A}}{\gamma^{B}}\frac{1-\tilde{\beta}}{\tilde{\beta}}\left(1-\tau^{I}\right). \tag{D.11}$$

Together, government budget constraint is given by

$$\begin{aligned} 0&=\tau^{I}+\tau^{F}\frac{1}{\gamma^{B}}ef^{F}\left(a^{F}\right)^{\gamma^{B}}N+\tau^{B}\frac{1}{\gamma^{B}}ef^{B}\left(a^{B}\right)^{\gamma^{B}}N+\tau^{S}\frac{1}{\gamma^{S}}ef^{S}\left(a^{S}\right)^{\gamma^{S}}N\\ \iff 0&=\frac{\tau^{I}}{1-\tau^{I}}+\frac{\tau^{F}}{1+\tau^{F}}\frac{\vartheta^{A}}{\gamma^{B}}+\frac{\tau^{B}}{1+\tau^{B}}\frac{\vartheta^{A}}{\gamma^{B}}\frac{1-\tilde{\beta}}{\tilde{\beta}}+\frac{\tau^{S}}{1+\tau^{S}}\frac{\vartheta^{A}}{\gamma^{S}}\frac{1-\beta}{\tilde{\beta}} \end{aligned} \tag{D.12}$$

The optimal set of taxes is given as the solution to the following problem:

$$\max_{\tau^{F},\tau^{B},\tau^{S},\tau^{I},a^{F},a^{B},a^{S},e,m^{S},m^{B},P^{F},C^{*}}\log\left(P^{F}\right)^{-1}\left(1-\tau^{I}\right)$$

subject to equations (D.1) - (D.8) and (D.12). Denoting the Lagrange multiplier of the constraints for the log of equations (D.1), (D.2), (D.3), (D.7), (D.8) by $\{\psi^{F},\psi^{B},\psi^{S},\psi^{C},\psi^{P}\}$, and that of equation (D.12) by ψ^{I}, the first-order conditions for $\{\tau^{F},\tau^{B},\tau^{S},\tau^{I}\}$ are:

$$1+\tau^{F}=\frac{\vartheta^{A}}{\gamma^{B}}\frac{\psi^{I}}{\psi^{F}}, \tag{D.13}$$

$$1+\tau^{B}=\frac{\vartheta^{A}}{\gamma^{B}}\frac{1-\tilde{\beta}}{\tilde{\beta}}\frac{\psi^{I}}{\psi^{B}}, \tag{D.14}$$

$$1+\tau^{S}=\frac{\vartheta^{A}}{\gamma^{S}}\frac{1-\beta}{\tilde{\beta}}\frac{\psi^{I}}{\psi^{S}}, \tag{D.15}$$

$$-\frac{1}{1-\tau^{I}}=-\frac{1}{1-\tau^{I}}\left(\psi^{F}+\psi^{B}+\psi^{S}\right)-\frac{1}{\left(1-\tau^{I}\right)^{2}}\psi^{I}\iff 1-\tau^{I}=\frac{\psi^{I}}{1-\left(\psi^{F}+\psi^{B}+\psi^{S}\right)}, \tag{D.16}$$

which characterizes the optimal taxes $\{\tau^{F},\tau^{B},\tau^{S},\tau^{I}\}$ given Lagrange multipliers $\{\psi^{F}$, ψ^{B}, ψ^{S}, ψ^{C}, $\psi^{P}\}$. Furthermore, the first-order conditions for $\{a^{F},a^{B},a^{S},C^{*},P^{F}\}$ are given by (substituting e,m^{S},m^{B}):

$$0=\left(\gamma^{B}-1\right)\psi^{F}-\psi^{P}, \tag{D.17}$$

$$0=\left(\gamma^{B}-1\right)\psi^{B}-\left(\lambda^{B}-1\right)\psi^{B}-\lambda^{B}\psi^{S}-\lambda^{B}\psi^{C}, \tag{D.18}$$

$$0=\left(\gamma^{S}-1\right)\psi^{S}-\left(\lambda^{S}-1\right)\psi^{S}-\lambda^{S}\psi^{B}-\lambda^{S}\psi^{C}, \tag{D.19}$$

$$0=-\left(1-\sigma\right)\psi^{F}-\beta\frac{\sigma-1}{1-\beta}\left(\psi^{B}+\psi^{S}\right)+\left(1-\mu\right)\left(\psi^{F}+\psi^{B}+\psi^{S}\right)+\frac{\beta\left(1-\sigma\right)}{1-\beta}\psi^{C}, \tag{D.20}$$

$$-1=\psi^{F}\left(1-\sigma\right)+\psi^{P}\left(1-\sigma\right). \tag{D.21}$$

Note that the last five equations is a linear system of $\{\psi^F, \psi^B, \psi^S, \psi^e, \psi^P\}$, with coefficient matrix depending on $\{\lambda^S, \lambda^B, \sigma, \mu, \beta\}$. Therefore, if the rank condition is satisfied, $\{\psi^F, \psi^B, \psi^S, \psi^C, \psi^P\}$ can be solved in terms of $\{\lambda^S, \lambda^B, \sigma, \mu, \beta\}$. Furthermore, a set of necessary condition for the optimality of the laissez-faire equilibrium is that the above equations are satisfied with no taxes, i.e., $\tau^F = \tau^B = \tau^S = \tau^I = 0$.

To further obtain a subset of necessary condition, notice that evaluating equations (D.14) and (D.15) at $\tau^B = \tau^S = 0$ yields

$$\frac{\psi^{S}}{\psi^{B}}=\frac{1-\beta}{1-\tilde{\beta}}\frac{\gamma^{B}}{\gamma^{S}}. \tag{D.22}$$

Furthermore, from equations (D.18) and (D.19), we have

$$\frac{\gamma^{B}-\lambda^{B}}{\lambda^{B}}\psi^{B}-\psi^{S}=\frac{\gamma^{S}-\lambda^{S}}{\lambda^{S}}\psi^{S}-\psi^{B}\iff\frac{\psi^{S}}{\psi^{B}}=\frac{\gamma^{B}}{\gamma^{S}}\frac{\lambda^{S}}{\lambda^{B}}. \tag{D.23}$$

Combining equations (D.22) and (D.23), and using our definition of aggregate labor share $\tilde{\beta} = \frac{\sigma-1}{\sigma}\beta + \frac{1}{\sigma}$, a necessary condition for the optimality of equilibrium search decision is

$$\frac{\lambda^{S}}{\lambda^{B}}=\frac{1-\beta}{1-\tilde{\beta}}=\frac{\sigma}{\sigma-1}. \tag{D.24}$$

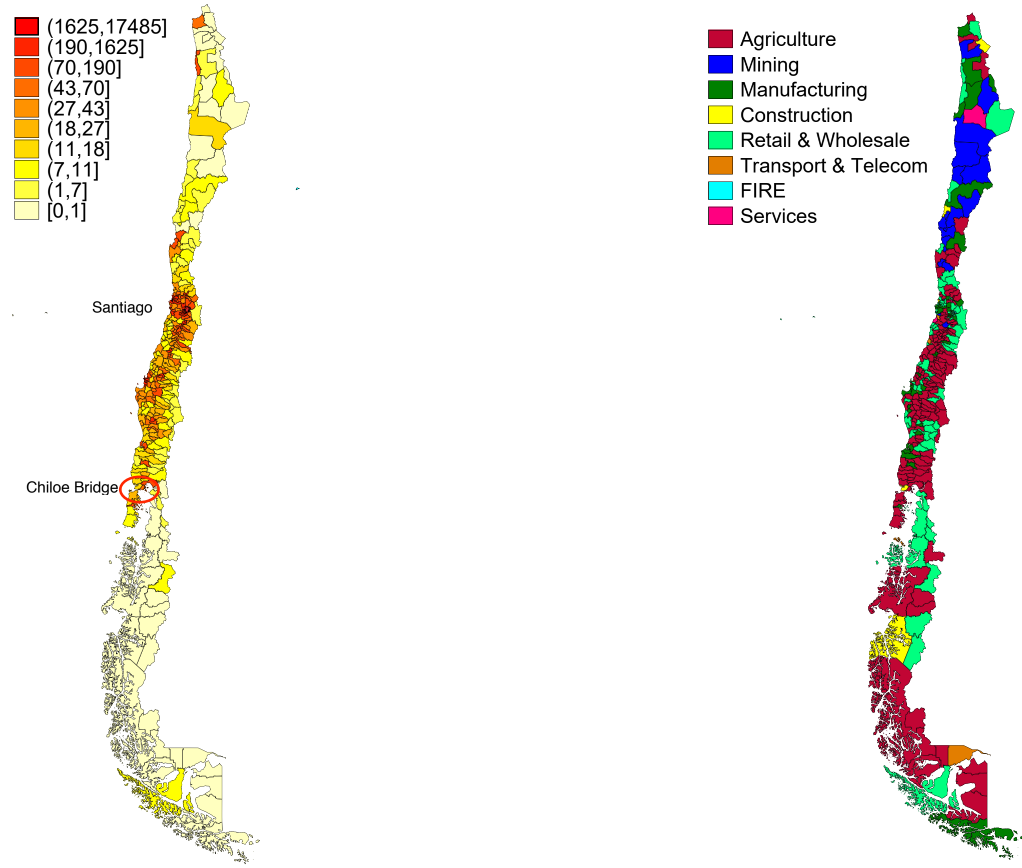
This condition resonates Hosios (1990), who provides a condition for equilibrium efficiency in two-sided search and matching models. To see the relationship, notice that $\frac{1-\beta}{1-\tilde{\beta}}$ corresponds to the ratio between the amount of resources that are used for supplier search and firm buyer search in the equilibrium, respectively (equations D.9 and D.11). In order for equilibrium to be efficient, this ratio has to coincide with the ratio of λ^S and λ^B that summarizes the search externality (thick-market and congestion externality) created by supplier and buyer search, respectively.

If equation (D.24) is not satisfied, equilibrium search decision is not socially efficient and there are welfare gains by imposing taxes. Notice also that this is only a necessary but not a sufficient condition. For example, if matching technology exhibits increasing returns to scale, e.g., $\lambda^S + \lambda^B > 1$, there is generically an under-supply of search even when equation (D.24) is satisfied (see Miyauchi (2021) for a related analysis).

E Additional Figures and Tables for Section 2

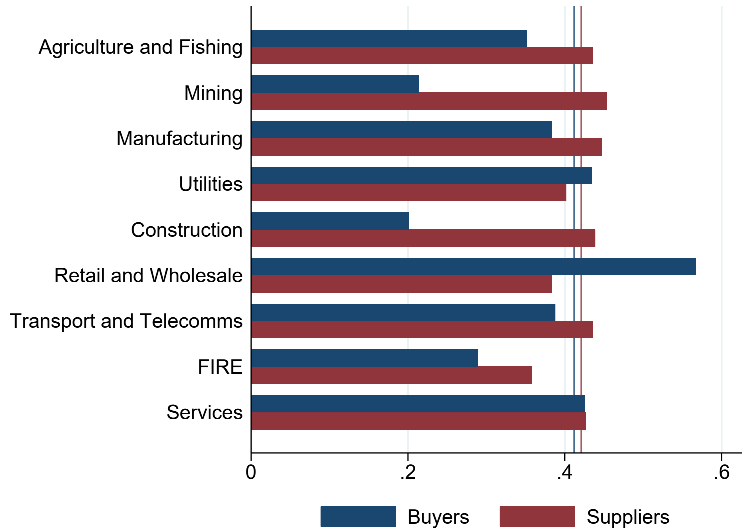
Figure E.1: Map of Chile with Population Density and Top Sectors

(a) Population Density (b) Top Sectors



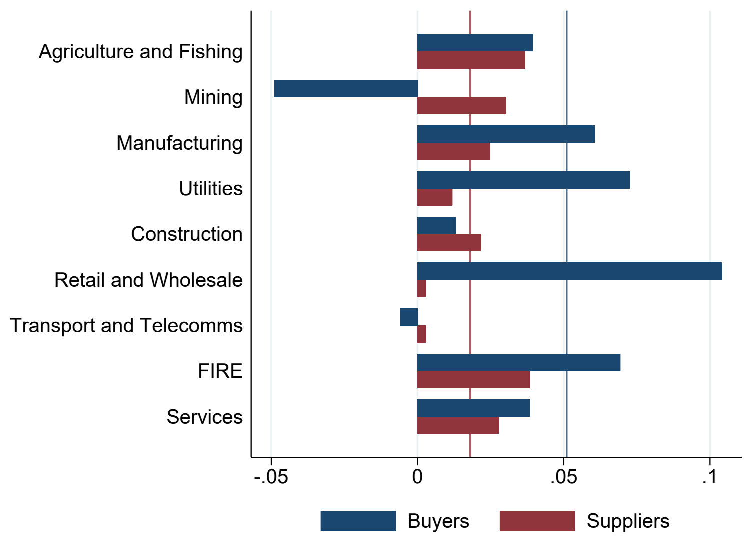
Notes: This figure shows the map of Chile at the municipality level. Panel (a) shows population density. Darker color indicates a higher population per squared kilometers. The map shows the location of the capital city of Chile, Santiago, and the new Chacao Bridge, which is planned to connect the mainland with the largest island of Chile, Chiloé, by 2025. Panel (b) shows which sector has the highest total domestic sales in 2018-2019.

Figure E.2: Number of Domestic Suppliers and Buyers and Firm Size: By Sector



Notes: This figure shows the regression coefficients of the log number of domestic suppliers and buyers per tax ID on log total sales for each sector, corresponding to the sectoral heterogeneity of the slopes in Figure 1.

Figure E.3: Number of Domestic Suppliers and Buyers and Geography: By Sector



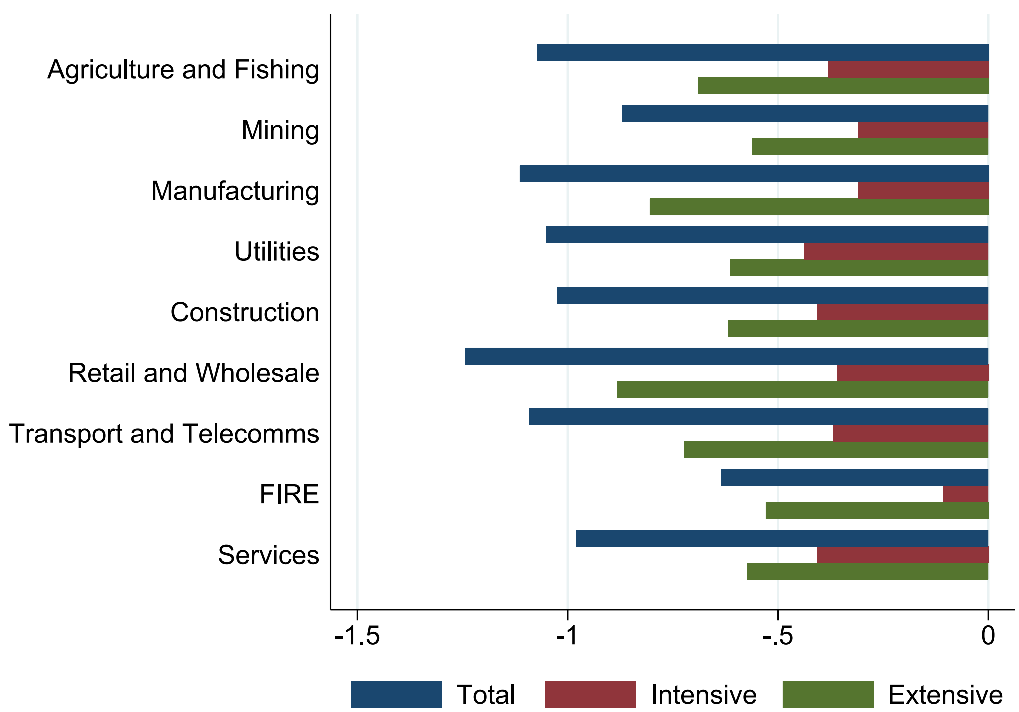
Notes: This figure presents the regression coefficients of log population density on the log average number of domestic suppliers and buyers per firm at the municipality level, corresponding to the sectoral heterogeneity of the regression slopes in Figure 2.

Table E.1: Gravity Regression with PPML: Total Trade Flows, Intensive and Extensive Margin

	Total		Intensive		Extensive	
	(1)	(2)	(3)	(4)	(5)	(6)
Log Distance	-0.487***		-0.137***		-0.634***	
	(0.023)		(0.022)		(0.015)	
Log Time Travel		-0.551***		-0.142***		-0.701***
		(0.026)		(0.022)		(0.026)
R^2						
Origin Municipality-Year FE	✓	✓	✓	✓	✓	✓
Destination Municipality-Year FE	✓	✓	✓	✓	✓	✓
Same Municipality-Year FE	✓	✓	✓	✓	✓	✓
N	237360	237360	237360	237360	237360	237360

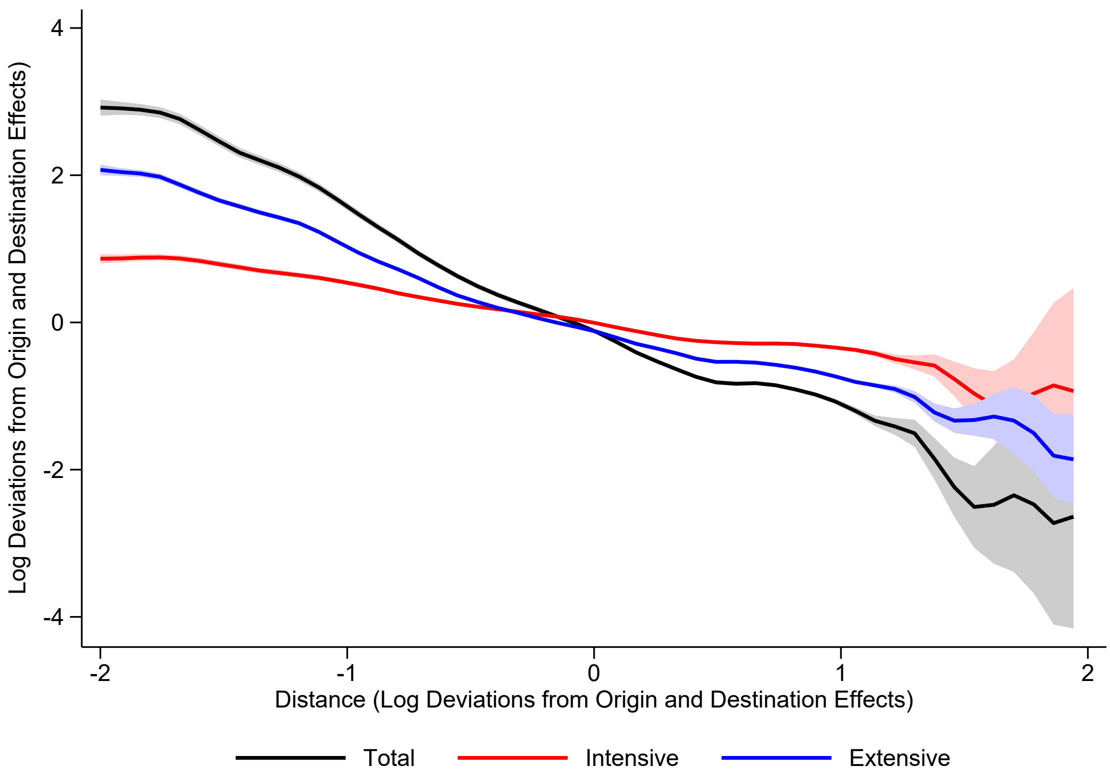
Notes: This table presents the results of the gravity regressions from Table 1, but with a Pseudo Poisson Maximum Likelihood (PPML) estimator to account for zero trade flows arising from granularity of firm-to-firm relationships (Silva and Tenreyro 2006, Dingel and Tiltenot 2020); where we regress total transaction volume between a pair of municipalities on the logarithm of the distance, controlling for origin-year, destination-year, and year fixed effects using SII data from 2018-2019. The dependent variable corresponds to total trade flow, average trade flow (intensive margin), and the number of links between municipalities (extensive margin). Distance (time travel) is measured with kilometers (minutes of time travel) between municipalities using the fastest land or water transportation method available within Chile.

Figure E.4: Gravity Regression by Sector: Total Trade Flows, Intensive and Extensive Margin



Notes: This figure presents the gravity equation from Equation 1, sector by sector. The sector is defined from the perspective of the seller.

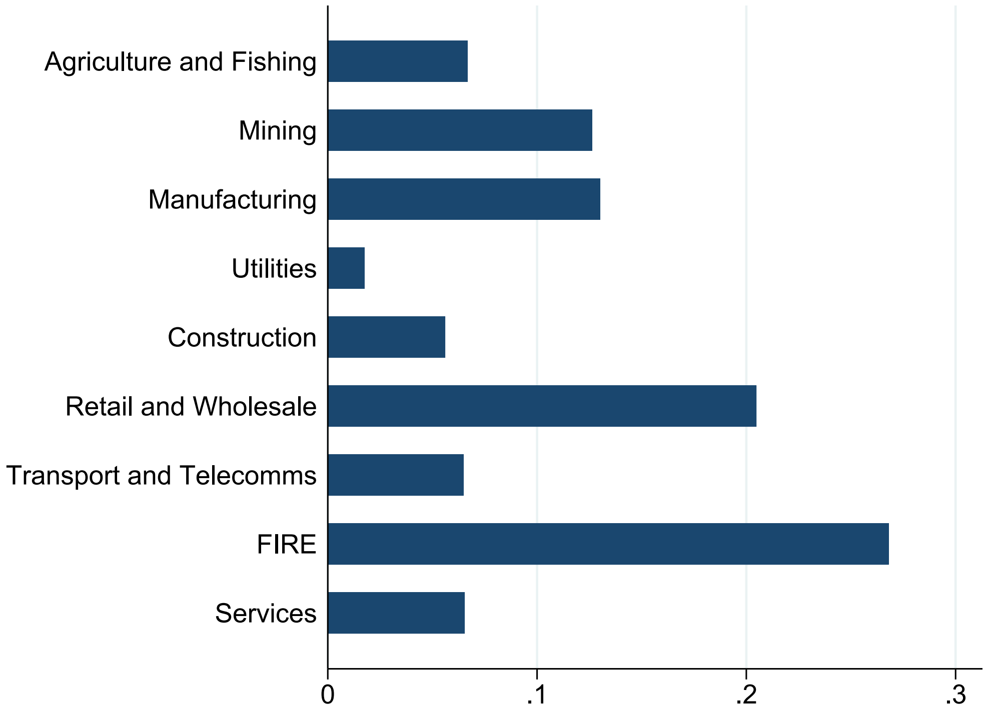
Figure E.5: Non-Parametric Gravity Regression: Total Flows, Intensive and Extensive Margin



Notes: This figure presents the non-parametric gravity correlation at the municipality-pair level between the log total trade flows (black curve), log intensive margin of trade (red curve) and log extensive margin of trade (blue curve), and log distance between municipalities, as in Equation 1. The non-parametric fit is implemented with local linear regressions after extracting municipality of origin and municipality of destination effects. Confidence intervals are presented at the 95% confidence. For exposition purposes, we trim the top 5% and bottom 5% percentiles of the distribution of bilateral trade.

F Additional Figures and Tables for Section 5

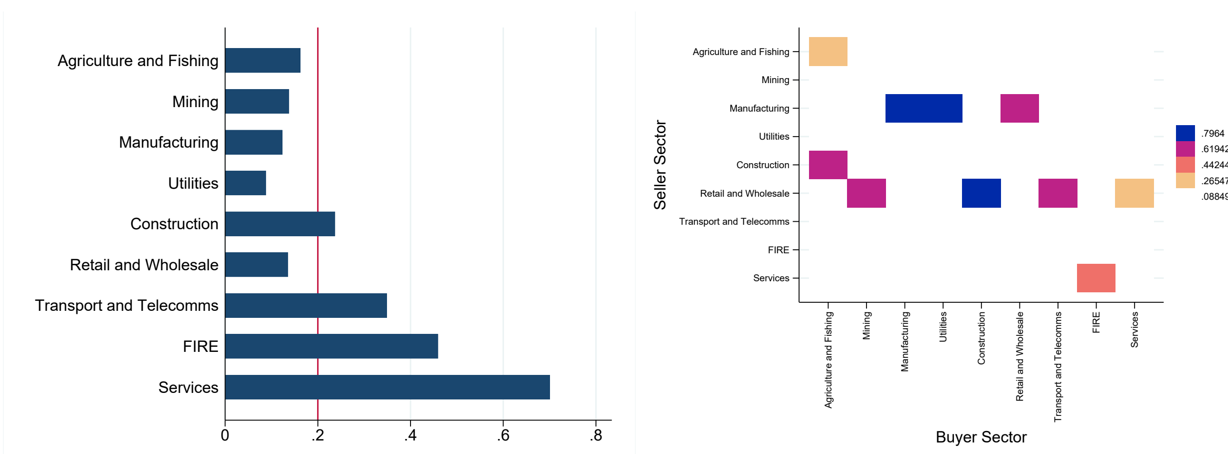
Figure F.1: Final Consumption Shares: By Sector (α_k)



Notes: This figure presents the calibration, at the sectoral level, of final consumption shares (α_k). The details of how these parameters are calibrated are presented in Section 5.2.

Figure F.2: Sectoral Input Share in Production: By Sector ($\beta_{k,L}$, β_{kl})

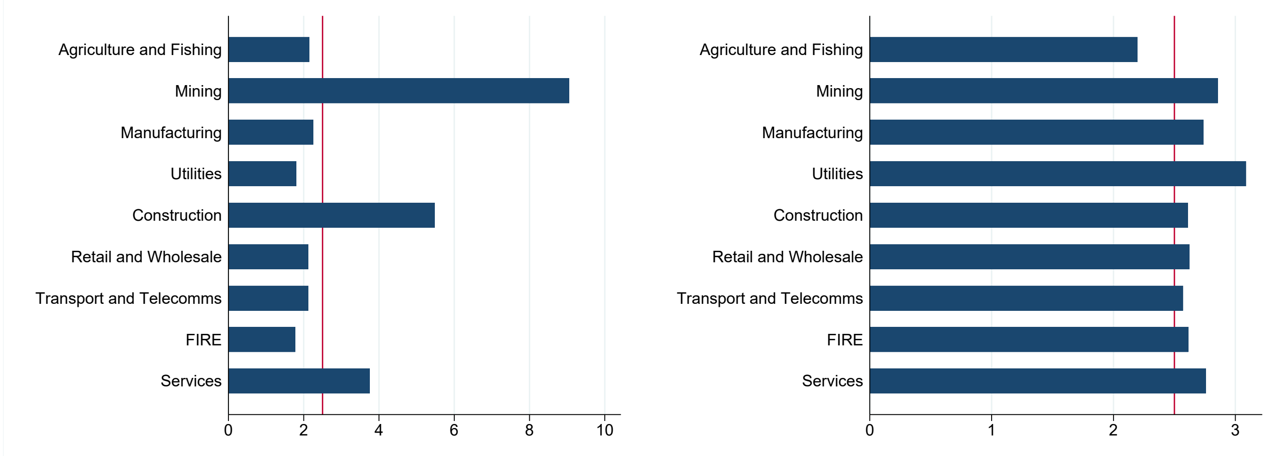
(a) Labor Shares of Cost: By Sector ($\beta_{k,L}$) (b) Intermediate Input Shares of Cost: By Sector (β_{kl})



Notes: This figure presents the calibration, at the sectoral level, of the input shares in production. Panel (a) presents the labor share of costs ($\beta_{k,L}$). The red vertical represents the average labor share across sectors. Panel (b) presents the intermediate input shares of production (β_{kl}). The details of how these parameters are calibrated are presented in Section 5.2.

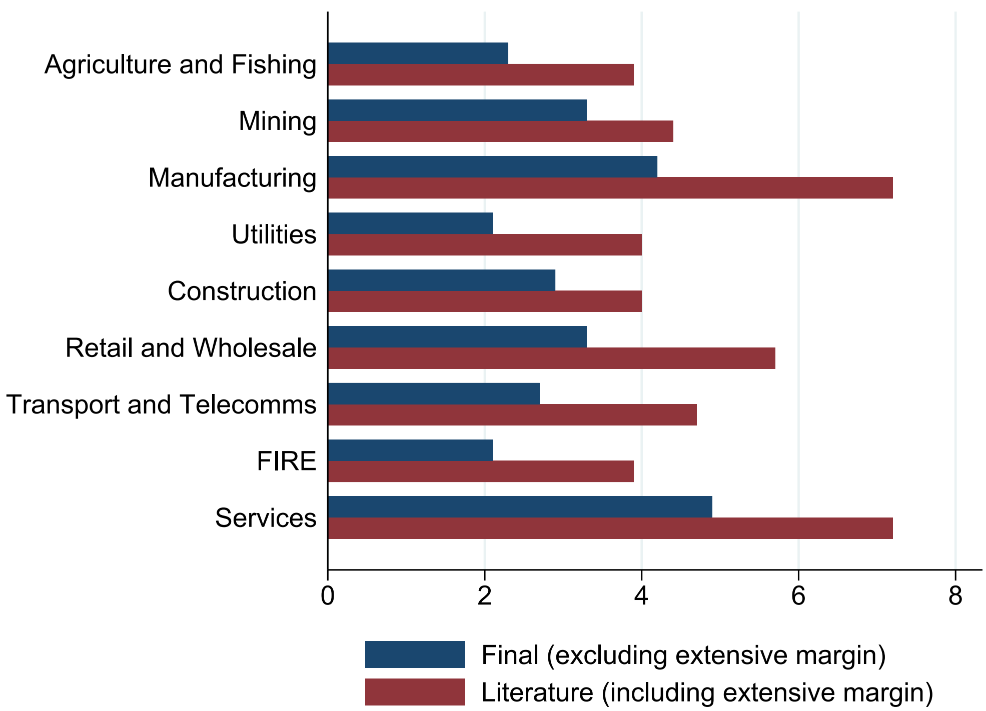
Figure F.3: Curvature of Advertisement Cost: By Sector

(a) Suppliers (γ_k^S) (b) Buyers (γ_k^B)



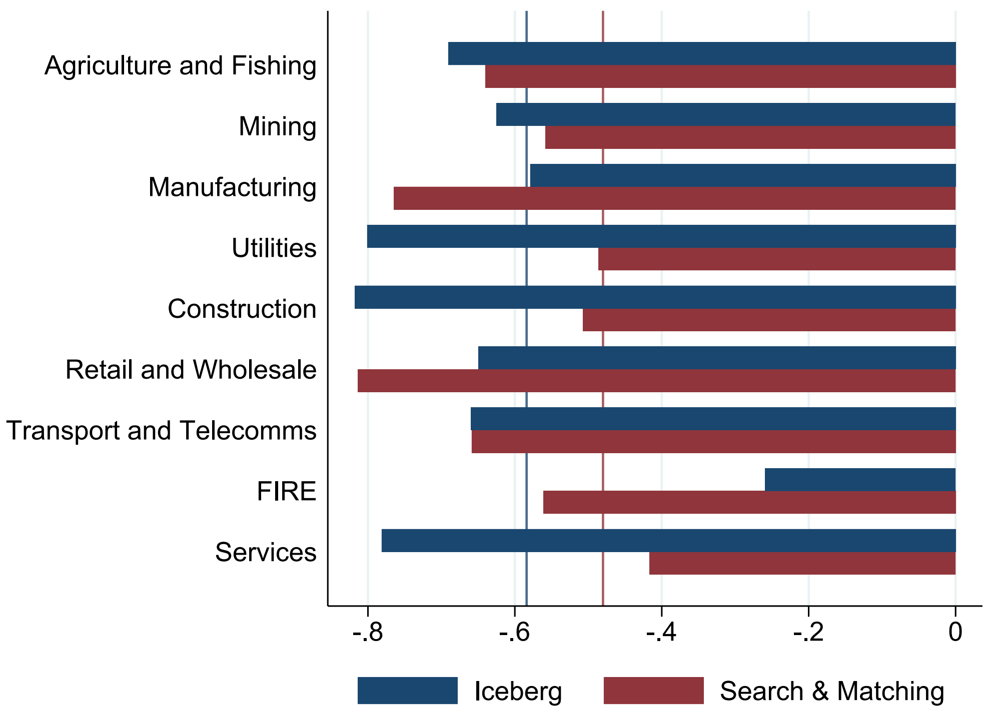
Notes: This figure presents the calibration, at the sectoral level, of the curvature of advertisement costs, for both suppliers (γ_k^S) and buyers (γ_k^B). The red vertical represents the average curvature across sectors, which is the same for suppliers and buyers. The details of how these parameters are calibrated are presented in Section 5.2.

Figure F.4: Elasticity of Substitution (σ_k): By Sector



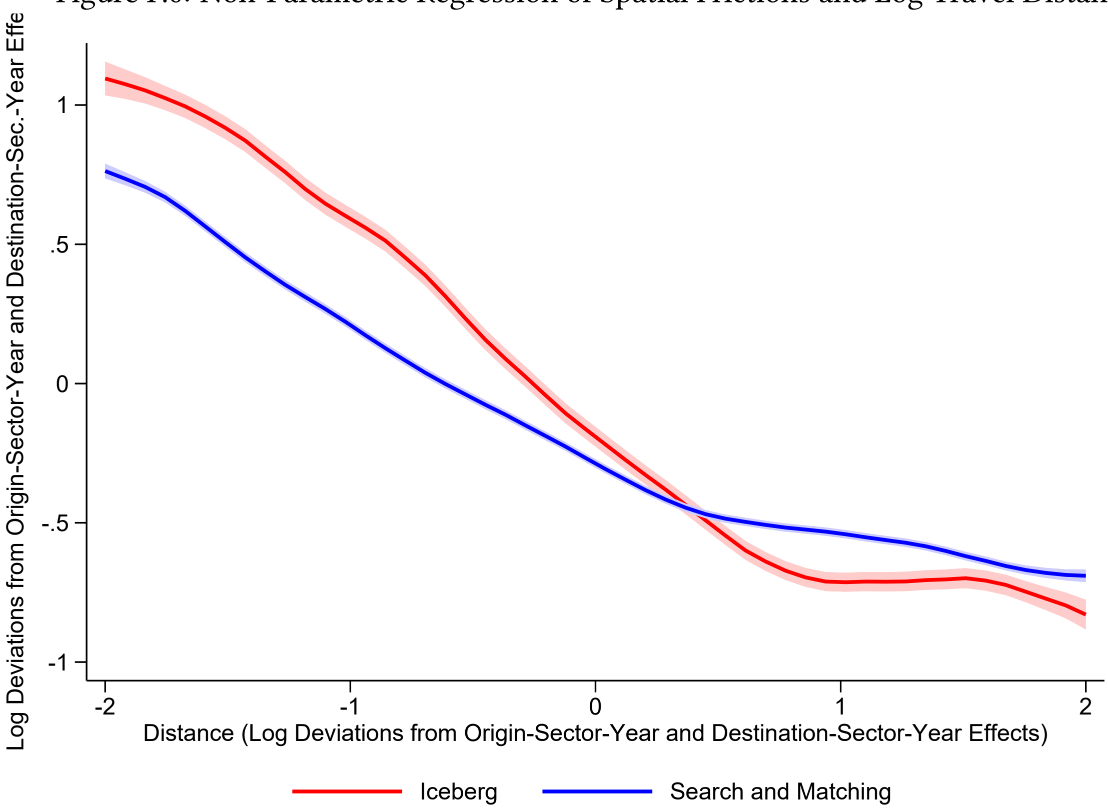
Notes: This table presents the results of the calibration of the demand elasticity of substitution for each sector, σ_k. The red bars present the estimates taken from Fontagné, Guimbard, and Orefice (2022). The blue bars present the calibration of our baseline model once we incorporate the role of the extensive margin of the production network, as explained in Section 5.2.

Figure F.5: Relationships between $\log(\tilde{\chi}_{ud,kl}^{\text{iceberg}})$ and $\log(\tilde{\chi}_{ud,kl}^{\text{matching}})$ and Geographic Proximity: Travel Time Regressor by Supplier Sector



Notes: This figure presents the regression coefficients of the regression of Panel (a) on travel time of Table 3 separately for each supplier sector.

Figure F.6: Non-Parametric Regression of Spatial Frictions and Log Travel Distance



Notes: This figure presents the non-parametric correlation at the municipality-sector-pair level between the log iceberg friction $\log(\tilde{\chi}_{ud,kl}^{iceberg})$ (red line) and the log search and matching friction $\log(\tilde{\chi}_{ud,kl}^{matching})$ (blue line), and log travel distance between municipalities. The non-parametric fit is implemented with local linear regressions after extracting municipality-sector of origin and municipality-sector of destination effects. Confidence intervals are presented at the 95% confidence.

G Additional Figures and Tables for Section 6

In this section, we provide additional results of our counterfactual simulation of international trade shocks as presented in Section 6.1.

Table G.1: Aggregate Welfare on International Trade Shocks: Sensitivity

	1) $\widehat{\text{Welfare}}$	2) Rel. to Baseline	3) $\hat{X}_{ui,u\in\{\text{US,China}\}}$	4) $\hat{X}_{ui,u\in\text{Chile}}$	5) $\hat{M}_{ui,u\in\{\text{US,China}\}}$	6) $\hat{M}_{ui,u\in\text{Chile}}$
a) Baseline	-0.67	48	-5.95	0.23	-2.69	-0.25
b) $\beta_{k,L}+0.2$	-0.41	51	-6.22	0.24	-2.73	-0.16
c) $\mu=0$	-1.27	28	-5.22	0.21	-2.67	-0.67
d) $\mu=1$	-0.47	63	-6.17	0.24	-2.68	-0.09
e) $\lambda^S=1,\lambda^B=0$	-0.65	49	-6.23	0.23	-2.95	-0.16
f) $\lambda^S=0,\lambda^B=1$	-0.70	46	-5.95	0.24	-2.66	-0.32
g) $\lambda^S=\lambda^B=0.6$	-0.89	36	-6.43	0.25	-3.56	-0.40
h) $\lambda^S=\lambda^B=0.3$	-0.45	71	-5.13	0.20	-1.35	-0.09
i) $\lambda^S/\lambda^B=\sigma/(\sigma-1),\lambda^S+\lambda^B=1$	-0.67	48	-5.98	0.23	-2.71	-0.23

Notes: The results of the counterfactual simulation of the tariff changes from and to the US and China as observed between 2001 and 2016 where we set alternative parameters as indicated in the left column. See the footnote of Table 5 for further details.

Table G.2: Aggregate Welfare on International Trade Shocks: Sensitivity of the Decrease of Tariffs

	1) $\widehat{\text{Welfare}}$	2) Rel. to Baseline	3) $\hat{X}_{ui,u\in\{\text{US,China}\}}$	4) $\hat{X}_{ui,u\in\text{Chile}}$	5) $\hat{M}_{ui,u\in\{\text{US,China}\}}$	6) $\hat{M}_{ui,u\in\text{Chile}}$
a) Baseline	0.99	40	7.55	-0.29	3.02	0.31
b) $\beta_{k,L}+0.2$	0.56	43	8.03	-0.30	3.06	0.17
c) $\mu=0$	1.92	23	6.41	-0.26	3.04	0.96
d) $\mu=1$	0.69	53	7.91	-0.31	3.00	0.08
e) $\lambda^S=1,\lambda^B=0$	0.94	42	8.00	-0.29	3.31	0.20
f) $\lambda^S=0,\lambda^B=1$	1.07	37	7.72	-0.31	3.16	0.45
g) $\lambda^S=\lambda^B=0.6$	1.49	27	8.37	-0.33	4.27	0.63
h) $\lambda^S=\lambda^B=0.3$	0.61	65	6.25	-0.24	1.43	0.11
i) $\lambda^S/\lambda^B=\sigma/(\sigma-1),\lambda^S+\lambda^B=1$	0.99	40	7.58	-0.29	3.03	0.30

Notes: This table presents the results of the counterfactual simulation of the tariff changes from and to the US and China as observed between 2001 and 2016 where we set alternative parameters as indicated in the left column. Instead of the baseline counterfactuals, here we implement a tariff reduction as in the data. Thus, this table presents the tariff changes in Table G.1. The last row is motivated by the necessary condition for optimal equilibrium search (see Appendix D).

Table G.3: Aggregate Welfare on International Trade Shocks: Export versus Import Tariff Changes

(a) Only Import Tariff Changes

	1) $\widehat{\text{Welfare}}$	2) Rel. to Baseline	3) $\hat{X}_{ui,u\in\{\text{US,China}\}}$	4) $\hat{X}_{ui,u\in\text{Chile}}$	5) $\hat{M}_{ui,u\in\{\text{US,China}\}}$	6) $\hat{M}_{ui,u\in\text{Chile}}$
a) Baseline	-0.59	100	-5.57	0.23	-2.52	-0.20
b) Exogenous Network: Low Sigma	-0.38	65	-2.14	0.10	0	0
c) Exogenous Network: Baseline Sigma	-0.32	55	-3.88	0.15	0	0
d) Exogenous Network: High Sigma	-0.32	54	-5.54	0.20	0	0

(b) Only Export Tariff Changes

	1) $\widehat{\text{Welfare}}$	2) Rel. to Baseline	3) $\hat{X}_{ui,u\in\{\text{US,China}\}}$	4) $\hat{X}_{ui,u\in\text{Chile}}$	5) $\hat{M}_{ui,u\in\{\text{US,China}\}}$	6) $\hat{M}_{ui,u\in\text{Chile}}$
a) Baseline	-0.08	100	-0.36	0.00	-0.13	-0.02
b) Exogenous Network: Low Sigma	-0.02	19	-0.22	0.00	0	0
c) Exogenous Network: Baseline Sigma	-0.00	1	-0.36	0.01	0	0
d) Exogenous Network: High Sigma	-0.00	2	-0.46	0.01	0	0

Notes: These tables present the results of the counterfactual simulation of the tariff changes from and to the US and China, but changing them from the value in 2016 to the value in 2001 (the inverse of the tariff reductions shown in Table G.1), where we change the import and export tariffs one by one. See the footnote of Table 5 for further details.

Table G.4: Different Signs and Magnitudes of the Tariff Changes

	1) $\widehat{\text{Welfare}}$	2) Exog. Network / Baseline (%)	3) $\hat{X}_{ui,u\in\{\text{US,China}\}}$	4) $\hat{X}_{ui,u\in\text{Chile}}$	5) $\hat{M}_{ui,u\in\{\text{US,China}\}}$	6) $\hat{M}_{ui,u\in\text{Chile}}$
a) Large Increase of Tariffs (Baseline Counterfactual)	-0.67	48	-5.95	0.23	-2.69	-0.25
b) Small Increase of Tariffs (10% of Row (a))	-0.07	40	-0.69	0.03	-0.31	-0.04
c) Large Decrease of Tariffs (Inverse of Row (a))	0.99	40	7.55	-0.29	3.02	0.31
d) Small Decrease of Tariffs (10% of Row (c))	0.06	41	0.66	-0.03	0.29	0.04

Notes: This tables presents the results of the counterfactual simulation to change tariffs from and to China and the US. Besides our baseline counterfactual in Row (a) (increases in tariff in the magnitudes of the inverse of the tariff reductions in Table G.1), Row (c) presents the observed tariff changes as they appear in Table G.1. Row (b) and (d) present a small fraction (10%) of the tariff changes of Row (a) and (c), respectively. See the footnote of Table 5 for further details.