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INDUSTRY LINKAGES FROM JOINT PRODUCTION

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Abstract

I develop a theory of joint production to quantify aggregate economies of scope. In US manufacturing data, increased export demand in one industry raises a firm's sales in its other industries that share *knowledge* inputs like R&D and software. I estimate that knowledge inputs contribute to economies of scope through their scalability and partial non-rivalry within the firm. On average a 10 percent increase in output in one industry lowers prices in other industries by 0.4 percent. Such economies of scope manifest disproportionately among knowledge-proximate industries and imply large spillover impacts of recent US-China trade policy on producer prices.

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Introduction

Multi-industry firms produce three-quarters of US manufacturing output. What explains this prevalence of joint production, and how does it shape the aggregate impact of industry-level shocks to foreign demand or tariffs? Despite a mature literature on the theory of joint production,[1] there is little empirical evidence on how a firm's output in one industry affects its marginal costs in another industry. Quantitative models in trade and macroeconomics instead assume that firms operate *nonjoint*, industry-specific production functions and remain silent on the aggregate consequences of joint production.[2]

This paper combines new theory and evidence to show that joint production within the firm generates economies of scope in the aggregate. On average producer prices fall with respect to output in not only the same industry but also other industries. I estimate that such economies of scope are due to the scalability and partial non-rivalry of shared knowledge-producing inputs such as R&D, software, and management. As a firm scales up shared knowledge inputs to produce more output in any one industry, the non-rivalrous nature of knowledge inputs also increases output in the firm's *other* industries. Economies of scope from joint production contribute to an aggregate elasticity of prices to output of -0.04, constituting one quarter of typical estimates of aggregate increasing returns to scale. Far from uniform, economies of scope are clustered among knowledge-intensive industries, indicating that shocks to industries such as electronics, aerospace, optical and medical equipment have disproportionate and widespread consequences.

I begin in Section 1 by testing and rejecting the usual assumption that a firm's production technology is nonjoint. I assemble panel data from the US Economic Census between 1997 and 2007 on the sales and exports of all US multi-industry firms in each of 206 manufacturing industries. I leverage variation in firms' exports by product and destination country along with changes in the size of these markets to construct plausibly exogenous demand shifters for each industry of the firm. If the production technology were nonjoint, a demand shock in one industry of a firm would have no impact on its sales in any another industry.

Instead, I find that a positive demand shock in one industry of a firm increases its sales in another the more that the two industries share *knowledge* inputs. I define knowledge inputs based on the NAICS classifications of headquarter services, professional and technical services, information, and the leasing of intangible assets, and I measure input-proximity using the BEA's input-output tables. The cross-industry impact of demand shocks on sales increases with proximity in the use of knowledge inputs but not proximity in any other type of input, such as agricultural or manufactured intermediates. These findings suggest that knowledge inputs have distinct properties

[1] A production technology is nonjoint if and only if the cost function can be written as $C(q_1, \ldots, q_J) = \sum_{j=1,\ldots,J} C_j(q_j)$ (see Shephard, 1953; Diewert, 1973; Lau, 1972; Hall, 1973; Baumol et al., 1982).

[2] For example, this assumption is adopted in influential models of firm heterogeneity in international trade (Bernard et al., 2010; Mayer et al., 2014), macroeconomics (Klette and Kortum, 2004), as well as industrial organization (Foster et al., 2008; De Loecker et al., 2016).

in production, consistent with recent evidence on their sharability and scalability within the firm (Atalay et al., 2014; Haskel and Westlake, 2017; Ding et al., 2022).

To interpret this empirical evidence, in Section 2, I develop a quantitative model of joint production where heterogeneous firms produce potentially multiple outputs using multiple inputs over two stages. In the first stage, the firm decides how much knowledge to accumulate in each industry. Accumulated knowledge is a proxy for the net contribution of any firm-level inputs such as R&D, software, management, or physical capital that are shared across the firm's industries. In the second stage, the firm takes this knowledge as given and uses a variety of industry-specific inputs to generate final output in each industry facing CES demand under monopolistic competition. Whereas the firm's second-stage production decisions are separable by industry, the firm's use of shared inputs in the first-stage generates interdependence across industries.

Two key properties of shared inputs—scalability and rivalry—parametrize interdependence in costs and generate cross-industry elasticities of sales to demand shocks that are heterogeneous and unrestricted in sign. The more scalable and the less rival are shared inputs, the more that a positive demand shock in one industry would increase sales in another. Consider how a shared input like R&D is used within General Electric in response to increased demand for aviation equipment. GE would scale up its expenditures on R&D, for example by hiring more scientists to improve the hot gas path of its aviation turbines. The more scalable is R&D, the more scientists GE would hire on the margin. And the less rival is R&D, the more likely are the additional scientists to create ideas (e.g., high-sensitivity scanning) that also improve productivity in GE's MRI scanners, or even to start up entirely new industries. On the other hand, if R&D is difficult to scale up and rival in use, increased demand for aviation turbines would cause GE to (optimally) reallocate R&D resources away from its other industries.

I parametrize scalability and rivalry as elasticities that potentially vary across different types of shared inputs. Outputs in two industries are complements whenever their shared inputs (like R&D) are on net more scalable and less rival. Expanding output in one industry would lower marginal costs and increase output in another. In contrast, output in two industries are substitutes when their shared inputs are less scalable and more rival. Consider, for example, a different shared input like real estate, which enters intensively in the production of both glass and metal hardware. If real estate is harder to scale and more rival in use, expanding output of glass would come at the expense of metal hardware. Technology coefficients in the model parametrize fundamental differences in how industries benefit from different types of shared inputs (like real estate versus R&D), allowing some industry pairs to be complements in production while others substitutes.

In Section 3, I estimate this joint production technology using variation *within* the firm. Rather than presuming that knowledge inputs (or other inputs) are scalable and non-rival, I let the data speak. Similar to how demand shocks trace out the supply curve in a textbook single-industry setting, in my multi-output setting, demand shocks identify both own- and cross-partial derivatives of the firm's cost function (and thus the scalability and rivalry of proximate shared inputs). I express this identification logic using micro moment conditions that set the same- and cross-industry covariances of demand shocks and sales growth within the firm to equal that in the model (conditional on firm observables). Same-industry covariances identify input scalability, whereas cross-industry covariances identify input rivalry.

I estimate these micro elasticities jointly with the model's macro-level parameters under a nested fixed-point algorithm. Conditional on micro elasticities (scalability and rivalry of shared inputs and parameters governing firm heterogeneity), I solve for the macro aggregates in the model to exactly match industry-level data in each year. For instance, data on industry-level output identify residual profitability in the model, and data on bilateral industry input expenditures identify technology coefficients behind shared inputs. These macro parameters in turn influence the micro moment conditions. For example, changes in industry residual profitability influence the model's predictions for firms' sales growth. And technology coefficients govern industries' proximity in relation to shared inputs, allowing me to separately identify the scalability and rivalry of knowledge inputs from other types of shared inputs like physical capital.

I estimate that knowledge inputs are scalable and partially non-rival within the firm. My estimates of scalability are consistent with Aghion et al. (2019) and Lashkari et al. (2019), who find that French firms increase R&D and IT expenditures in response to positive demand shocks. Whereas this existing literature treats the entire firm as the unit of analysis (and models output at the level of the firm), my estimates of input non-rivalry are new and imply that outputs are complements across knowledge-proximate industries *within* the firm.[3] I also find that other shared inputs (like physical capital) are in comparison less scalable and more rival, which causes output to be substitutes across industries more proximate in these other types of inputs. The co-existence of complementarities and substitutabilities imply that joint production can be responsible for both economies as well as diseconomies of scope.

In Section 4, I use these estimates to quantify the macroeconomic implications of joint production. I compute the aggregate elasticity of the producer price index in any one industry to demand shocks in any other industry. In general equilibrium, any initial industry demand shock could change firms' joint production decisions, affect price indices and residual demand in other industries, and trigger yet further responses among other firms. I show that the total impact of a demand shock as it percolates across industries can be computed by inverting a matrix of demand elasticities and joint production parameters—the scalability and rivalry of shared inputs in the first stage and same-industry production returns to scale in the second stage.

Under a calibration to US manufacturing data from 2017, I simulate a proportional increase in foreign demand in each industry and decompose the equilibrium change in the US producer

[3] Numerous papers study the effects of specific knowledge inputs in isolation, for example R&D in Aw et al. (2011), marketing in Arkolakis (2010), management in Bloom et al. (2019), and ICT in Fort (2016). These papers focus on the scalability of knowledge inputs in single-output production rather than their rivalry in use *across* multiple outputs.

price index into (i) within-industry economies of scale and (ii) cross-industry economies of scope. I find that both within- and cross-industry elasticities of price to output are on net negative. My estimates of within-industry elasticities match those in recent papers such as Bartelme et al. (2019) and Lashkaripour and Lugovskyy (2019) that feature external economies of scale. However, unique to my model, negative *cross*-industry elasticities are internal to the firm and occur because shared knowledge inputs are scalable and partially non-rival under joint production.

I estimate that economies of scope from joint production contribute to an aggregate elasticity of price to output of -0.04, nearly one quarter of typical estimates of aggregate increasing returns in manufacturing. Far from uniform, economies of scope are concentrated among knowledge-proximate industries. For example, a demand shock in the computer electronics industry that raises aggregate output by 10 percent would lower prices in other industries by on average 1.4 percent. In comparison, mild diseconomies of scope exist across industries that predominantly share other types of inputs (and very little knowledge). For example, in the production of flavoring syrup, the industry with the highest diseconomies of scope, a demand shock that raises aggregate output by 10 percent would *raise* prices in other industries, but only by 0.1 percent on average.

The endogeneity of producer prices under joint production suggests that trade policy and market access are determinants of a nation's comparative advantage. As a proof of concept, I show that joint production amplifies the producer price impact of the recent US-China trade war. Unilateral import tariffs increase domestic market share for US firms, triggering scale and scope economies that mute (by more than one third) the direct adverse impact of higher import prices on the CPI. In contrast, retaliatory tariffs restrict US market access in China. The loss of foreign demand works in reverse to *increase* producer prices, especially among knowledge-proximate and export-intensive industries. Retaliation offsets more than half of the initial producer price decline from unilateral US import tariffs, but also widens the initial distributional impact across industries. Producer prices in import-competing sectors like textiles and furniture retain their benefit from import protection, whereas prices in export-oriented and-knowledge intensive sectors like transportation equipment, chemicals and computers face the most dramatic increases after retaliation.

Related Literature. This paper contributes to a vast literature on multi-output firms. I develop a tractable, quantifiable model where shared knowledge inputs give rise to joint production across 206 manufacturing industries, a relevant margin at which tariffs and other shocks occur. Papers in international trade typically model interdependence across a firm's product varieties due to demand-side cannibalization (e.g., Eckel and Neary, 2009; Feenstra and Ma, 2007; Dhingra, 2013) or span-of-control (e.g., Nocke and Yeaple, 2014; Bernard et al., 2018). Whereas these models predict negative and homogeneous cross-product impacts of demand shocks on sales (and entry), I find that cross-*industry* impacts increase with knowledge input-proximity. A related paper by

Boehm et al. (2022) finds that economies of scope from shared *manufactured* inputs determine Indian firms' industry entry choices. In comparison, I find that shared knowledge inputs generate economies of scope in US firms and my model endogenizes both intensive and extensive margin changes within the firm. Another strand of empirical research has documented spillovers within internal firm networks (e.g., Giroud and Mueller, 2019) but does not consider joint production as the mechanism.[4] On the other hand, papers in industrial organization and agricultural economics have provided flexible estimates of joint production but only in settings limited to two or three types of outputs.[5]

My paper provides evidence to corroborate a common *presumption* that knowledge is non-rival within the firm. Knowledge-sharing is a key premise in many literatures, from early theories of the firm boundary (Penrose, 1959; Gort, 1962; Rubin, 1973; Teece, 1980), the multinational enterprise (Helpman, 1984; Markusen, 1984), to macroeconomic models of firm scope (Jovanovic, 1993; Hsieh and Rossi-Hansberg, 2020; Argente et al., 2021). Research in corporate finance has also suggested that the non-rivalry of intangible capital (formed out of knowledge inputs like R&D) might be responsible for recent changes in firm scope and concentration (Hoberg and Philips, 2022; Crouzet et al., 2022b). Different from these papers, my model is *a-priori* agnostic on the role of knowledge. I contribute an econometric framework where properties of knowledge (and other shared inputs) can be estimated in the data. I impose no restrictions on the sign, magnitude, or symmetry of same- and cross-industry price elasticities of supply. This level of generality is typically limited to non-parametric models of joint production (Färe and Primont, 1995), where identification requires data on real inputs and outputs at the level of the firm. In contrast, my model requires less onerous data (on only sales, demand shocks and input expenditures) without sacrificing generality.

My estimates of scale and scope economies contribute to a macroeconomic literature estimating aggregate production functions. Hall (1973) provides an econometric framework for testing joint production, but subsequent empirical tests using maco data at the level of two or three sectors have remained inconclusive (Burgess, 1976; Kohli, 1981). I circumvent the empirical challenge confronting these papers through a different micro-founded model and estimation strategy. My estimates of economies of scope also resolve the 'aggregation puzzle' noted by Caballero and Lyons (1992) and Basu and Fernald (1997). I find that aggregate increasing returns are 28 percent higher than the sum of within-industry increasing returns to scale precisely because industry-level estimates miss cross-industry price declines in equilibrium.

[4]For example, mechanisms for intra-firm spillovers range from internal capital markets (Stein, 1997; Lamont, 1997), multinational knowledge transfer (Keller and Yeaple, 2013; Cravino and Levchenko, 2016; Bilir and Morales, 2019), vertical supply linkages (Desai et al., 2009; Boehm et al., 2019), to distance (Giroud, 2013; Gumpert et al., 2019). None of these papers consider industry heterogeneity or how the input-sharing might give rise to joint production.

[5]For example, Dhyne et al. (2017) estimate pairwise relationships between manufacturing industries, and Grieco and McDevitt (2016) estimate tradeoffs between the quality and quantity of dialysis care. The Cobb-Douglas functional form in these papers, however, presumes that different types of outputs are substitutes in production. Pokharel and Featherstone (2019) non-parametrically estimate the cost frontier but limit their analysis to four types of outputs among agricultural cooperatives.

Besides aggregate price impacts, joint production in my model provides a wholly technological explanation for industry linkages. A shock in any one industry can affect outcomes in another. This finding complements existing papers where other mechanisms *external* to the firm shape industry interactions. In the neoclassical trade literature that estimates similar aggregate supply functions (Harrigan, 1997), general equilibrium factor price movements determine cross-industry impacts. Other external mechanisms include agglomeration externalities (Ellison et al., 2010), innovation spillovers (Bloom et al., 2013), input complementarity (Jones, 2011), and production networks (Hulten, 1978).[6] These unrelated mechanisms operate *across* firms and would most likely amplify the quantitative relevance of joint production *within* the firm. For example, I find that embedding joint production within an input-output production structure à la Caliendo and Parro (2014) more than doubles the baseline cross-industry price elasticity of output.

1 Data and Empirical Evidence

1.1 Multi-Industry Firms in US Manufacturing

I assemble data on the universe of US manufacturing firms' sales by industry, every five years between 1997 and 2012. First, I obtain sales by product line at each establishment of the firm using product trailer files from the US Census of Manufactures. Next, I aggregate sales across a firm's products and establishments to the level of 206 industries $j \in \mathcal{J}$ (roughly 5-digit NAICS, the most disaggregated level at which BEA input-output data are available).[7] Finally, I combine this dataset with the Longitudinal Foreign Trade Transaction Database (LFTTD), which contains data on each firm's exports and imports (if any), by product and country.

Table 1 highlights the prevalence of multi-industry firms and the stability of these trends over time. One-fifth of all US manufacturers produce in two or more manufacturing industries, accounting for a disproportionate three-quarters of manufacturing sales, exports, imports, and employment in each year. The second and third rows of Table 1 reveal that this dominance stems not only from each firm's primary (highest sales) industry. Output from these firms' remaining industries still contribute about one-quarter of overall manufacturing activity.

Table 1 also reveals considerable limits to firm scope. The median multi-industry firm produces in only two industries, and the two industries are sufficiently dissimilar that they belong in different *sectors* (3-digit NAICS). Given the already-coarse definition of an industry ($j \in \mathcal{J}$ is a 5-digit NAICS

[6] Research on production networks (e.g. Gabaix, 2011; Acemoglu et al., 2012, 2016; di Giovanni et al., 2018; Baqaee and Farhi, 2019; Liu, 2019; Lim, 2018) has focused on how productivity shocks propagate across industries, typically under constant returns to scale, whereas my paper focuses on how demand shocks propagate under non-constant returns.

[7] By aggregating over plants and products within an industry, I abstract from the plant-dimension and product-variety dimension of the firm, the subject of much existing research. Further, to avoid the possibility that sales in multi-plant firms would be over-counted (relative to that of a single-plant firm) when output is shipped from one plant for use as an input in another, I subtract intra-firm shipments from a plant's total shipments and use the resulting value as my measure of (external) sales. This matters little in practice because intra-firm shipments constitute a trivial fraction (between 1 and 2 percent) of aggregate shipments in my data, consistent with Atalay et al. (2014).

Table 1: Summary Statistics on US Multi-Industry Manufacturing Firms

	1997	2002	2007	2012
Share of aggregate outcome (out of all manufacturing firms):				
Number of firms	.19	.20	.20	.20
External manufacturing sales	.74	.74	.74	.75
- from firm's primary (highest grossing) industry	.47	.48	.51	.52
- from firm's remaining industries	.27	.25	.23	.23
Employment	.62	.63	.61	.60
Exports	.84	.80	.81	.76
Imports	.82	.79	.79	.77
Mean and median scope (among multi-industry firms):				
Mean number of industries	2.69	2.73	2.63	2.65
Median number of industries	2	2	2	2
Mean number of sectors	1.69	1.74	1.69	1.70
Median number of sectors	2	2	2	2

Notes: A firm is multi-industry if it manufactures goods under at least two distinct industry classifications in that year. An industry is defined at roughly the 5-digit NAICS level, of which there are 206 in manufacturing. A sector refers to a 3-digit NAICS code, of which there are 21 in manufacturing. External manufacturing sales is equal to the firm's gross manufacturing sales less its total inter-plant shipments reported. Employment refers to the firm's employment at manufacturing establishments.

code), I interpret multi-industry firm activity in my data as reflecting the production of sufficiently distinct categories of goods (like MRI scanners versus jet engines in General Electric) rather than closely substitutable product varieties, the subject of much existing research.[8]

1.2 An Empirical Test of Nonjoint Production

By itself, the cross-sectional data in Table 1 reveals nothing about whether output is jointly determined across a firm's industries. The simplest way to rationalize the data is to model the multi-industry firm as a random collection of industry lines each operating independently (i.e., under nonjoint production, as in Bernard et al., 2010 or Klette and Kortum, 2004).

I thus turn to within-firm variation over time to test this conventional assumption of nonjoint production. Under the null hypothesis, a firm's sales in a given industry would increase in response to a demand shock in that same industry but would be unaffected by demand shocks in any of its other industries. I carry out this test using variants of the reduced-form specification in equation (1). I regress the sales growth of each firm f in a given industry j ($\Delta \log X_{fjt}$) on its demand shock in the same industry j ($\Delta \log S_{fjt}$), and its demand shocks in other industries $k \neq j$ ($\Delta \log S_{fjt}^{OTHER}$):

$$\Delta \log X_{fjt} = \psi^{SAME} \Delta \log S_{fjt} + \psi^{CROSS} \Delta \log S_{fjt}^{OTHER} + \text{Controls}_{fj,t-1} + FE_{jt} + \epsilon_{fjt}, \quad (1)$$

[8]See, e.g., Feenstra and Ma (2007), Arkolakis et al. (2019), and Macedoni and Xu (2019), who focus on interdependence across substitutable and symmetric product varieties, highlighting a demand-side rather than supply-side mechanism.

where Δ is a five-year first-difference operator between t and $t-1$, and $t \in \{1,2,3\}$ maps to years 1997, 2002, and 2007 in the data.[9] Industry-year fixed effects (FE_{jt}) control for unobserved supply and demand shocks common to all firms within each given industry j, while Controls$_{fj,t-1}$ include initial-period firm-level and firm-industry-level characteristics (such as size and export intensity) that might explain non-parallel growth trends.

Constructing Export Demand Shocks. My identifying assumption requires demand shocks ($\Delta \log S_{fkt}$) in any other industry $k \neq j$ of the firm to be conditionally uncorrelated with the error term ϵ_{fjt}, which comprises unobserved supply and demand-side shocks for the same firm in industry j.

I construct plausibly exogenous firm-industry-level demand shocks by leveraging differential exposure of US firms to changes in foreign market size. First, using the BACI Comtrade dataset, I measure each foreign destination n's import growth in each HS 6-digit product h (excluding imports from the US). I denote this five-year market-size change by $\Delta \log IMP_{nht}$ (a destination n and product h pair).[10] Next, I average these market size shifters to the firm-industry level using the firm's initial-period export shares ($s_{fjnh,t-1}$) across these markets as weights:

$$\Delta \log S_{fjt} = s^*_{fj,t-1} \sum_n \sum_{h \in H_j} s_{fjnh,t-1} \Delta \log IMP_{nht}, \tag{2}$$

where $s_{fjnh,t-1}$ is the firm's exports of HS product h to destination n as a share of its total exports in industry j (containing HS6 products $h \in H_j$), and the share $s^*_{fj,t-1}$ scales the demand shock by the firm's export intensity in industry j in year $t-1$ (so firms that sell predominantly at home receive appropriately smaller export demand shocks). I use data from the LFTTD on firm exports by destination and product to construct the export intensity and share variables.

A novel aspect of my empirical strategy is that demand shocks vary across industries *within* the firm. A large literature has constructed *firm*-level export demand shocks by aggregating over variation across all products and destinations among a firms' exports (see Hummels et al., 2014; Mayer et al., 2016; Aghion et al., 2019; Garin and Silverio, 2018, who use firm-level data from various countries in Europe). I build on this approach to extract variation at the level of different

[9] I do not use the time period 2007-2012 because (i) the financial crisis generated correlated shocks across countries, industries, as well as firms, jeopardizing independent variation in demand shocks, and (ii) variation in expenditure shares from the 1997 input-output table used to construct other-industry demand shocks would have become less relevant by 2012.

[10] A market's import growth $\Delta \log IMP_{nht}$ could reflect both (i) changes in the level of demand, and (ii) changes in relative foreign producer prices. Both sources of variation are relevant shifters of a US firm's residual demand, though they would move residual demand in opposite directions. Column (1) of Table 2 shows that empirically, the first force dominates. Nevertheless, one threat to my identification assumption is that import growth $\Delta \log IMP_{nht}$ could reflect idiosyncratic supply-side shocks within a sufficiently large US exporter. For example, if GE, a major exporter, became more productive as a firm and exported more MRI scanners to India, Indian imports of MRI scanners from *non-US* countries could fall. At the same time, the firm-wide productivity shock would raise GE's output in other industries. To mitigate this possibility, I construct export demand shocks in equation (2) using only variation from export markets where the firm's market share is below 10%. None of my results hinge on this choice or on the value of the threshold.

industries *within* a firm. Much to my advantage, in the US data, manufacturers have extensive export networks that vary by destination and product. The median number of destination-product (nh) export markets within a single industry of a firm in my sample is 6.2, and the mean is 24.1, from among tens of thousands of observed destination-product pairs.

Input Proximity and Cross-Industry Impacts. Under the null hypothesis that the production technology is nonjoint, a firm's sales in an industry j would be unaffected by a demand shock in *any* of its other industries $k \neq j$. While in principle I can test whether all 206×205 pairwise cross-elasticities ψ_{jk}^{CROSS} are zero (by setting $\Delta \log S_{fjt}^{OTHER} = \Delta \log S_{fkt}$ in equation 1), I lack statistical power given my limited sample size and sparsity in firms' industry presence (the median firm is active in only two industries). Instead, I test two simpler, necessary conditions for the null to hold: whether (i) cross-elasticities are zero on average, and (ii) cross-elasticities do not vary with industries' input proximity. A rejection of either condition suffices for rejecting the null hypothesis of nonjoint production.

First, to test condition (i), I average demand shocks in each of the firm's other industries $k \neq j$ using each industry's share in firm sales as weights:

$$\Delta \log S_{fjt}^{OTHER} \equiv \sum_{k \neq j} \left(\frac{X_{fk,t-1}}{\sum_{k' \neq j} X_{fk',t-1}} \right) \Delta \log S_{fkt}. \tag{3}$$

The coefficient on $\Delta \log S_{fjt}^{OTHER}$ represents the average cross-elasticity of sales to demand shocks. A non-zero coefficient would imply that this necessary condition for nonjoint production does not hold.

However, condition (i) is not sufficient for production to be nonjoint. Joint production can yield positive cross-elasticities for some industry pairs and negative cross-elasticities for others, such that the average effect measured above will wash out to zero. Such heterogeneity could come from properties of different *types* of shared inputs. For example, suppose real estate is scarce and rival across different uses. To expand output in a real estate-intensive industry, the firm would have to reallocate real estate inputs away from its other real estate-intensive industries. On the other hand, suppose another input, information technology (IT), is scalable and non-rival in use. To expand output in an IT-intensive industry, the firm would purchase incremental IT resources, and the non-rival nature of IT would increase output in the firm's other IT-intensive industries.

I test for such heterogeneity using condition (ii), which stipulates that cross-elasticities are uncorrelated with industries' input proximity. Using data from the BEA's input-output (I/O) and capital flow tables, I define input-proximity, $Prox_{fjkm}$, to measure how an industry-k shock might affect industry-j output through a potentially shared input m:

$$Prox_{fjkm} \equiv \beta_{jm} \left(\frac{\beta_{km} X_{fk}}{\sum_{k' \neq j} \beta_{k'm} X_{fk'}} \right), \tag{4}$$

where β_{jm} is the share of industry j expenditures on input m, and X_{fk} is the firm's output in industry k. Input proximity is the product of two share terms and is intentionally asymmetric and firm-specific. $Prox_{fjkm,t-1}$ is increasing in both share terms: (i) industry j's expenditures on input m (relative to other inputs m'), and (ii) expenditures on input m by shocked industry k (relative to the firm's other industries k'). The first share reflects how much industry j output might benefit from a marginal change in input m, and the second share reflects how the firm's overall use of inputs m might change with respect to a demand shock in k.

To test condition (ii), I estimate a triple-differences specification. In addition to the average other-industry shock $\Delta \log S_{fjt}^{OTHER}$, I interact the firm's demand shock in each other industry k with its proximity to industry j in relation to various sets of inputs $m \in \mathcal{M}$:

$$\Delta \log S_{fjt}^{OTHER \times \mathcal{M}} \equiv \sum_{k \neq j} \left(\sum_{m \in \mathcal{M}} Prox_{fjkm,t-1} \right) \Delta \log S_{fkt}. \tag{5}$$

A non-zero regression coefficient on this interaction variable for *any* input set $\mathcal{M}$ would imply that this necessary condition for nonjoint production does not hold. A negative coefficient is consistent with shared inputs $\mathcal{M}$ being scarce and rival within the firm (e.g., real estate), while a positive coefficient would imply that shared inputs $\mathcal{M}$ are scalable and non-rival (e.g., software).

I cluster inputs m based on their assigned BEA NAICS root codes, since inputs within a root code are likely to have similar properties. I create the following categories of inputs: agriculture (NAICS 1); construction, mining, and utilities (NAICS 2); manufacturing (NAICS 3); transportation, wholesale, and retail (NAICS 4); finance, insurance, and real estate (NAICS 52, 531, 532); information, intellectual property, management, and professional, scientific and technical services (NAICS 51, 533, 54, 55); administrative services (NAICS 56); other service inputs (NAICS 6, 7, 8, and 9); labor; and capital. The triple-differences specification also allows me to estimate whether properties of one category of inputs differs from that of another category.

1.3 Cross-Industry Impact of Demand Shocks on Output

I estimate these double- and triple-differences variants of equation (1) on a regression sample of all exporting multi-industry firms in each base year, $t-1$. An observation is a continuing industry of one of these firms over a five-year period from $t-1$ to t. This regression sample of roughly 5000 multi-industry firms per year accounts for over half of all US manufacturing gross output. Appendix Table A.2 provides summary statistics on regression variables and other attributes of firms in the sample.

Table 2 presents estimates of the same- and cross-industry impacts of demand shocks on firm sales. First, column (1) reveals that the same-industry impact is positive and statistically significant,

Table 2: Same- and Cross-Industry Impacts of Demand Shocks on Sales within the Firm

Change in firm-industry sales, $\Delta \log X_{fjt}$	(1)	(2)	(3)	(4)	(5)
Same-industry demand shock	0.45	0.46	0.46	0.51	0.37
$\Delta \log S_{fjt}$	(0.10)	(0.10)	(0.09)	(0.10)	(0.19)
Other-industry demand shocks					
(i) Average effect		-0.08	-0.83	-0.81	-1.67
$\Delta \log S_{fjt}^{OTHER}$		(0.12)	(0.24)	(0.26)	(0.52)
(ii) $\times$ knowledge input-proximity			8.00	8.26	13.31
$\Delta \log S_{fjt}^{OTHER \times KLG}$			(2.25)	(2.22)	(3.58)
Industry-year fixed effects	✓	✓	✓	✓	✓
Firm-wide and firm-industry controls				✓	
Firm-year fixed effects					✓
Observations	21,500	21,500	21,500	21,500	17,500
R^2	0.06	0.06	0.06	0.12	0.39

Notes: This table estimates variants of regression equation (1): the response of sales in one industry of the firm to demand shocks in the same industry and also other industries, in 5-year differences over the period 1997-2007. Two measures of other-industry demand shocks are considered. Measure (i) averages demand shocks in other industries using sales weights (equation 3). Measure (ii) interacts demand shocks in each other-industry k with its knowledge input-proximity to industry j (equation 5 where $\mathcal{M}$ is NAICS 51, 533, 54, and 55). Firm-wide controls include initial period firm size and firm export intensity. Firm-industry level controls include firm-industry size, firm-industry export status, firm-industry export intensity, as well as averages of these characteristics in other industries, and interactions of these characteristics in other industries interacted with knowledge input-proximity. Standard errors in parentheses are clustered at the firm level.

suggesting the constructed shocks are empirically relevant as shifters of demand.[11] Next, column (2) tests and does not reject condition (i) for nonjoint production. Cross-elasticities are on average statistically insignificant from zero.

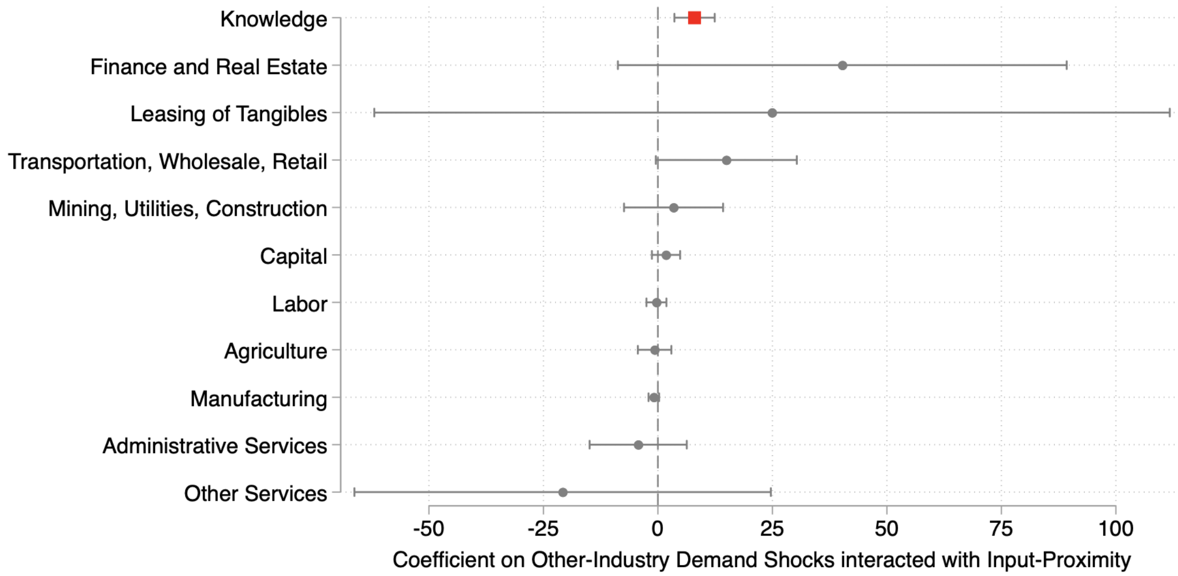
In the remaining columns, I test condition (ii)—that cross-elasticities do not vary with input-proximity. To allow for differences in cross-elasticities across input categories, I estimate a triple-differences specification separately for each category $\mathcal{M}$. I regress the firm's sales growth in industry j on other-industry demand shocks interacted by input-m proximity ($\Delta \log S_{fjt}^{OTHER \times \mathcal{M}}$), controlling for the average (homogeneous) impact of other-industry demand shocks ($\Delta \log S_{fjt}^{OTHER}$) and the same-industry demand shock ($\Delta \log S_{fjt}$).[12]

Knowledge Input-Proximity. Figure 1 plots the estimated interaction coefficients on $\Delta \log S_{fjt}^{OTHER \times \mathcal{M}}$ across input categories $\mathcal{M}$, and Appendix Table A.4 provides the results in tabular form. I find that

[11] Appendix Table A.3 shows that this effect is not driven by any correlation between the export intensity variable ($s^*_{fj,t-1}$) embedded in the shock and unobserved pre-trends in growth rates (e.g., if more export-intensive industries of the firm grow faster). Results are robust to controlling for a full set of industry-year dummies interacted with the export intensity variable, following Borusyak et al. (2021)'s recommendation for specifications with 'incomplete' shares.

[12] There is no stand-alone coefficient on the input-proximity interaction terms because they sum to industry j's expenditure share on inputs $m \in \mathcal{M}$ and are absorbed by industry-year fixed effects ($\sum_{k \neq j} \sum_{m \in \mathcal{M}} Prox_{fjkm,t-1} = \beta_{j\mathcal{M}}$).

Figure 1: Cross-industry Impact of Demand Shocks depend on Input-Proximity



Notes: This figure displays point estimates and 95% confidence intervals of cross-industry elasticities $\psi^{CROSS \times \mathcal{M}}$, where $\mathcal{M}$ references a different input category (in each row of the figure) in the triple-differences regression specification in column (3) of Table 2 (which gives the coefficient on the knowledge interaction). See Appendix Table A.4 for the corresponding regression table and the exact industry codes of these input categories.

cross-elasticities increase with industries' proximity in use of inputs from the information, intellectual property, management, and professional, scientific, and technical services sectors (NAICS 51, 533, 55, 54). I abbreviate these 21 input industries as *knowledge* inputs in the rest of the paper, and summarize their use in manufacturing in Appendix Table A.1.[13] Besides from knowledge, cross-elasticities do not vary with proximity in the use of any other category of inputs. The estimated coefficients relating to all other input categories are insignificant from zero, suggesting that these inputs may indeed be industry-specific as commonly assumed.[14]

These results suggest knowledge inputs have distinct properties under joint production compared to other inputs. This is consistent with the existing literature. Knowledge inputs produce much of the intangible capital hypothesized to be transferrable (Atalay et al., 2014) and non-rival (Crouzet et al., 2022a) within the boundary of the firm. As of 1997, knowledge industries constitute 15 percent of US GDP and are used intensively by manufacturing firms. Manufacturing firms' expenditures on knowledge inputs constitute 9 percent of their gross output as a whole, and vary

[13]Examples of such input industries include data processing services, scientific R&D, engineering, consulting, architectural, advertising, and legal services. Knowledge input industries in my model relate closely to the classification of 'professional and technical services' in Ding et al. (2022), 'skilled scalable services' in Eckert et al. (2020) and 'tradable services' in Gervais and Jensen (2019) and Eckert (2019).

[14]The results do suggest a mildly significant (at the 10 percent level) positive cross-elasticity across industries proximate in their use of inputs from the 'transportation, wholesale, and retail' sector, which is consistent with cost savings from shared warehousing and distribution.

greatly by the input-output industry pair. For example, organic chemical production (NAICS 325190) has the highest expenditure share (2.4 percent) on architectural, engineering and related services (NAICS 541300), while semiconductor manufacturing has the highest expenditure share (1 percent) on computer systems design services (NAICS 541512).

Column (3) of Table 2 reproduces this key finding from Figure 1: cross-elasticities increase with knowledge input-proximity. The exact coefficient estimates suggest that cross-elasticities are negative for industry-pairs that do not use any knowledge inputs in common, while positive for industry-pairs that have high expenditure shares on the same knowledge inputs.[15] The existence of both positive and negative cross-elasticities is consistent with the estimate of a zero average cross-elasticity in column (2).

The economic magnitudes of these reduced-form elasticities are hard to assess because the regressions are unweighted and fixed effects absorb any correlated changes across firms. While the rest of the paper develops and structurally estimates a model to quantify the impact of joint production, I first use the estimates in column (3) to provide a back-of-the-envelope calculation. Consider two firms that operate in the same industry j but in different other industries k. In the first firm, industry j has knowledge input-proximity equal to 0.06 (one standard deviation below the sample mean), whereas in the second firm, industry j has a higher knowledge input-proximity of 0.12 (one standard deviation above the mean). Suppose both firms receive a 10 log point demand shock in all their industries, roughly one standard deviation in the sample. In the first firm, the demand shock in industry j alone would increase sales in that industry by 4.6 log points ($= 0.1 \times 0.46$), while the demand shocks in its other industries would *decrease* sales in industry j by 3.5 log points ($= 0.1 \times -0.83 + 0.1 \times 0.06 \times 8.00$) due to the lack of knowledge proximity. Combining the same- and cross-industry impacts, sales in industry j would increase on net by only 1.1 log points for the first firm.

In comparison, the second firm would see a greater increase in industry j sales. The same-industry effect still increases sales in industry j by the same 4.6 log points, but now other-industry demand shocks would additionally increase (rather than decrease) sales in industry j by 1.3 log points ($= 0.1 \times -0.83 + 0.1 \times 0.12 \times 8.00$). Combining the direct and cross-industry impacts, output in industry j would increase on net by 5.9 log points, more than five times that in the first firm.

Robustness to Additional Covariates. The estimates in Table 2 are robust to controlling for an exhaustive set of initial-period firm-industry-level characteristics that explain subsequent growth in a firm's industries. These controls mitigate omitted variables bias if the assignment of demand shocks is correlated with covariates. I control for same-industry as well as *other-industry* measures

[15] I also estimate specifications where $\Delta \log S_{fjt}^{OTHER \times \mathcal{M}}$ for multiple input categories $\mathcal{M}$ appear jointly. In all specifications the coefficient on the knowledge input-proximity interacted demand shock is positive and significant. See, for example, specification (4) of Appendix Table A.5, which simultaneously estimates coefficients on three measures of other-industry demand shocks: (i) average, (ii) interaction with knowledge-input proximity, and (iii) interaction with other input-proximity.

of these covariates, aggregated in the same way as demand shocks. For example, an other-industry control for size represents the sales-weighted average log sales among the firm's other industries. I also interact these other-industry characteristics with the same knowledge input-proximity terms used to construct $\Delta \log S_{fjt}^{OTHER \times \mathcal{M}}$. For example, the control variable for any characteristic $Y_{fk,t-1}$ in the firm's other-industries interacted with knowledge input-proximity is:

$$Control_{fjt}^{OTHER \times KLG}(Y) \equiv \sum_{k \neq j} \left(\sum_{m \in KLG} Prox_{fjkm,t-1} \right) Y_{fk,t-1}.$$

These other-industry controls address the potential for omitted variables bias if a firm's growth in one industry j is correlated with pre-existing covariates $Y_{fk,t-1}$ in its *other* knowledge input-proximate industries.

In column (4) of Table 2, I saturate the triple-differences specification with an exhaustive list of controls for pre-period characteristics: firm log sales, firm export intensity, firm-industry-level log sales, firm-industry export intensity, firm-industry export status, the value of these characteristics in the other industries of the firm ($Control_{fjt}^{OTHER}(Y)$), as well as these other-industry covariates interacted with knowledge input proximity ($Control_{fjt}^{OTHER \times KLG}(Y)$). Controlling for all these covariates affects neither the significance nor magnitude of the key regression coefficients ψ^{CROSS} and $\psi^{CROSS \times KLG}$. Since the same proximity measures, sales shares, and export intensities used to construct other-industry demand shocks are also used to construct these other-industry covariates, this specification with additional controls also provides reassurance that the cross-elasticities are identified from changes in foreign market size rather than the shares.[16]

Finally, column (5) of Table 2 shows that results remain robust to controlling for firm-year fixed effects, which soak up any unobserved supply and demand changes common to each industry of the firm. This demanding specification shows that a given firm's sales in industry j rises more compared to its sales in industry j' whenever j is more knowledge-proximate to the firm's other positive demand-shocked industries.[17]

1.4 Mechanisms and Discussion of Results

Table 2 provides evidence on *how* knowledge inputs might be used in production. One mechanism consistent with positive cross-elasticities is that knowledge is scalable and partially non-rival. A

[16]The latest research on shift-share analyses (Adão et al., 2019; Borusyak et al., 2021) recommends adjusting standard errors to address the mismatch between the levels at which the shocks are observed (destination-market nh) versus applied (firm-industry fj). My empirical setting falls outside of these frameworks, because I construct and utilize multiple shift-share shocks with differing shares. In practice I find that standard errors clustered by firm are conservative. Results are robust to other forms of clustering as well as heteroskedasticity-robust standard errors. Moreover, the null coefficients on other shock-interactions in Figure 1 provide reassurance that my choice of standard errors does not lead to an abundance of false positives.

[17]Results from the triple-differences specification are also robust to further including firm-industry fixed effects, which limits identifying variation to *changes* in growth rates and demand shocks between the period 1997-2002 and the period 2002-2007.

positive demand shock in one industry causes the firm to scale up its use of knowledge inputs, and the partial non-rivalry of the additional knowledge increases the firm's sales in its other knowledge-proximate industries. In contrast, other potential mechanisms for explaining interdependence across industries, such as demand cannibalization or credit constraints, would have resulted in cross-elasticities that are negative and homogeneous rather than increasing with knowledge input-proximity. The null result in column (2) rules out these alternative mechanisms as the primary driver of cross-industry impacts in my empirical setting.[18]

Scalability of Knowledge Inputs. I provide evidence that firms' knowledge input expenditures increase with a shock to demand, a necessary condition behind my proposed mechanism. While each firm's total knowledge input expenditures (which include, for example, expenditures on in-house knowledge inputs) are hard to measure, the CMF provides data on a particular subset: firms' purchases of professional services, which comprise software, data processing, management, and advertising services. Table 3 estimates the elasticity of these knowledge input expenditures with respect to firm-level demand shocks, and compares the elasticity to that of other firm-level outcomes Y_{ft}: capital expenditures, payroll, and sales. I run the following firm-level regressions:

$$\Delta \log Y_{ft} = \tilde{\nu} \sum_{k} \eta_{fk,t-1} \Delta \log S_{fkt} + \epsilon_{ft}, \tag{6}$$

where I use weights $\eta_{fk,t-1}$ (the share of industry k in the firm's initial-period outcome $Y_{f,t-1}$) to construct the relevant firm-wide average demand shock.[19]

I estimate these regressions at the firm-level because unlike sales, there is no data on firms' input expenditures by industry of use. Of course, this data limitation arises naturally in the context of joint production when inputs are shared. Both the reduced-form evidence in this section and model estimation in Section 3 rely only on industry-level rather than firm-level input expenditure data. There is an econometric advantage to doing so. Input proximity constructed from aggregate expenditure shares β_{jm} is unlikely to be correlated with firm-specific unobservables (e.g., unequal access to input markets), therefore mitigating a potential source of endogeneity bias.

Column (1) of Table 3 shows that firms increase their expenditures on these professional services in response to positive firm-wide demand shocks. The coefficient of 0.65 comprises the product of two elasticities: (i) a "first-stage" elasticity of marginal revenue to the empirically measured demand shocks, and (ii) the elasticity of knowledge input expenditures to the shift in marginal revenue. The remaining columns of Table 3 repeat the analysis for other firm-wide outcomes. While each coefficient shares the same "first-stage" elasticity, the second elasticity is different.

[18] This null result is consistent with Borusyak and Okubo (2016), who also find that average intra-firm, cross-segment impacts of demand shocks are insignificant from zero in Japanese firm-level data.

[19] Appendix A.3.4 provides the precise definitions. Results are robust to using simple averages or any other common type of weight across all four outcome variables in Table 3.

Table 3: The Impact of Demand Shocks on Firm-level Input Expenditures

	(1) Purchased Prof. Services	(2) Capex	(3) Payroll	(4) Sales
Outcome-relevant demand shock	0.65	0.47	0.25	0.37
$\sum_k \eta_{fk,t-1} \Delta \log S_{fkt}$	(0.22)	(0.37)	(0.11)	(0.15)
Year-FE	✓	✓	✓	✓
Observations	3,900	3,900	3,900	3,900
R^2	0.02	0.04	0.01	0.05

Notes: This regression table estimates the impact of demand shocks (averaged across the firm's industries) on firm-level variables, in 5-year differences over the period 1997-2007. Weights used for the average, $\eta_{fk,t-1}$, are defined in Appendix A.3.4. Standard errors in parentheses are clustered at the firm level. Number of observations are rounded for disclosure avoidance. The sample of firms is limited to those reporting non-zero purchased professional services.

Columns (2) and (3) show that the elasticity of expenditures on capital and payroll with respect to the same shift in marginal revenue is lower, consistent with knowledge inputs being more scalable than these other inputs. Finally, column (4) estimates the firm-level elasticity of sales to demand shocks at 0.37. This coefficient lies in between the elasticity of various input expenditures in the prior columns, consistent with the assumption of constant markups taken up in my model.

Other Interpretations and Threats to Identification. My identification assumption requires demand shocks in a firm's *other* industries $k \neq j$ to be conditionally uncorrelated with unobserved demand and supply shifters in a given industry j of the firm.[20] I entertain and rule out various potential threats to this identification assumption.

First, the lack of statistical significance on $\Delta \log S^{OTHER}_{fjt}$ in column (2) of Table 2 rules out the possibility of a simple, symmetric correlation structure between export demand shocks and unobserved shifters across industries. Therefore, correlations between demand shocks in industry k and unobservable shifters in industry j would be problematic for the main results in columns (3)-(5) *only if* the correlation happens to be precisely stronger among knowledge-proximate industries.

To entertain the possibility of such a correlation, suppose that import demand in each foreign country happens to be more correlated across more knowledge-proximate industries. Under this scenario, the cross-elasticities in Table 2 could reflect firms receiving correlated shocks across industries *within* their export markets and taking advantage of 'demand-scope' complementarities (Bernard et al., 2018) or shared market access costs (Arkolakis et al., 2019). A related threat occurs if knowledge-intensive industries are disproportionately demand-complementary within a firm's set of buyers. Under any of these scenarios, a positive demand shock in one industry would raise the firm's sales in its other knowledge-proximate industries.

[20]Demand shocks can be arbitrarily correlated with own-industry unobserved shocks without affecting the use of cross-industry coefficients ψ^{CROSS} to test for nonjoint production. Demand shocks also do not need to be unanticipated. The possibility that a particular shock in k is anticipatable t years ahead of time simply changes the interpretation of the relevant time horizon for a supply-side response in j to materialize.

I address these concerns in three ways. First, in Appendix A.3.5 I find no evidence that import growth within a foreign destination is positively correlated among knowledge-proximate industries. Second, the regressions in Table 2 control directly for same-industry demand shocks ($\Delta \log S_{fjt}$), which would contain any correlated shocks in a given export destination as long as the firm already exports in industry j to that destination n. Third, my results are robust to excluding from the outcome variable $\Delta \log X_{fjt}$ any of the firm's exports of j to destination countries where demand shocks in its *other* industries k originated.[21]

A second threat to identification comes from selection on correlated supply-side shocks. If such shocks were anticipatable by the firm and, again, happen to be positively correlated in knowledge input-proximate industries, selection on industry and export market entry could generate spurious positive cross-industry impacts of demand shocks specifically among knowledge input-proximate industries. However, this hypothesis does not survive the following placebo exercise. I re-assign firm-industry exporters in each industry k different export demand shocks drawn from the empirical distribution of shocks received by other firms in that same industry. Keeping all remaining firm variables (e.g. firm-industry sales weights, and other controls) unchanged in these placebo regressions, I do not find a statistically significant share of positive cross-industry coefficients.[22]

A third threat to identification comes from the possibility that input-proximity is contaminated by other mechanisms that explain co-production. For example, suppose firms (and their plants) are more likely to co-produce in more demand-complementary industries. Since the BEA constructs input-output tables by apportioning reported plant-level input expenditures across each plant's output industries, demand-complementary industries could mechanically have greater input proximity. However, the lack of statistical significance in Figure 1 on any other interaction besides from knowledge-proximity rules out this possibility.

Finally, because the regression outcome is a firm's *external* sales, cross-elasticities cannot be explained by increased intra-firm shipments of goods, for example, to supply a shocked downstream industry. Nevertheless, it could be that increased intra-firm shipments trigger subsequent productivity gains upstream that then induce firms to sell externally. I use data on intra-firm shipments to test and reject this mechanism in Appendix A.3.6. I find no evidence that intra-firm shipments upstream respond to demand shocks in a downstream industry (or vice versa).

[21] As a fourth step, I find that results are also robust to controlling for *latent* demand shocks—a measure of demand for industry j of the firm not from where it is *currently* exporting its products in industry j (which is $\Delta \log S_{fjt}$) but from any other destinations in which it currently exports products in other industries k. These results are undisclosed but can be provided upon request.

[22] These results are undisclosed but can be provided upon request. This placebo exercise also provides further reassurance that pre-existing variation in Bartik weights or bilateral industry characteristics such as knowledge input-proximity are not picking up correlated industry trends in the error terms.

2 Model of Joint Production

I develop a theory of joint production that rationalizes the heterogeneous cross-industry transmission of demand shocks within the firm. Because inputs are shared under joint production, a firm's marginal cost in a given industry depends on its output in not only the same industry but also others. The theory illustrates two key properties of shared inputs—*scalability* and *rivalry*—that parametrize this interdependence in costs and allow for arbitrary returns to scale and scope.

I embed this joint production technology within a conventional monopolistic competition setting featuring CES industry demand and endogenous entry and exit of firms across industries. The model relaxes the assumption of constant-returns and nonjoint production found in workhorse models of heterogeneous firms (e.g., Melitz, 2003; Bernard et al., 2010) and nests their general equilibrium predictions as special cases.

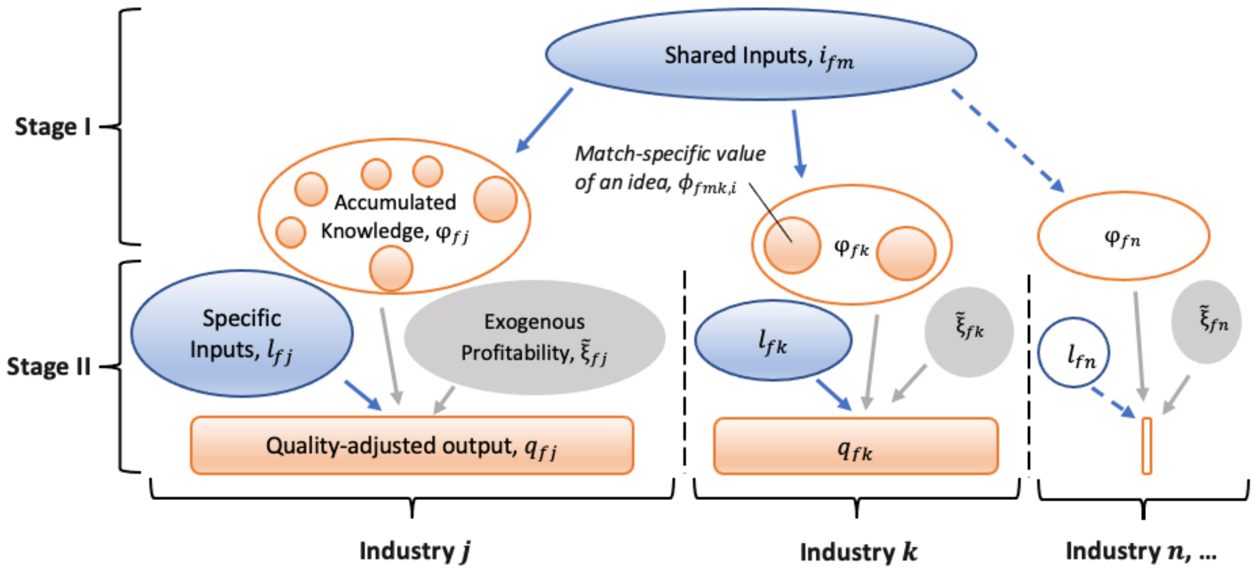
2.1 Production Technology and Market Structure

Figure 2 illustrates the static joint production technology of the firm. I model production as taking place sequentially over two stages, where inputs are denoted in blue and outputs are denoted in orange. I interpret the first stage as the firm's production of knowledge (or intangible) capital φ_{fj} across its industries using shared inputs ι_{fm} like information technology, intellectual property, and professional services. The composite φ_{fj} encompasses any intermediate output of the firm created from any type of shared input, but motivated by the empirical evidence, I use the specific label of knowledge to aid exposition. Knowledge (or any other intermediate output) accumulated in this ex-post form φ_{fj} is proprietary and (unlike shared inputs) cannot be bought or sold on the market. By the second stage, accumulated knowledge φ_{fj} acts as a revenue productivity shifter. The firm takes φ_{fj} as given and combines it with a bundle of industry-specific inputs l_{fj} like assembly-line labor, materials, and energy to produce its differentiated variety q_{fj} in each industry.

Using this technology, a continuum of firms compete across a set of industries $j \in \mathcal{J}$ under monopolistic competition facing CES demand (with elasticity σ_j) within each industry. In both stages of production, firms face constant input prices and know aggregate demand conditions as well as their own fundamental (exogenous) profitability shifters $\tilde{\xi}_{fj}$, which reflect, for example, idiosyncratic differences in product appeal, production know-how, or access to foreign output and input markets. Firms choose inputs in each stage to maximize total expected profits. I describe the two production stages in reverse order.

Stage II: Use of Industry-specific Inputs. By stage II, production of quality-adjusted output $\{q_{fj}\}_{j \in J}$ is independent across industries given that stage II inputs are industry-specific (and available at constant unit prices). I assume each industry operates a standard Cobb-Douglas production function given by equation (7).

Figure 2: An Illustration of the Firm's Joint Production Technology



Notes: This figure illustrates the firm's two-stage joint production technology that transforms inputs (in blue) to quality-adjusted outputs across multiple industries, $j, k, n, \ldots$ (in orange). Inputs used in stage I are shared across the firm's industries, whereas inputs used in stage II are industry-specific. The firm uses shared inputs to develop discrete ideas (displayed by the small orange circles) that, when adapted to an industry j, increases the value of knowledge capital φ_{fj}. Profitability shifters (shaded grey) are exogenous and observable by the firm in both stages. In the example, the firm does not produce output (does not enter) in industry n because it has not accumulated any knowledge capital in that industry ($\varphi_{fn} = 0$).

Assumption 1 (Stage II Industry Production Function) *In each industry j, quality-adjusted output q_{fj} is a Cobb-Douglas function over (i) a homothetic index l_{fj} of industry-specific inputs, (ii) an index φ_{fj} of accumulated knowledge determined in stage I, and (iii) an exogenous profitability shifter, $\tilde{\xi}_{fj}$:*

$$q_{fj} = l_{fj}^{\gamma_j} \, \varphi_{fj} \, \tilde{\xi}_{fj}, \qquad \forall j \in \mathcal{J}, \tag{7}$$

where $\gamma_j \in [0, \frac{\sigma_j}{\sigma_j - 1})$ is the elasticity of final output with respect to stage-II inputs l_{fj}.

My production technology is a generalization of that found in standard models of heterogeneous firms. For example, when stage-II returns to scale are constant ($\gamma_j = 1$) and stage-I knowledge accumulation is exogenous (so the combined revenue productivity term $\varphi_{fj}\tilde{\xi}_{fj}$ is exogenous), equation (7) recovers the constant-returns, nonjoint-production benchmark of Bernard et al. (2010). Relative to this benchmark, my production technology provides more flexibility in two dimensions. First, properties of stage-II inputs allow for arbitrary increasing ($\gamma_j > 1$) as well as decreasing ($\gamma_j < 1$) *within*-industry returns to scale. Second, properties of shared inputs in stage I, described below, generate marginal cost interdependence *across* industries and therefore arbitrary economies or diseconomies of scope.

Stage I: Use of Shared Inputs. Assumption 2 completes the description of the firm's production technology. The firm uses various shared knowledge-producing inputs (indexed by type $m \in \mathcal{M}$) to develop ideas and adapt them across different industries to improve knowledge capital (i.e., increase revenue-productivity in stage II). For example, inputs like scientists contribute to knowledge by developing automation techniques, managers contribute by configuring factory floor space, and advertisers contribute by raising product awareness. The combined value of all these ideas adapted in an industry make up the index of accumulated knowledge, φ_{fj}. Because φ_{fj} is essential for production in stage II, knowledge accumulation also determines the firm's extensive margin—the set of industries $j \in \mathcal{J}$ in which it sells final output.

Assumption 2 (Stage I Stochastic Accumulation of Knowledge) *Shared inputs ι_{fm} of each type m increase the firm's Poisson rate of development of ideas of that type:*

$$A_{fm} \sim \mathit{Poisson}\left(Z\left(\frac{\rho_m}{\rho_m - 1}\iota_{fm}\right)^{\frac{\rho_m - 1}{\rho_m}}\right), \qquad \forall m \in \mathcal{M}, \tag{8}$$

where parameter Z governs the average arrival rate of ideas, and $\rho_m \in (1, \infty)$ measures input m's scalability. Each idea $i \in \{1, ..., A_{fm}\}_m$ has match-specific value $\phi_{fmi,j}$ when adapted in an industry j. Match-specific values are drawn i.i.d. from a Fréchet distribution with shape parameter $\theta_m \in (1, \infty)$:

$$Pr(\phi_{fmi,j} \le x) = e^{-x^{-\theta_m}}, \qquad \forall j \in \mathcal{J}. \tag{9}$$

The firm chooses the industry j in which to adapt each idea (denoted by indicator $\mathbf{1}_{fmi,j}$) after observing the idea's match-specific values in each industry. Total accumulated knowledge φ_{fj} in an industry is a power sum over the value of all ideas adapted in that industry:

$$\varphi_{fj} = \left(\sum_{m \in \mathcal{M}} \sum_{i=1}^{A_{fm}} \tilde{\alpha}_{mj} \phi_{fmi,j} \mathbf{1}_{fmi,j}\right)^{\frac{\sigma_j}{\sigma_j - 1} - \gamma_j}, \qquad \forall j \in \mathcal{J}, \tag{10}$$

where technology coefficients $\{\tilde{\alpha}_{mj}\}_{m,j}$ denote the average value of type-m ideas when adapted in industry j.

Equation (8) specifies the development of ideas within the firm as an endogenous Poisson process. The more shared inputs ι_{fm} the firm uses, the greater the arrival rate $\mathbb{E}[A_{fm}]$ of type-m ideas. Input scalability, $\rho_m \in (1, \infty)$, parametrizes the elasticity of the arrival rate with respect to inputs used. The lower is ρ_m, the less responsive are the firm's input m expenditures to changes in demand (profitability) conditions. In the limit as $\rho_m \to 1$, the arrival rate of ideas is inelastic to input use, and knowledge accumulation becomes exogenous: $A_{fm} \sim$ Poisson(Z).

Each idea $i_m = 1, ..., A_{fm}$ that the firm develops has an idiosyncratic i.i.d. value $\phi_{fmi,j}$ when adapted to improve knowledge capital in a given industry j. The firm observes $\{\phi_{fmi,j}\}_{j \in \mathcal{J}}$ and chooses the most suitable industry j in which to adapt that idea. Equation (9) parametrizes variation in match-specific values using a Fréchet distribution with shape parameter θ_m. This variability is natural in the context of knowledge creation (and more generally, in the use of any non-routine, cognitive input). Consider, for example, General Electric, which employs ceramics scientists to develop R&D ideas. A particular idea could represent an invention like gemstone scintillators, which are more valuable when adapted in GE's CT medical scanners than in GE's aviation turbines. Whereas some shared inputs like scientists may generate more variable ideas (indicated by a lower θ_m), other shared inputs like legal and accounting services could generate more predictable ideas (indicated by a higher θ_m).

Finally, equation (10) combines the value of all ideas adapted in an industry j into a single index of accumulated knowledge, φ_{fj}. The functional form assumes that the marginal profit contribution of each idea is additively separable from that of other ideas.[23] In addition, exogenous technology coefficients $\tilde{\alpha}_{mj}$ allow the ideas of a given type m to be either more or less valuable on average when adapted in a given industry j. Similar to conventional input-output coefficients, $\{\tilde{\alpha}_{mj}\}_{m\in\mathcal{M},j\in\mathcal{J}}$ generate variation across industries in their use of stage-I shared inputs m and facilitate the quantitative mapping between the model and the data. For example, software inputs are intensively used in computer manufacturing while management consulting inputs are intensively used in petrochemicals production. Because the two industries are not proximate in the types m of shared inputs used, the cross-industry transmission of demand shocks from petrochemicals to computers would be close to zero.

2.2 Solution of the Firm

Given the production technology described by Assumptions 1 and 2, it is easy to solve for the firm's profit-maximizing decisions in reverse order.

In stage II, conditional on accumulated knowledge φ_{fj}, the firm's gross profit maximization problem is separable by industry. Equation (11) describes the well-known solution under monopolistic competition and CES industry demand. The firm's revenues X_{fj} and gross profits π_{fj} (revenues less stage-II industry-specific input costs) in each industry can be expressed in terms of accumulated knowledge (φ_{fj}), an exogenous profitability shifter (ξ_{fj}), and an industry-level profitability index (B_j) common to all firms:

$$\pi_{fj} = (1-\varsigma_j)X_{fj} = B_j \xi_{fj} \varphi_{fj}^{\frac{\sigma_j - 1}{\sigma_j(1-\varsigma_j)}}, \tag{11}$$

[23]In addition to generating additive separability, the index $\sigma_j/(\sigma_j-1)-\gamma_j$ in equation (10) serves as a normalization that, when combined with equation (7), limits the overall returns to scale in production to be smaller than $\sigma_j/(\sigma_j-1)$. This normalization ensures that the firm's profit-maximization problem is well-defined, i.e., the supply curve cannot be more steeply downward sloping than the demand curve. Note that since stage-I inputs have arbitrary scalability γ_j and stage-II inputs have arbitrary scalability ρ_m, the firm's production technology is not dependent on demand elasticities σ_j (oustide of the edge-case values of either $\rho_m = \infty$ or $\gamma_j = \sigma_j/(\sigma_j-1)$).

where $\xi_{fj} \equiv \tilde{\xi}_{fj}^{\frac{\sigma_j-1}{\sigma_j(1-\varsigma_j)}}$ is a convenient re-normalization and $\varsigma_j \equiv \gamma_j \frac{\sigma_j-1}{\sigma_j} < 1$ is an industry-level parameter equal to the share of sales expensed on stage-II inputs l_{fj}. Industry-wide profitability B_j is an equilibrium object that depends on gross profit margins $1-\varsigma_j$, the unit cost c_j of the industry-specific input composite l_{fj}, and two shifters of residual demand: P_j (the CES price index), and Y_j (total expenditures):

$$B_j = (1-\varsigma_j)\left(\frac{\varsigma_j}{c_j}\right)^{\frac{\varsigma_j}{1-\varsigma_j}} \left(P_j^{\sigma_j-1} Y_j\right)^{\frac{1}{\sigma_j(1-\varsigma_j)}}. \tag{12}$$

Having optimized over the firm's stage-II input use, the only remaining endogenous variable is the index value for accumulated knowledge (φ_{fj}), the outcome of the firm's decisions in stage I.

In stage I, the firm faces a multi-dimensional profit maximization problem. It chooses (i) the industry j in which to adapt each idea $\{\mathbf{1}_{fmi,j}\}_{\{i=1,\dots,A_{fm}\}_m}$, (ii) overall quantities of shared inputs $\{\iota_{fm}\}_m$, and (iii) its "extensive margin"—the set of industries $j \in \mathcal{J}$ to produce output in. Replacing knowledge capital φ_{fj} in the firm's stage-II gross profit function (11) with its definition in equation (10) yields firm gross profits as an additively separable function of the number and value of all ideas adapted to that industry:

$$\pi_{fj} = \sum_{m\in\mathcal{M}} \sum_{i=1}^{A_{fm}} B_j \xi_{fj} \tilde{\alpha}_{mj} \phi_{fmi,j} \mathbf{1}_{fmi,j}, \qquad \forall j \in \mathcal{J}.$$

Given this additive separability, the firm's various decisions in Stage I can be solved for in isolation. First, the choice of which industry j in which to adapt a given idea $\{i_m = 1, \dots, A_{fm}\}_m$ becomes a repeated discrete choice problem. The firm observes the Fréchet-distributed industry match-specific values $\{\phi_{fmi,j}\}_{j\in\mathcal{J}}$ and chooses the industry where adapting that idea would yield the greatest (additive) increase in profits. The firm adapts each type-m idea to industry j with probability μ_{fmj}:

$$\mu_{fmj} \equiv \frac{\delta_{fmj}^{\theta_m}}{\sum_{k\in\mathcal{J}} \delta_{fmk}^{\theta_m}}; \qquad \delta_{fmj} \equiv B_j \xi_{fj} \alpha_{mj} Z, \tag{13}$$

which is increasing in δ_{fmj}, an index of input-by-industry-level exogenous profitability terms: the firm's stage-II profit shifters ($B_j\xi_{fj}$ in equation 11), stage I technology coefficients (renormalized as $\alpha_{mj} \equiv \tilde{\alpha}_{mj}\Gamma(1-1/\theta_m)$), and the exogenous rate of arrival of ideas (Z).

Second, the firm chooses its overall level of each shared input ι_{fm} such that the marginal benefit of that input equals its marginal cost, w (a constant). Altogether, trading off a constant marginal cost against a diminishing marginal benefit yields a unique interior solution for shared inputs $\{\iota_{fm}\}_m$. The marginal benefit is the product of two terms: (i) the effect of the marginal input towards increasing the Poisson arrival rate of ideas, and (ii) the expected profit contribution of a given idea that arrives. The first term is decreasing in ι_{fm} given concavity in the production of ideas ($(\rho_m - 1)/\rho_m < 1$ in equation 8). The second term, the expected profit contribution of an idea from input m, is a constant and given by:

$$\Delta_{fm} \equiv \mathbb{E}\left[\max_j B_j \xi_{fj} \tilde{\alpha}_{mj}\, \phi_{fmi,j}\right] = \left(\sum_{j \in \mathcal{J}} \delta_{fmj}^{\theta_m}\right)^{\frac{1}{\theta_m}}, \tag{14}$$

a θ_m-norm of the vector of exogenous industry profitabilities, δ_{fmj}.

The θ_m-norm captures the firm's *option value* from being able to observe $\{\phi_{fmi,j}\}_j$ and then choose the most profitable industry to adapt that idea. Equation (14) ranges from a low of $\max_j \delta_{fmj}$ (in the limit as $\theta_m \to \infty$) to a high of $\sum_j \delta_{fmj}$ (when $\theta_m \to 1$). I show in Section 2.3 that θ_m acts as a microfoundation for input rivalry. The lower is θ_m, the more variable are the match-specific values of type-m ideas, and the greater is the option value from input m. In the limit as $\theta_m \to 1$, option value is so high that the firm values inputs as if they are fully non-rival across industries. Each idea is valued *as if* it is used simultaneously in all the firm's industries.[24]

Lastly, the industry entry decision of the firm is simple in the absence of fixed costs. Each firm can adapt ideas and produce output in any of the $\mathcal{J}$ industries. But because industry knowledge capital φ_{fj} is essential for production in stage II, the choice of whether to adapt the first idea to an industry (which causes $\varphi_{fj} > 0$) *is in fact* the choice of whether to "enter" that industry. The first idea adapted to an industry marks entry, while adaptations of subsequent ideas to the same industry improve knowledge capital (and thus output) on the intensive margin. Given their additive separability in profits, each idea is adapted to industry j with probability μ_{fmj} regardless of whether it is the first or a subsequent idea.[25]

Lemma 1 puts these results together and derives closed-form expressions for the firm's *ex-ante* expected industry sales, probability of industry entry, and net profits.

Lemma 1 (The Firm's Solution) *Let w denote the (normalized) constant unit cost of each shared input. The firm's expected gross profits in each industry $j \in \mathcal{J}$ is a constant fraction $(1 - \varsigma_j)$ of expected sales:*

$$\mathbb{E}[\pi_{fj}] = (1 - \varsigma_j)\ \mathbb{E}[X_{fj}] = \sum_m \delta_{fmj}^{\theta_m} \Delta_{fm}^{\rho_m - \theta_m} w^{1-\rho_m}, \tag{15}$$

[24]The Fréchet microfoundation here plays a similar role as in Eaton and Kortum (2002), where θ governs how goods are sourced from different outlets (countries) at the lowest cost. In that paper lower values of θ indicate greater dispersion of comparative advantage across countries (for a given input variety) and thus a lower expected cost. In my model, θ governs how ideas are adapted to different outlets (industries) at the highest profit. Lower values of θ indicate greater dispersion of match-specific values (for a given idea) and thus a higher expected profit.

[25]The use of discrete stochastic processes to explain 'zeros' (the absence of firm entry) is inspired by Klette and Kortum (2004), Eaton et al. (2013), and Armenter and Koren (2014), and presents theoretical and computational advantages over settings with literal fixed costs. Fixed costs presume a certain degree of economies of scale, and generate non-convexities from the point of view of not just firms but also the aggregate economy. Recent work by Jia (2008), Antràs et al. (2017), and Arkolakis and Eckert (2017) provide algorithms that reduce the computational burden of fixed-cost models but operate under a partial equilibrium framework where industry profitability is fixed. Instead, in my stochastic setting each individual firm's profit maximization problem is convex, which guarantees a unique solution for industry profitability $\{B_j\}_j$ in multi-industry equilibrium.

and the probability of industry entry (denoted $\chi_{fj} = 1$) is one minus the probability the firm does not adapt any idea to industry j:

$$Pr(\chi_{fj} = 1) = 1 - \exp\left(-Z \sum_{m \in \mathcal{M}} \delta_{fmj}^{\theta_m} \Delta_{fm}^{\rho_m - 1 - \theta_m} w^{1-\rho_m}\right). \tag{16}$$

The firm's ex-ante expected net profit (revenues less stage-I and stage-II input costs) is given by:

$$\mathbb{E}[\Pi_f] = \sum_j \mathbb{E}[\pi_{fj}] - \sum_{m \in \mathcal{M}} w \iota_{fm} = \sum_{m \in \mathcal{M}} \frac{1}{\rho_m} \Delta_{fm}^{\rho_m} w^{1-\rho_m}. \tag{17}$$

2.3 Interdependence from the Scalability and Rivalry of Shared Inputs

In Lemma 1, the firm's expected sales in a given industry j (both the intensive and extensive margin) depends on profitability shifters in not only the same industry but also other industries $k \neq j$ (contained in the Δ_{fm} terms). The precise direction of interdependence is governed by parameters ρ_m and θ_m of shared inputs.

I define ρ_m as input scalability and θ_m as input rivalry. These two properties of shared inputs generate scale and rivalry effects that have opposing effects on cross-elasticities. The more scalable are shared inputs (the higher is ρ_m), the more that the firm increases these inputs in response to an industry-k-specific demand shock. As long as inputs are not fully rival, this increase in shared inputs benefits knowledge accumulation (and thus sales) in other industries j of the firm. But the more rival are shared inputs (the higher is θ_m), the more that the firm will optimally substitute its use of shared inputs away from other industries j to meet the increase in demand in industry-k. As long as inputs are not fully scalable, this substitution comes at the expense of knowledge accumulation (and sales) in its other industries j.

I describe these two effects analytically before combining them in Proposition 1. First, the firm's sales in each industry j (equation 15) can be re-written as (see Appendix B.2 for the derivations):

$$\mathbb{E}[X_{fj}] = \frac{B_j \xi_{fj}}{1 - \varsigma_j} \mathbb{E}\left[\varphi_{fj}^{\frac{\sigma_j - 1}{\sigma_j(1-\varsigma_j)}}\right] = \frac{B_j \xi_{fj}}{1 - \varsigma_j} \sum_m \tilde{\alpha}_{mj} \underbrace{\mathbb{E}[A_{fm}]}_{\text{scale effect}} \underbrace{\mu_{fmj}\ \mathbb{E}[\phi_{fmi,j} \,|\, \mathbf{1}_{fmi,j} = 1]}_{\text{rivalry effect}}, \tag{18}$$

where the last equality decomposes expected knowledge φ_{fj} into a term determined by input scalability (the total number of ideas of each type m developed), and a term determined by input rivalry (the share of all ideas and expected value of each idea that the firm adapts in industry j). A demand shock in another industry k can raise industry-j output through the scale effect but also lower it through the rivalry effect.

Rivalry effect is parametrized by $\theta_m \in (1, \infty)$. The rivalry effect consists of two terms. First, in response to increased demand in industry k, the firm will find it more profitable to adapt a greater share of ideas to industry k and a smaller share μ_{fmj} to industry j. From equation (13), the higher is θ_m, the lower the share of ideas adapted in industry j. Besides from μ_{fmj}, the second term contributing to input rivalry measures the expected value of an idea *conditional* on the firm adapting it in industry j. This second term provides a small offset due to selection. When demand is higher in industry k, the remaining ideas the firm (optimally) chooses to adapt to industry j must be on average higher quality than before. Overall the rivalry effect can be expressed as:

$$\mu_{fmj}\ \mathbb{E}[\phi_{fmi,j}\,|\,\mathbf{1}_{fmi,j}=1]=\left(\frac{\delta_{fmj}}{\Delta_{fm}}\right)^{\theta_m-1}\Gamma(1-1/\theta_m),\qquad \forall m\in\mathcal{M},$$

which causes sales to decrease with other-industry demand shocks (Δ_{fm}) with elasticity $\theta_m - 1$.

In the limit as $\theta_m \to 1$, shared inputs m become fully non-rival within the firm. In response to a demand shock in industry k, the slight decline in the share of ideas μ_{fmj} adapted in industry j is fully offset by the increase in the expected value of ideas still being adapted in industry j, and the rivalry effect in equation (18) disappears. In the other limit as $\theta_m \to \infty$, relative adaptation shares are so sensitive that a slight increase in profitability in another industry k can cause virtually all ideas to be adapted instead in that industry, and shut down production in other industries $j \neq k$.

Scale effect is parametrized by $\rho_m \in (1, \infty)$. In response to increased demand in industry k, the firm will also find it more profitable to increase its overall arrival rate of ideas, $\mathbb{E}[A_{fm}]$. From equation (8), the higher is ρ_m, the more elastic is this arrival rate to the firm's use of shared inputs. The firm chooses quantities of shared inputs ι_{fm} to equate marginal expected gross profits with its marginal input cost. The firm's expected number of adaptable ideas can be expressed as:

$$\mathbb{E}[A_{fm}]=\Delta_{fm}^{\rho_m-1}w^{1-\rho_m}Z,$$

which is increasing in other-industry demand shifters (Δ_{fm}) with elasticity $\rho_m - 1$.

In the limit as $\rho_m \to 1$, shared inputs are not scalable and knowledge accumulation within the firm is exogenous. Firms do not adjust their use of shared inputs, and the scale effect in equation (18) disappears. In the other limit as $\rho_m \to \infty$, shared inputs are so scalable that the slightest increase in profitability in industry k causes the firm's profits in each industry to increase infinitely (this virtuous cycle is possible in partial equilibrium as the firm moves down its cost curve and lower prices invite even more demand).

Net effect on cross-elasticities. Proposition 1 combines these two effects to derive the net cross-industry elasticities of sales ($\mathbb{E}[X_{fj}]$) with respect to demand shocks ($\xi_{fk}B_k$). Cross-elasticities are increasing in the scalability (ρ_m) and decreasing in the rivalry (θ_m) of proximate shared inputs m, such that the net effect can be either positive or negative. The model-relevant measure of shared-input-m proximity is $\lambda_{fjm}\mu_{fmk}$, the theoretical counterpart to $Prox_{fjkm}$ in Section 1. Finally, the last term in equation (19) includes a strictly positive term whenever the outcome industry is the same as the shocked industry ($k = j$). A positive demand shock always increases same-industry sales because scale and rivalry effects push in the same direction.

Proposition 1 (Cross-Industry Elasticities within the Firm) *The elasticity of expected firm sales in any industry j, $\mathbb{E}[X_{fj}]$, to a change in profitability in any industry k, $\xi_{fk}B_k$, is given by:*

$$\psi_{fjk} \equiv \frac{\mathrm{d}\log \mathbb{E}[X_{fj}]}{\mathrm{d}\log \xi_{fk}B_k} = \sum_{m\in\mathcal{M}} (\rho_m - \theta_m)\,\lambda_{fjm}\,\mu_{fmk} + \mathbf{1}_{j=k}\sum_{m\in\mathcal{M}} \theta_m\,\lambda_{fjm}, \tag{19}$$

where industry adaptation shares μ_{fmk} are given by equation (13) and input utilization shares λ_{fjm} denote the share of industry j gross profits attributable to shared input m:

$$\lambda_{fjm} \equiv \frac{\mu_{fmj}\Delta_{fm}^{\rho_m} w^{1-\rho_m}}{\sum_{m'} \mu_{fm'j}\Delta_{fm'}^{\rho_{m'}} w^{1-\rho_{m'}}}.$$

The theory nests two edge cases where the null hypothesis of nonjoint production would hold ($\psi_{fjk} = 0$). The first, trivial, case occurs when all shared inputs in stage-I are in fact industry-specific: each input m is only ever useful when adapted to improve knowledge in a given industry j, so $\alpha_{mk} = 0$ for all $k \neq j$. In this case $\lambda_{fjm}\mu_{fmk} = 0$, so the cross-elasticity is zero. The second case occurs on a knife's edge when scale and rivalry effects perfectly cancel out ($\rho_m = \theta_m\ \forall m$). For example, if all proximate shared inputs (e.g., brand capital) were completely unscalable and also perfectly non-rival ($\rho_m = \theta_m = 1$), knowledge accumulation is fixed in each industry, and the model is isomorphic to the firm receiving exogenous firm-industry 'productivity draws' of φ_{fj}.

Outside of these edge cases, cross-elasticities are asymmetric and heterogeneous across firms and industry-pairs. They depend flexibly on the scalability and rivalry of shared inputs as well as the technology coefficients α_{mj} that govern input-proximity, allowing me to estimate these parameters from the observed elasticities of sales to demand shocks in the data.[26]

3 Model Estimation

This section connects the empirical evidence in Section 1 with the theory in Section 2. I leverage the conditional exogeneity of demand shocks at the firm-industry level to estimate the firm's joint production technology. I base inference on exact model-implied moment conditions, allowing

[26]The elasticities in Proposition 1 include both extensive and intensive margin responses in expectation. In Appendix B.4, I show that the total elasticity of sales to demand shocks occurs mostly on the extensive margin for smaller firms and mostly on the intensive margin for larger firms (such as those in my regression sample), providing support for the reduced-form regressions in Section 1.

demand shocks and other general equilibrium controls to affect firm outcomes non-linearly. I leverage variation in the data from not only the intensive margin (e.g., decrease in sales) but also the extensive margin (e.g., closure of an industry), consistent with the model.

3.1 Overview and Assumptions

Notationally, I use variables in boldface to refer to vectors and matrices, for example $\boldsymbol{B} \equiv \{B_{jt}\}_{j,t}$.

I develop a nested fixed-point algorithm to jointly estimate the model's micro and macro parameters. First, conditional on micro parameters (scalability $\boldsymbol{\rho}$ and rivalry $\boldsymbol{\theta}$), I set the model's macro parameters (industry profitability levels $\boldsymbol{B}$ and technology coefficients $\boldsymbol{\alpha}$) to exactly match BEA industry-level data on output and input expenditures. Second, conditional on macro parameters, I compute the model's structural residuals—the difference between the model and the data in each firm's output growth in each industry, ΔX_{fj}. I exploit the orthogonality of these structural residuals with respect to same- and cross-industry demand shocks to identify micro parameters.

Input Taxonomy. Estimation requires taking a stance on which inputs in the data are shared. Given the reduced-form evidence in Section 1, I classify inputs from the knowledge sector into three categories of shared inputs in the model: (i) leasing of intangibles (NAICS 533), (ii) headquarters services (NAICS 55), and (iii) information and professional services (NAICS 51, 54). I specify a pair of scalability and rivalry parameters (ρ_{KLG}, θ_{KLG}) common to these knowledge inputs.

In addition, I create a fourth residual category of shared inputs in the model to accommodate regression evidence of negative cross-elasticities among industries that are the least knowledge-proximate. I map spending on this residual category to the following inputs where uncertainty around interaction effects in Figure 1 is high: finance and real estate (NAICS 52), leasing of tangibles (NAICS 531, 532), administrative services (NAICS 56), other services (NAICS 6, 7, 8, and 9), and capital. I aggregate all these inputs in the data into one composite residual input in the model to speed up computation (leaving only $4 \times |\mathcal{J}|$ technology coefficients to identify). I let the scalability and rivalry of this residual shared input (ρ_{RES}, θ_{RES}) differ from those of knowledge inputs. In practice, this residual category allows the model to quantitatively account for other mechanisms that generate interdependence, including, for example, span-of-control or internal capital markets.

I assume that production in stage II uses inputs from all remaining BEA sectors: agriculture, mining, construction, utilities, manufactures, wholesale, retail and transportation industries, as well as labor value added. Since these inputs are industry-specific by assumption, their impact on firms' production decisions are absorbed in the estimation of industry profitability $\boldsymbol{B}$.

Altogether $\boldsymbol{\Theta} \equiv \{\rho_{KLG}, \theta_{KLG}, \rho_{RES}, \theta_{RES}\}$ represents the key micro parameters to be estimated.

Firm Profitability Shifters and Demand Shocks. Similar to the reduced-form regressions, I exploit variation within the firm over time for identification. Firms in the model compete under a separate static equilibrium in each period $t \in \{1, 2, 3\}$ (corresponding to years 1997, 2002, and 2007 in the data). Firms optimize their input expenditures, knowledge accumulation, and final output in each period t after observing profitability conditions $\boldsymbol{B}_t, \boldsymbol{\xi}_{ft}$.[27]

In the first period, firms draw their exogenous profitability shifters $\{\xi_{fj}\}_j$ in each industry from a joint lognormal distribution specified by Assumption 3. Profitability shifters $\boldsymbol{\xi}_{ft}$ stand in for any firm-specific demand and supply conditions (e.g., differences in product appeal, non-depreciating capital stocks) unobservable to the econometrician. I assume that the firm-industry-specific demand shocks constructed in Section 1 shift $\boldsymbol{\xi}_{ft}$ over time,[28] thereby triggering changes in firm sales in the model (as a function of $\boldsymbol{\Theta}$). Besides from the impact of these idiosyncratic demand shocks, I assume that firms retain their initial exogenous profitability shifters over time, allowing the model to explain persistence in a firm's size and industry specialization in the data.

Assumption 3 (Demand Shocks as Profitability Shifters) *Each firm's fundamental profitability shifters are distributed joint lognormal in period $t = 1$ according to:*

$$\xi_{fj,t=1} = \zeta_{fj}\,\zeta_f, \quad \log\zeta_{fj} \sim_{i.i.d.} \mathcal{N}(0, \upsilon_0), \quad \log\zeta_f \sim_{i.i.d.} \mathcal{N}(0, \upsilon_1), \quad \forall f \in \mathcal{F}, j \in \mathcal{J}.$$

In years $t = \{2, 3\}$, a measure-zero set of firms $\mathcal{F}_t^D$ (corresponding to the regression sample) receive demand shocks $\{\Delta \log S_{fjt}\}_{j\in\mathcal{J}}$ as constructed in Section 1, which affect their profitability shifters according to:

$$\Delta \log \xi_{fjt} = \nu\, \Delta \log S_{fjt}, \qquad \forall f \in \mathcal{F}_t^D,\ j \in \mathcal{J},\ t = \{2, 3\}. \tag{20}$$

Other firms retain their initial-period profitability shifters over time.

The variance parameters $\boldsymbol{\upsilon} \equiv \{\upsilon_0, \upsilon_1\}$ control firm-level comparative and absolute advantage respectively. First, the higher is υ_0, the more dispersed is profitability across industries *within* a firm, and the more persistent is a firm's pattern of specialization over time. This occurs as the firm is more likely to repeatedly accumulate knowledge in industries with very high ξ_{fj}. I estimate υ_0 by matching the share of industries in multi-industry firms that survive over 5-year intervals to that in the data, equal to 0.42. Second, the higher is υ_1, the more dispersed is size *across* firms. I estimate υ_1 by matching the aggregate share of sales by multi-industry firms in 1997 to that in the data, equal to 0.74. I normalize the means of the lognormal distributions to zero because they are isomorphic to shifters of industry profitability $\boldsymbol{B}$.

[27]In other words, I assume that accumulated knowledge completely depreciates across periods. Using the BEA's estimates of knowledge capital depreciation rates of 0.33, only 14 percent of accumulated knowledge would remain across five-year intervals, so this assumption is not far off. In addition, by allowing profitability shifters $\boldsymbol{\xi}_f$ to persist over time, I account for the impact of any (unobserved) capital that does not depreciate across periods.

[28]This assumption can be explicitly micro-founded in a multi-destination export setting in which firms draw different initial taste shifters across destinations. These shifters inform pre-existing patterns of exporting, and subsequent changes in foreign market size across destinations will manifest in changes in firms' profitability shifters in the model.

I estimate the 'first-stage' elasticity ν in equation (20) by leveraging the model's log-linear relationship between a firm's expenditures on professional services M_{ft}^{PROF} (one of the four categories of shared inputs) and the firm's average demand shock:

$$\Delta \log M_{ft}^{PROF} = \nu \ \rho_{KLG} \sum_{k \in \mathcal{J}} \eta_{fk,t-1}^{PROF} \Delta \log S_{fkt},$$

where model-consistent expenditure shares $\eta_{fk,t-1}^{PROF}$ are approximated using BEA industry-level expenditure shares on professional services $\beta_{k,PROF}$:

$$\eta_{fk,t-1}^{PROF} \equiv \frac{\mu_{fmk,t-1}\Delta_{fm,t-1}^{\rho_m}}{\Delta_{fm,t-1}^{\rho_m}} \approx \frac{\beta_{k,PROF} X_{fk,t-1}}{\sum_k \beta_{k,PROF} X_{fk,t-1}}, \qquad \text{for } m = PROF.$$

Intuitively, the elasticity of professional service input expenditures with respect to demand shocks depends on the product of two elasticities (i) ν, the elasticity of firm profitability with respect to demand shocks, and (ii) ρ_{KLG}, the elasticity of professional service input expenditures with respect to firm profitability. Column (1) of Table 3 provides a regression estimate of the combined elasticity $\nu\rho_{KLG} = 0.65$, allowing identification of ν conditional on knowledge input scalability ρ_{KLG}. Before turning to identification of micro scalability and rivalry parameters $\mathbf{\Theta}$, I describe identification of macro variables conditional on $\mathbf{\Theta}$.

3.2 Identification of Macro Variables

The first half of Table 4 summarizes the macro variables and their sources of identification. First, I set the mass of potential entrants at $N = 318000$, the total number of firms (including administrative and inactive records) in the 1997 Census of Manufactures.[29] Second, $\varsigma_j = \gamma_j(\sigma_j - 1)/\sigma_j$ in the model is equal to the share of gross output expensed on stage-II inputs, which is readily available in BEA input-output data.[30] Third, I calibrate the average arrival rate of ideas, Z_t, by matching the share of multi-industry firms in the model to that in the data in each year (0.2).

Finally, I calibrate industry profitability $\boldsymbol{B}_t$ and technology coefficients $\boldsymbol{\alpha} \equiv \{\alpha_{mj}\}_{m,j}$ so that the model exactly matches BEA data on gross output $\boldsymbol{X}_t$ and expenditures on shared (stage-I) inputs $\boldsymbol{M}_t$ by industry. In the model, firms' expenditures on stage-I inputs are shared across industries, whereas the BEA reports these expenditures (e.g., on R&D) separately by industry. I assume that an equivalent statistical agency in the model registers each firm's total expenditures on a given shared input m under the industry j where the firm adapts its *first* type-m idea. Given a continuum of firms, this is equivalent under aggregation to apportioning each firm's expenditures on input m

[29]Firms in my model move in and out of active status due to stochasticity in knowledge accumulation. An inactive firm is any firm that, despite positive stage-I input expenditures, has accumulated zero knowledge. In any period in which this happens, the firm will register zero sales and fall out of the observed sample, i.e., become an inactive record.

[30]Estimation is invariant to values of actual production returns to scale γ_j or demand elasticities σ_j. Under monopolistic competition, the sufficient equilibrium parameter is ς_j, a combination of the two.

Table 4: Overview of Model Parameters and Sources of Identification

Variable and Description		Source of Identification
Macro Variables		
N	Mass of all firms $\mathcal{F}$	All active and inactive firms (318,000)
ς_j	Stage-II input expenditures as share of output	Corresponding shares in BEA I/O Table
Z_t	Average arrival rate of ideas	Share of multi-industry firms (0.2)
B_{jt}	Industry profitability	BEA Industry Gross Output X_{jt}
α_{mj}	Average value of shared input m in industry j	BEA Input-by-industry Expenditures M_{mj}
Micro Parameters		
v_0	Variance in ξ_f across industries within the firm	Share of industries that continue (0.42)
v_1	Variance in ξ_f across firms	Share of sales by multi-industry firms (0.74)
ν	Responsiveness of ξ_f to demand shocks	Assumption 3 and Table 3 ($\nu\rho_{KLG} = 0.65$)
$\boldsymbol{\Theta}$	Scalability and rivalry of shared inputs	Proposition 2

across industries according to adaptation probabilities μ_{fmj}.[31]

Equation (21) provides the national accounting identities that define industry-level output (X_{jt}) and input expenditures (M_{mjt}) as a sum over respective firm-level variables in the model:

$$\begin{aligned} X_{jt} &= \frac{1}{1-\varsigma_j} N \int \sum_{m \in \mathcal{M}} \delta_{fmjt}^{\theta_m} \Delta_{fmt}^{\rho_m - \theta_m} \, \mathrm{d}G(\boldsymbol{\xi}), \qquad \forall j \in \mathcal{J}, \ \ t \in \{1,2,3\}, \\ M_{mjt} &= \frac{\rho_m - 1}{\rho_m} N \int \delta_{fmjt}^{\theta_m} \Delta_{fmt}^{\rho_m - \theta_m} \, \mathrm{d}G(\boldsymbol{\xi}), \qquad \forall j \in \mathcal{J}, \ \ t = 1, \ \ m \in \mathcal{M}, \end{aligned} \tag{21}$$

where $G(\boldsymbol{\xi}; \boldsymbol{v})$ is the joint-lognormal distribution parametrized by Assumption 3.[32] I normalize the unit cost of shared inputs w_t to equal one in each year (differences across types of shared inputs m are absorbed by $\boldsymbol{\alpha}$) and match this normalization in the data by deflating X_{jt} and M_{mjt} in each year by wage inflation. I calibrate $\boldsymbol{\alpha}$ to match input-by-industry expenditure data from the first cross-section, 1997, and assume that $\boldsymbol{\alpha}$ is time-invariant. Equation (21) is a system of $|\mathcal{J}| \times (3 + |\mathcal{M}|)$ equations with as many unknowns. I develop a fast recursive computational algorithm that inverts the system of equations to solve for $\boldsymbol{B}_t$ and $\boldsymbol{\alpha}$ (contained in $\boldsymbol{\delta}$ and $\boldsymbol{\Delta}$) given data on $\boldsymbol{X}_t$, $\boldsymbol{M}$ and other macro and micro parameters. I provide more details in Appendix C.1.

Notice that identification of these macro variables does not require specifying other general equilibrium details of the model (such as trade or vertical input-output linkages) as long as such features affect all firms equally. Industry-level profitability $\boldsymbol{B}_t$ encapsulates the combined effect of export market access, import market competition, as well as prices of industry-specific intermediate

[31]Most papers on multi-output firms assume nonjoint production (e.g., De Loecker et al., 2016; Orr, 2019) and devise some weighting scheme to apportion a firm's input use across industry lines. This however does not allow for inputs to be shared across industries; for example, a firm could book the initial use of inputs in one industry while reaping benefits in other industries. I circumvent this challenge by aggregating input use over firms. The apportioning shares μ_{fmj} would not be correct for any single firm but would be under when aggregating over a continuum of firms.

[32]With an abuse of notation, a variable subscripted with f indicates that it is dependent on ξ_f.

inputs in each year, as long as they are common to all firms.

3.3 Identification of Scalability and Rivalry Parameters

Lastly, I exploit within-firm variation over time and use the vector of demand shocks $\Delta \log \boldsymbol{S}_{ft}$ as instruments to identify scalability and rivalry parameters $\boldsymbol{\Theta}$. By Assumption 3, demand shocks shift firms' profitability $\boldsymbol{\xi}_{ft}$ between t and $t-1$, allowing me to identify $\boldsymbol{\Theta}$ from changes in firms' sales across industries. Proposition 1 shows that same and cross-industry elasticities of sales inform values of ρ_m and θ_m. The moment estimator makes use of similar covariances except over exact five-year changes in the data, accounting for the effects of other changes in macro aggregates.

An immediate challenge for mapping firms in the model to the data is the non-random assignment of demand shocks to firms. Export demand shocks $\Delta \log \boldsymbol{S}_{ft}$ from Section 1 are constructed only for firms that are already selling in a given industry j. Firms active in industry j would have a higher-than-average fundamental profitability $\xi_{fj,t-1}$ in that industry, and comparing the outcomes of such a firm in the data against a those of a *randomly* drawn firm in the model would lead to selection bias.

I address this potential selection bias using Assumption 4. I assume that demand shocks are uncorrelated with initial-period unobserved profitability $\boldsymbol{\xi}_{f,t-1}$ and sales $\boldsymbol{X}_{f,t-1}$ *conditional on* the firm's initial industry presence, $\boldsymbol{\chi}_{f,t-1}$. In other words, identification only requires that shocks are as good as randomly assigned among firms with identical initial-period extensive margins.

Assumption 4 (Conditional Independence) *Demand shocks are randomly assigned to firms conditional on pre-existing industry presence:*

$$\Delta \log \boldsymbol{S}_{ft} \perp \boldsymbol{\xi}_{f,t-1}, \boldsymbol{X}_{f,t-1} \mid \boldsymbol{\chi}_{f,t-1}, \qquad t \in \{2,3\}.$$

Proposition 2 describes the moment conditions I use to estimate $\boldsymbol{\Theta}$. Equation (22) stipulates that under true values of $\boldsymbol{\Theta}$, conditional covariances between demand shocks and sales growth in the data should equate that in the model for any pair of industries j, k. This yields a $|\mathcal{J}|^2$ matrix of moment conditions for each of the two time-differenced periods $t = 2, 3$. In the sample analogs of each moment condition, I include firms with industry activity in j and k in the initial period, $t-1$ and include endogenous exit as an outcome of the firm (whereby $X_{fjt} = 0$). By conditioning model predictions on the firm's initial-period extensive margin $\boldsymbol{\chi}_{f,t-1}$, I am able to correct for potential selection bias arising from a firm having higher-than-average ξ_{fk} and ξ_{fj} whenever it is observed to be jointly active in those industries.

Proposition 2 (Identification of Scalability and Rivalry) *Define structural residuals $\Delta\epsilon_{fjt}$ as functions of initial-period fundamental profitability $\boldsymbol{\xi}_{f,t-1}$, initial-period extensive margin $\boldsymbol{\chi}_{f,t-1}$, and demand* *shocks* $\Delta \log \boldsymbol{S}_{ft}$*:*

$$\Delta\epsilon_{fjt} \equiv \left(X_{fjt} - X_{fj,t-1}\right) - \left(\mathbb{E}_t[X_{fjt} \mid \boldsymbol{\xi}_{f,t-1}, \boldsymbol{\chi}_{f,t-1}, \Delta \log \boldsymbol{S}_{ft}] - \mathbb{E}_{t-1}[X_{fj,t-1} \mid \boldsymbol{\xi}_{f,t-1}, \boldsymbol{\chi}_{f,t-1}, \Delta \log \boldsymbol{S}_{ft}]\right),$$

the difference between a firm's change in sales in a given industry j *in the data (the first bracketed term) and its expected change in sales in the model (the second bracketed term, where* $\mathbb{E}_t$ *is an expectation operator expressing the firm's ex-ante expected sales conditional on* Θ *as well as macro parameters* $\boldsymbol{B}_t, Z_t, \boldsymbol{\alpha}, \boldsymbol{\varsigma}$*, as in equation 15). Under Assumptions 3 and 4, the following moments hold in expectation (for any* j, k*):*

$$\mathbf{E}_f \left[\Delta\epsilon_{fjt} \; \Delta \log S_{fkt} \;\middle|\; \boldsymbol{\chi}_{f,t-1}\right] = 0, \qquad \forall t = \{2, 3\}, \qquad \forall j, k \in \mathcal{J}. \tag{22}$$

In Appendix C.2, I prove Proposition 2 and derive analytical sample analogs for these moments as functions of the data and the micro and macro parameters in Table 4. By estimating micro and macro parameters jointly, I account for the effect of equilibrium changes in industry demand and supply conditions on firm sales growth. Macro parameters $\boldsymbol{B}_t, Z_t, \boldsymbol{\alpha}, \boldsymbol{\varsigma}$ behave in my micro moment conditions as non-linear fixed effects. For example, any changes in industry-wide demand (or supply) conditions between t and $t-1$ are reflected in differences between $\boldsymbol{B}_{jt}$ and $\boldsymbol{B}_{j,t-1}$, which affect firms' output growth $\mathbb{E}_t[X_{fjt} \mid \boldsymbol{\xi}_{ft}] - \mathbb{E}_{t-1}[X_{fj,t-1} \mid \boldsymbol{\xi}_{f,t-1}]$ in the model.

Given the sparsity in firms' extensive margins in the data, I create four groupings of moments in each year $t = \{2, 3\}$. Each grouping contains an average over the following elements from the $\mathcal{J} \times \mathcal{J}$ matrix of moments: (i) main-diagonals $j = k$ for industries j with higher-than-average expenditure shares on knowledge inputs, (ii) remaining main-diagonals, (iii) off-diagonals $j \neq k$ for industry pairs j, k with higher-than-average knowledge-input proximity, and (iv) remaining off-diagonals.

Grouping moments according to same- versus cross-industry covariances helps identify scalability (ρ_{KLG}, ρ_{RES}) separately from rivalry ($\theta_{KLG}, \theta_{RES}$). Recall from Proposition 1 that scale and rivalry effects push in opposite directions for the response of sales to cross-industry shocks, but push in the same direction for the response of sales to same-industry shocks. For example, if a positive demand shock in industry k increases sales in another industry j of the firm, shared inputs used by k and j can be either more scalable (ρ_m is high) or less rival (θ_m is low) to match this covariance in the data. If, however, the same demand shock also raises sales in the same industry k of the firm, shared inputs must be sufficiently scalable on an absolute basis (i.e., ρ_m is high).

Next, grouping moments according to their knowledge-proximity helps identify parameters associated with shared knowledge inputs (ρ_{KLG}, θ_{KLG}) separately from that of residual shared inputs (ρ_{RES}, θ_{RES}). If industries with greater knowledge-proximity (parametrized by technology coefficients $\boldsymbol{\alpha}$ in the model) exhibit greater covariances of sales growth to demand shocks, it must be that the shared inputs used intensively by those industries (i.e., knowledge inputs) are more scalable and less rival than the residual shared input. Altogether these four groupings of moments provide

Table 5: Estimates of Scalability, Rivalry, and Firm Heterogeneity Parameters

Parameter	Description	Estimate	S.E.
ρ_{KLG}	Scalability of shared knowledge inputs	12.64	(0.39)
θ_{KLG}	Rivalry of shared knowledge inputs	3.61	(0.07)
ρ_{RES}	Scalability of residual shared inputs	2.63	(0.05)
θ_{RES}	Rivalry of residual shared inputs	4.06	(0.12)
v_0	Degree of comparative advantage within the firm	0.85	(0.03)
v_1	Variation in absolute advantage across firms	0.99	(0.05)
Test of Over-identifying Restrictions: $7.28 \sim \chi^2_4$			$p = 0.12$

Notes: This table reports estimates of micro parameters in the model. The six parameters are estimated on the sample of all multi-industry firms and all pairwise industries in which they are initially active, over years 2002-2007 and 1997-2002, using 10 moments. There are 13,000 (rounded) firm-year observations used in the sample. Standard errors of estimates are computed based on results from 21 bootstrap samples, where I re-draw over both the data and the simulated ξ samples.

sufficient identifying variation to estimate the four micro elasticities ($\mathbf{\Theta} = \rho_{KLG}, \theta_{KLG}, \rho_{RES}, \theta_{RES}$).

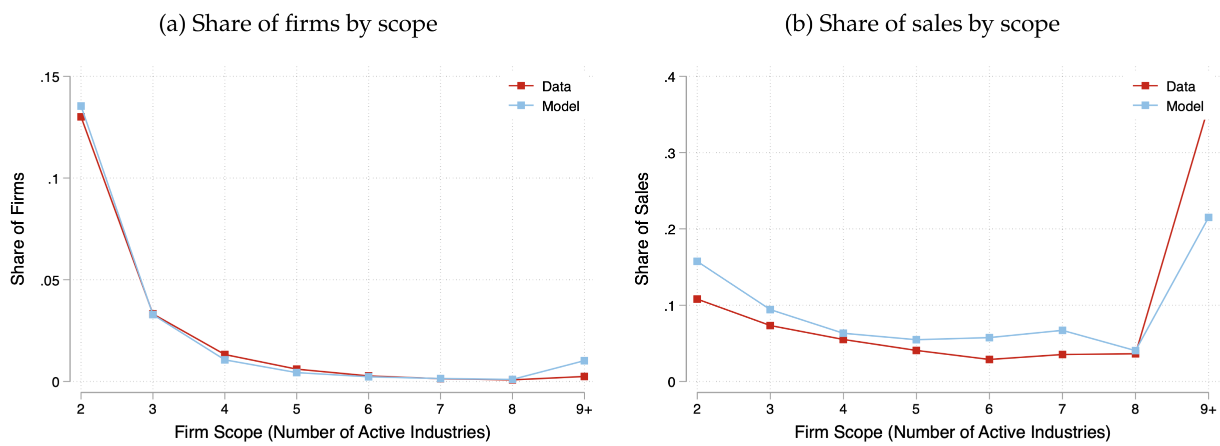
Table 5 presents estimates of input scalability and rivalry $\mathbf{\Theta}$ as well as the variances $\boldsymbol{v}$ of firms' fundamental profitability shifters. I use ten moment conditions weighted equally: the four grouped moments above repeated for each of two years $t = 2, 3$, and two remaining cross-sectional moments (from the initial year $t = 1$, shown in Table 4), which identify the variances v_0, v_1 in the joint-lognormal distribution. Consistent with evidence in Section 1, the ranking $\rho_{KLG} > \theta_{RES} > \theta_{KLG} > \rho_{RES}$ implies that knowledge inputs induce stronger scale effects and weaker rivalry effects within the firm. In contrast, residual shared inputs induce weaker scale effects and stronger rivalry effects, consistent with negative cross-elasticities among industries that are not knowledge-proximate. Estimates do not change by much when using the optimal weighting matrix under two-step GMM. In the last row, I show that a test of over-identifying restrictions does not reject the null that the identifying moment conditions are jointly valid, suggesting the estimated parameters provide a good fit to these micro moments.

3.4 External Validation: Scale, Scope, and Industry Co-Production

Despite its limited number of (six) micro parameters, the estimated model reproduces other extensive-margin moments in the data not targeted in estimation. First, the model matches the distribution of the number of firms and their sales by firm scope in the data, shown in Figure 3. Both the data and the model attribute a significant size premium to the right tail of the firm scope distribution, though for firms with nine or more industries, the model somewhat undershoots the data because it cannot account for the scale of true conglomerates. Overall, the close fit between the model and data validate the Poisson and Fréchet functional form assumptions.

Next, I conduct a more demanding validation test by showing that knowledge proximity in the

Figure 3: Model versus Data: Distribution of Firms and Sales by Scope in 1997



Notes: Panel (a) plots the distribution of firms by scope. Panel (b) plots the share of total sales accounted for by firms across the scope distribution. Data in 1997 is shown in red and model outcomes (computed using 1997 macro aggregates $\boldsymbol{B}_t, \boldsymbol{\alpha}$) are shown in blue.

model can explain non-random patterns of co-production in the data documented by Bernard et al. (2010). I create an asymmetric jk-level measure of co-production (pairwise industry entry) as the share of industry j sales by firms that also produce in k:

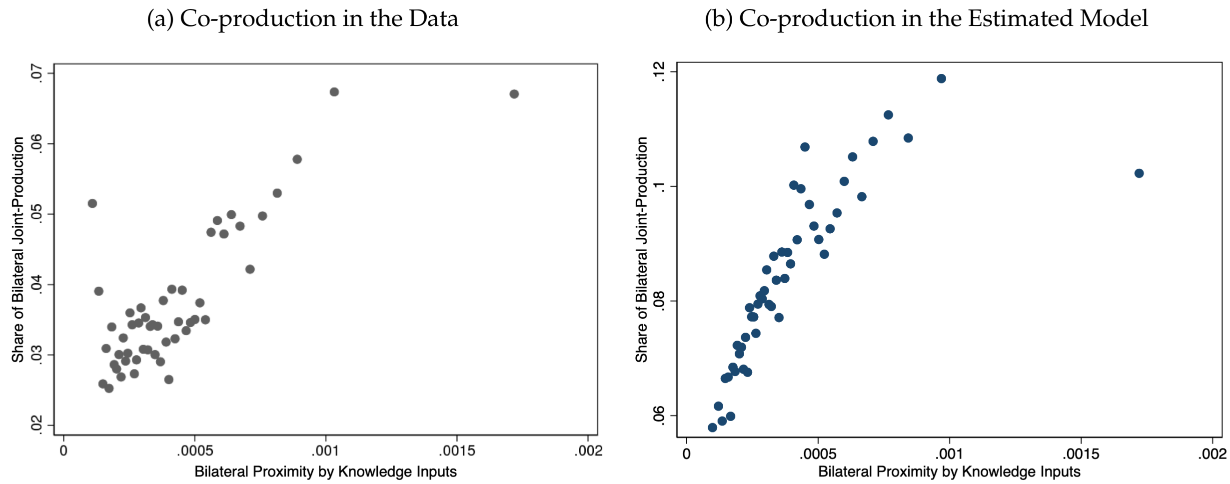
$$CoProd_{jk} \equiv \frac{\sum_f X_{fj} \mathbf{1}_{(X_{fk}>0)}}{\sum_f X_{fj}},$$

and I measure an industry-pair's knowledge-proximity in a similar way as equation (5) but with industry-level output X_j in place of firm-level output X_{fj}:

$$Prox_{jk}^{KLG} \equiv \sum_{m \in \mathcal{M}^{KLG}} \beta_{jm} \frac{\beta_{km} X_k}{\sum_{k' \in \mathcal{J}} \beta_{k'm} X_{k'}}. \tag{23}$$

Panel (a) of Figure 4 finds that, in the data, co-production increases with knowledge input-proximity. In panel (b), I show that the estimated model reproduces this strong positive correlation. Co-production is more prevalent across pairs of knowledge-proximate industries because their shared inputs are more scalable (resulting in a higher overall arrival rate of ideas) and less rival (resulting in ideas being adapted to improve knowledge in both industries). These results provide external validation to the model estimates given that the cross-sectional patterns of co-production were not targeted during estimation.

Figure 4: External Validity: Co-production in the Data and the Model in 1997



Notes: These panels correlate bilateral industry co-production (share of industry j sales by firms with activities in k) with bilateral knowledge-proximity, in (a) the data, and (b) the model.

4 The Aggregate Implications of Joint Production

The scalability and non-rivalry of knowledge inputs is a source of economies of scale and scope in the aggregate. Increasing output in one industry will reduce producer prices in not only the same industry but also others on average. I analytically characterize and quantify these industry linkages in general equilibrium using a matrix of same- and cross-industry elasticities of the producer price index to demand shocks.

4.1 Joint Production in General Equilibrium

To isolate the role of joint production, I introduce bare-bones general equilibrium assumptions that rule out unrelated mechanisms for cross-industry interdependence. Definition 1 in Appendix D.1 specifies the exact equilibrium conditions. Consumer demand is Cobb-Douglas across manufacturing industries, which shuts down demand-side linkages. Production in stage II uses only labor, which shuts down conventional input-output linkages.[33] The US economy (denoted u) trades with a set of foreign partners (denoted $d \in \mathcal{D}^F$) each with exogenous macro aggregates (demand levels and foreign firm costs), which shuts down cross-country linkages. There is a large non-manufacturing sector in which US exporters face infinitely elastic foreign demand. This

[33] This assumption is still consistent with prior sections of the paper. Estimation of scale and rivalry elasticities in Section 3 is invariant to the input-output structure of the economy. Any changes in either stage-II input costs or factor prices are absorbed by the industry profitability shifter $\boldsymbol{B}_t$, the sufficient macro variable for micro moment conditions. Nevertheless, in Appendix D.2, I prove a more general version of Proposition 3 that accommodates arbitrary stage-II input-output linkages across manufacturing industries. In a robustness exercise in Appendix Table D.8 input-output linkages I find that the presence of economies of scope doubles the equilibrium response of prices to demand shocks.

assumption keeps wages fixed across counterfactuals, since overall trade can balance via changes in the non-manufacturing sector's net exports. The total stock of firms (including latent ones) is fixed at N (which still allows for firm entry and exit across industries, as well as in and out of active status). Finally, all firm profits and tariff revenues are spent on the non-manufacturing sector, which shuts down expenditure-driven feedback effects.

The key endogenous macro aggregates are domestic producer price indices in each industry j:

$$\mathcal{P}_j^{1-\sigma_j} \equiv N \int \mathbb{E}\left[p_{fj}^{1-\sigma_j}\right] \mathrm{d}G(\xi), \qquad \forall j \in \mathcal{J},$$

which depend on firms' joint production decisions. Proposition 3 derives the equilibrium elasticity of PPI and output in each industry with respect to exogenous shifters of industry market size $\mathrm{d}\log \boldsymbol{S}$ (e.g., shocks to population size, foreign demand, or foreign costs).

Proposition 3 (Industry Linkages from Joint Production) *Under the open economy general equilibrium conditions provided in Definition 1, domestic producer price indices* $\mathrm{d}\log \boldsymbol{\mathcal{P}}$ *and output* $\mathrm{d}\log \boldsymbol{X}$ *respond to exogenous shocks to industry market size* $\mathrm{d}\log \boldsymbol{S}$ *(defined in equation 33) according to:*

$$\mathrm{d}\log \boldsymbol{\mathcal{P}} = diag\left(\frac{1}{\boldsymbol{\sigma}-1}\right)\left(\mathbb{I} + \boldsymbol{\Psi}\; diag(\boldsymbol{\lambda}^{cpt})\right)^{-1} \boldsymbol{\Psi}\,\mathrm{d}\log \boldsymbol{S}, \tag{24}$$

$$\mathrm{d}\log \boldsymbol{X} = \mathrm{d}\log \boldsymbol{S} \;+\; diag(\boldsymbol{\lambda}^{cpt}(1-\boldsymbol{\sigma}))\,\mathrm{d}\log \boldsymbol{\mathcal{P}} \tag{25}$$

where (i) $\mathbb{I}$ *is the identity matrix, (ii)* $\boldsymbol{\Psi}$ *is a macro joint production matrix containing inverse cross-industry supply-side elasticities* $\boldsymbol{\Upsilon}$ *for the 'average' firm:*

$$\begin{aligned}
[\boldsymbol{\Psi}]_{jk} &\equiv \sigma_j(1-\varsigma_j)[\boldsymbol{\Upsilon}^{-1}]_{jk} \;-\; \mathbf{1}_{j=k},\\
[\boldsymbol{\Upsilon}]_{jk} &\equiv \sum_{m\in\mathcal{M}} (\rho_m - \theta_m)\,\bar{\lambda}_{jm}\,\bar{\mu}_{jmk} \;+\; \mathbf{1}_{j=k}\sum_{m\in\mathcal{M}} \theta_m\,\bar{\lambda}_{jm}, \qquad \forall j,k \in \mathcal{J},
\end{aligned}$$

where industry choice shares $\bar{\mu}_{jmk}$ *indicate the average propensity for type m-ideas to be adapted in industry k (relative to k') among firms that produce in j, and input utilization shares* $\bar{\lambda}_{jm}$ *indicate the average profit-contribution to industry j of shared input m (relative to m'):*

$$\bar{\mu}_{jmk} \equiv \int \frac{\mathbb{E}[X_{fj}]\lambda_{fjm}}{\int \mathbb{E}[X_{fj}]\lambda_{fjm}\,\mathrm{d}G(\xi)}\mu_{fmk}\,\mathrm{d}G(\boldsymbol{\xi}), \qquad \bar{\lambda}_{jm} \equiv \int \frac{\mathbb{E}[X_{fj}]}{\int \mathbb{E}[X_{fj}]\,\mathrm{d}G(\xi)}\lambda_{fjm}\,\mathrm{d}G(\boldsymbol{\xi}),$$

and (iii) $\boldsymbol{\lambda}^{cpt}$ *reflects the potential for US firms to gain share from foreign competitors in each market d:*

$$\lambda_j^{cpt} \equiv \sum_{d\in\{u,\mathcal{D}^F\}} \lambda_{dj}^X(1-\lambda_{dj}^M) \qquad \forall j \in \mathcal{J},$$

where λ_{dj}^M *is the share of country d's industry j consumption originating from US firms, and* λ_{dj}^X *is the share* *of US firms' industry j sales exported to d.*

The joint production matrix $\mathbf{\Psi}$ encapsulates the equilibrium impact of demand shocks on industry-level PPI and highlights two within-firm sources of aggregate increasing returns to scale.[34] First, economies of scale (same-industry elasticities of price with respect to output) are governed by the scalability of stage-II industry-specific inputs ($\boldsymbol{\gamma}$) and stage I shared inputs ($\boldsymbol{\rho}$). Second, economies of scope (cross-industry elasticities of price with respect to output) are governed by the *relative* scalability and rivalry of shared inputs ($\boldsymbol{\rho} - \boldsymbol{\theta}$).

For intuition on how these forces manifest in equilibrium, first consider an economy under autarky, so $\boldsymbol{\lambda}^{cpt} = 0$ and equation (24) simplifies to:

$$\mathrm{d}\log \boldsymbol{\mathcal{P}} = \mathrm{diag}\left(\frac{1}{\boldsymbol{\sigma}-1}\right)\mathbf{\Psi}\,\mathrm{d}\log \boldsymbol{S}.$$

The relative magnitudes of economies of scale and scope depend on main-diagonal versus off-diagonal elements of the matrix $\mathbf{\Psi}$. I analyze each in turn.

Economies of Scale. When off-diagonal elements of $\mathbf{\Psi}$ are zero, the only force present is within-industry economies of scale. This occurs when either (i) shared inputs are in fact industry-specific (so that $\bar{\lambda}_{jm}\bar{\mu}_{jmk} = 0\ \forall m$), or (ii) scale and rivalry effects offset each other on a knife's edge ($\rho_m = \theta_m\ \forall m$). Whereas producer price indices in each industry are unaffected by demand shocks in any other industry, the main-diagonals of $\mathbf{\Psi}$ still allow for arbitrary within-industry returns to scale. Equation (24) simplifies further to (using the fact that $\gamma_j = \varsigma_j \frac{\sigma_j}{\sigma_j-1}$):

$$\mathrm{d}\log \mathcal{P}_j = \frac{1}{\sigma_j-1}\left(\frac{\sigma_j-\gamma_j(\sigma_j-1)}{\sum_m \bar{\lambda}_{jm}\rho_m} - 1\right)\mathrm{d}\log S_j, \qquad \forall j \in \mathcal{J},$$

with an elasticity bounded between $(-\frac{1}{\sigma_j-1}, 1)$, nesting the range of scale elasticities in Kucheryavyy et al. (2019). The more scalable are stage I inputs (ρ_m) and stage II inputs (γ_j), the more negative is this elasticity, and the stronger are industry-level economies of scale.

Two limit cases are worth highlighting. First, when $\rho_m \to 1$, knowledge accumulation in stage I is exogenous and non-responsive to changes in industry profitability. The response of industry prices to demand shocks depends only on the scalability γ_j of stage II industry-specific inputs. Under, for example, constant returns to scale in stage II production ($\gamma_j = 1$), the matrix $\mathbf{\Psi}$ is element-wise 0 (since $\sigma_j - \gamma_j(\sigma_j-1) = 1$), and prices are affected by neither same-industry nor cross-industry demand shocks. Outside of this knife-edge value of $\gamma_j = 1$, higher values ($\gamma_j > 1$)

[34] While Proposition 3 suffices for the analysis in the rest of the section (evaluating the impact of counterfactual shocks to foreign demand), I provide more theoretical results in Appendix D.3. I show how the joint production matrix $\mathbf{\Psi}$ characterizes (i) partial-equilibrium aggregate cross-price elasticities of *supply*, and (ii) how supply-side shocks (e.g., industry-level TFP) propagate.

generate economies of scale, and lower values ($\gamma_j < 1$) generate diseconomies of scale.

Industry-level economies of scale also increase with the scalability (ρ_m) of shared inputs in stage I. The higher is ρ_m, the more easily can firms respond to an increase in market size by accumulating more knowledge capital and lowering marginal production costs without running into potential decreasing returns to scale in stage II. For any value of γ_j, in the limit as $\rho_m \to \infty$ for any m, industry-level economies of scale reaches its maximum strength (at $-1/(\sigma_j - 1)$). This limit reproduces the same scale elasticity as that in a standard multi-industry Krugman (1980) model. One difference, of course, is that scale economies generated by both γ_j and ρ_m are internal to the firm. They come from (quality-adjusted) cost improvements rather than new varieties by entrants, but the two are isomorphic from the lens of the CES price index.

Economies of Scope. Cross-elasticities (off-diagonal elements of $\mathbf{\Psi}$) reflect the impact of any economies of scope. Economies of scope increase with the scalability and non-rivalry ($\rho_m - \theta_m$) of stage-I shared inputs. Consider a numerical example: an economy with two symmetric industries, ex-ante identical firms, identical demand elasticities ($\boldsymbol{\sigma} = 5$), constant returns to scale in stage-II production ($\boldsymbol{\gamma} = 1$), and a single type of shared input that is scalable and partially non-rival ($\rho = 7$ and $\theta = 3$). Using the fact that $\sigma(1 - \varsigma) = 1$, $\bar{\lambda} = 1$, and $\bar{\mu} = 0.5$, equation (24) reduces to:

$$\mathrm{d}\log \boldsymbol{\mathcal{P}} = \underbrace{\begin{pmatrix} 0.25 & 0 \\ 0 & 0.25 \end{pmatrix}}_{\mathrm{diag}\left(\frac{1}{\sigma-1}\right)} \underbrace{\left[\frac{1}{21}\begin{pmatrix} 5 & -2 \\ -2 & 5 \end{pmatrix} - \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\right]}_{\mathbf{\Psi}=\mathbf{\Upsilon}^{-1}-\mathbb{I}} \mathrm{d}\log \boldsymbol{S} = \begin{pmatrix} -0.191 & -0.024 \\ -0.024 & -0.191 \end{pmatrix} \mathrm{d}\log \boldsymbol{S}.$$

A positive demand shock in one industry would lower firms' marginal costs in other industries that share scalable and non-rival inputs ($\rho > \theta$). Off-diagonal elements of $\mathbf{\Upsilon}$ would be positive across such industries, corresponding to positive *firm-level* elasticities of sales with respect to demand shocks from Proposition 1. In equilibrium, these changes within each firm affect output price indices (PPI) and therefore competition in other industries, triggering additional changes in firms' production decisions. Similar to the Leontief inverse, $\mathbf{\Psi} = \mathbf{\Upsilon}^{-1} - \mathbb{I}$ captures the total impact of an industry demand shock as it percolates across industries and firms. In the example above, a 1 percent increase in demand in the first industry lowers the PPI in the same industry by 0.19 percent and lowers the PPI in the other industry by 0.02 percent, one-eight the size of the own-industry effect. Outside of this toy setting, of course, cross-elasticities are asymmetric and unrestricted in sign. Cross-elasticities can even be positive if shared inputs were on net more rival and less scalable, indicating *diseconomies* of scope.

Open Economy Effects. Finally, with trade, a decline in producer prices allows domestic firms to win market share against foreign producers (both at home and abroad). The equilibrium impact of demand shocks on producer prices (equation 24) depends on an additional term,

$\left(\mathbb{I} + \boldsymbol{\Psi}\ \mathrm{diag}(\lambda^{cpt})\right)^{-1}$. Once again, the matrix inverse captures how, given non-constant returns to scale and scope, changes in market share in turn trigger further rounds of changes in the PPI.

4.2 Quantifying the Impact of Joint Production

I use Proposition 3 to decompose aggregate increasing returns in the US manufacturing sector into that generated by economies of scale (within-industry) versus economies of scope (across industries). Whereas numerous models feature industry-level increasing returns (e.g., monopolistic competition with free entry, or perfect competition with external economies of scale), economies of scope represent a new, intra-firm channel specific to joint production.

Making headway on this question requires parametrizing CES industry demand elasticities σ. Up until now, estimation of supply-side parameters ρ, θ *conditioned* on observable general equilibrium changes and firm input expenditure shares ς (a combination of σ and γ), absolving the need for σ. However, Proposition 3 illustrates that *counterfactual* changes in the economy in response to shocks depend separately on values of σ and γ. Under monopolistic competition, σ mediates firms' profit incentives and therefore the extent to which they increase output (and input use) in response to a demand shock.

In my baseline estimates, I calibrate demand elasticities σ so that my model generates the same sector-level increasing returns to scale as estimates in Bartelme et al. (2019). In Appendix D.5, I show that the contribution of economies of scope towards aggregate increasing returns is not sensitive to alternative calibration strategies. I calibrate remaining aggregate parameters of the model to fit data on the US economy trading with two foreign regions: China, and the rest of the world. I calibrate exogenous foreign price and expenditure levels so the model's equilibrium exactly matches industry-level production and trade data from 2017, as detailed in Appendix D.4.

Under this calibration, I simulate a proportional change in foreign demand in each industry k (so $\mathrm{d}\log S_k = 1 - \lambda^X_{uk}\ \forall k$, the share of industry sales that are exported), and use Proposition 3 to compute the aggregate scale elasticity, defined as the elasticity of overall manufacturing PPI with respect to total manufacturing output given demand shocks $\mathrm{d}\log \mathbf{S}$:[35]

$$\left.\frac{\mathrm{d}\log PPI_{\text{manuf}}}{\mathrm{d}\log X_{\text{manuf}}}\right|_{\mathrm{d}\log S} \equiv \left.\frac{\sum_{j\in\mathcal{J}} \lambda^X_j \,\mathrm{d}\log \mathcal{P}_j}{\sum_{j\in\mathcal{J}} \lambda^X_j \,\mathrm{d}\log X_j}\right|_{\mathrm{d}\log S}$$
$$= \underbrace{\left.\frac{\sum_{j\in\mathcal{J}} \lambda^X_j \,\mathrm{d}\log \mathcal{P}^{(\text{same})}_j}{\mathrm{d}\log X_{\text{manuf}}}\right|_{\mathrm{d}\log S}}_{\text{economies of scale}_k} + \underbrace{\left.\frac{\sum_{j\in\mathcal{J}} \lambda^X_j \,\mathrm{d}\log \mathcal{P}^{(\text{cross})}_j}{\mathrm{d}\log X_{\text{manuf}}}\right|_{\mathrm{d}\log S}}_{\text{economies of scope}_k}, \tag{26}$$

[35] Note that since industries differ in economies of scale and scope, the overall scale elasticity depends on compositional make-up of different industries in the economy as well as the levels of shocks received in each industry $\mathrm{d}\log S_j$. Different combinations of shocks would induce different "scale elasticities'.

where weights λ_j^X represent industry j's share of sales within manufacturing. The second line decomposes the overall change in prices into those due to same-industry elasticities versus cross-industry elasticities (i.e., main and off-diagonals of the transmission matrix in equation 24).

The top panel of Figure 5 illustrates that economies of scope from joint production constitute one-quarter of aggregate increasing returns in US manufacturing. Aggregate manufacturing prices fall with respect to output with an elasticity of -0.16, with -0.04 (the darker bar) resulting from cross-elasticities and -0.12 (the lighter bar) coming from same-industry elasticities. In other words, economies of scope cause aggregate US producer prices to fall by 0.4 percent for every 10 percent increase in output induced by foreign demand. This quantitatively large spillover would have been absent in a model that does not take into account joint production.

Moreover, I find that economies of scope are disproportionately concentrated among knowledge-proximate industry clusters. In the bottom panel of Figure 5, I provide a disaggregated visualization of how demand shocks impact price indices. I display within- and cross-sector elasticities aggregated to the level of NAICS 4-digit sectors (there are 86 such sectors in manufacturing, each containing one or more *industries*). Each cell in the matrix measures the contribution to overall increasing returns from price changes within a row sector m induced by demand shocks to industries in a column sector n. For example, the mn cell measures

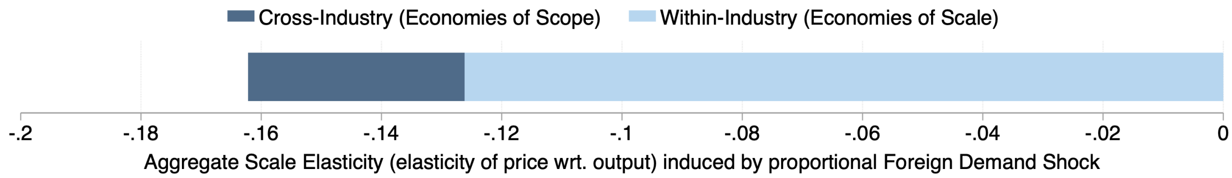
$$\left.\frac{\sum_{j\in m}\lambda_j^X \,\mathrm{d}\log\mathcal{P}_j}{\mathrm{d}\log X}\right|_{\mathrm{d}\log \boldsymbol{S_n}},$$

where $\mathrm{d}\log \boldsymbol{S_n}$ is the vector containing $(1-\lambda_{uk}^X)$ for all industries $k \in n$ and 0 everywhere else. For each given column n (a given demand-shocked sector), off-diagonal cells over rows $m \neq n$ sum to the net effect of economies of scope that manifest *across sectors*, while the main diagonal cell $m = n$ measures the net effect of within-industry economies of scale as well as economies of scope that manifest across industries *within* that sector n.

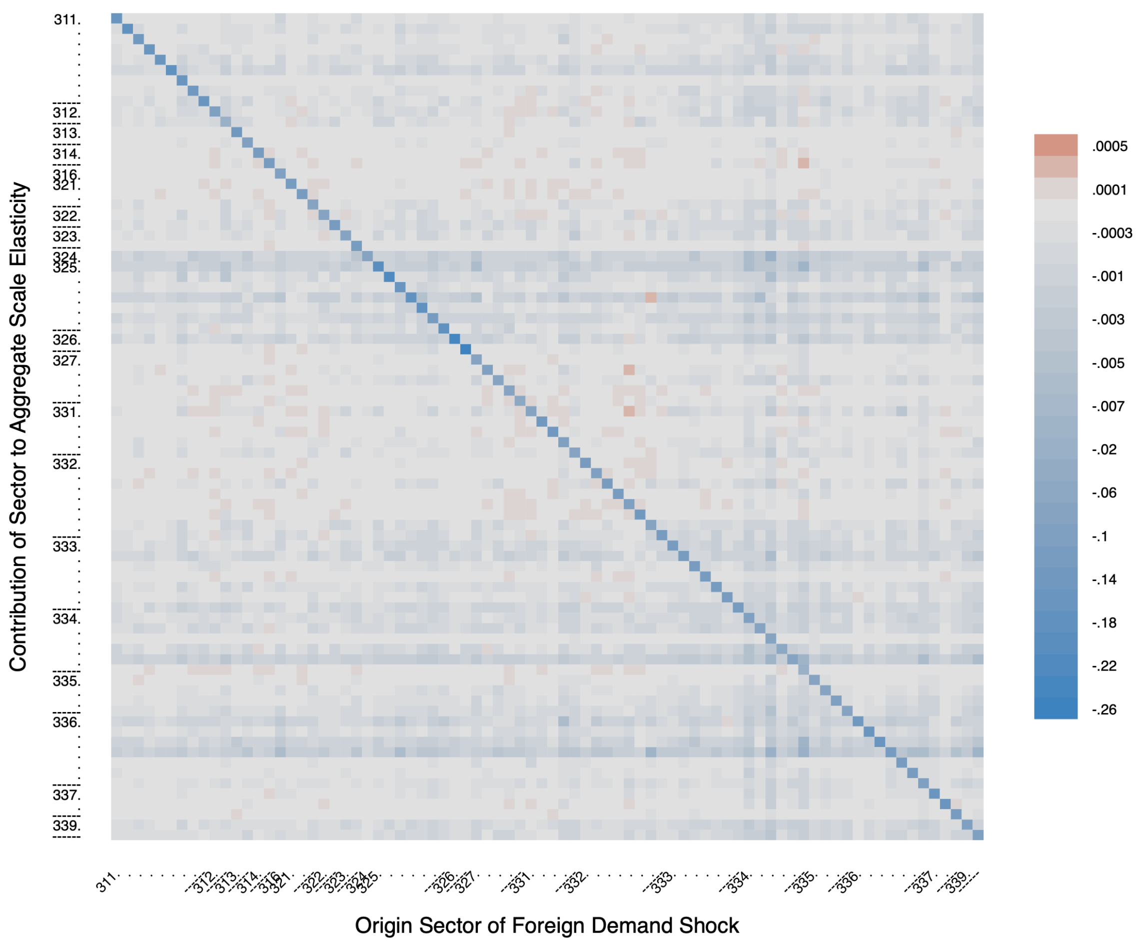
While the main diagonals are strongly negative and reflect the contribution of within-sector economies of scale and scope, a substantial amount of economies of scope manifest even *across* four-digit manufacturing sectors. Cross-sector elasticities, indicated by the off-diagonal values, are heterogeneous and asymmetric. Sectors such as computers and electronics are strong contributors to aggregate increasing returns via economies of scope, while other sectors such as aerospace products are strong beneficiaries. For example, a demand shock to computers and peripherals (NAICS 3341) that raises manufacturing output by 10 percent triggers price declines in other sectors that lower the PPI by a total of 1 percent. This is indicated by the strong negative *column* for NAICS 3341 in the matrix. On the other hand, prices changes in just the aerospace products (NAICS 3364) sector lower the PPI by 2.4 percent for every 10 percent increase in manufacturing output caused by demand shocks in other sectors. This is indicated by the strong negative *row* for NAICS 3364 in the matrix.

Figure 5: Decomposition of Aggregate Scale Elasticity under Joint Production

(a) Economies of Scale versus Scope



(b) Same- and Cross-Sector Elasticities of Price to Output



Notes: Panel (a) decomposes the aggregate scale elasticity (the total elasticity of the manufacturing PPI with respect to manufacturing output in the US) caused by a proportional increase in foreign demand in all industries into within-industry versus net cross-industry impacts. Panel (b) illustrates shock propagation at the bilateral sector (NAICS 4-digit) level. Each cell highlights the contribution to the aggregate scale elasticity by price changes in industries in the row sector given proportional foreign demand shocks to all industries in the column sector.

Other sectors that are less knowledge-proximate have cross-elasticities that are much smaller in magnitude. In fact, cross-elasticities are mildly positive for 264 sector pairs (four percent of all pairs), indicating *diseconomies* of scope, which I shade in red. For example, glass manufactures (NAICS 3272) and metal hardware (NAICS 3325)—two industries with the strongest diseconomies of scope—share more residual inputs such as capital and administrative services than they do knowledge. Prices of glass manufactures are predicted to *rise* in response to a demand shock for metal hardware as firms reallocate scarce and rivalrous residual shared inputs m ($\rho_m < \theta_m$) away from glass products and towards metal hardware.

Figure 6 offers more evidence that joint production among more knowledge-intensive industries entails stronger economies of scope. I decompose the equilibrium scale elasticity induced by foreign demand shocks into within- and cross-industry components for one shocked industry k at a time. Panel (a) of Figure 6 plots the contribution of cross-elasticities (economies of scope) towards the aggregate scale elasticity (y-axis) against the shocked industry's expenditure share on knowledge inputs (x-axis). I find that economies of scope increase with in knowledge intensity. Panel (b) plots the same estimates of economies of scope on the y-axis against economies of scale on the x-axis. Interestingly, industries with stronger economies of scope (computers and electrical equipment) tend to have lower economies of scale, and vice versa for industries like chemicals and plastics. Focusing only on within-industry returns to scale would thus overweigh the contribution toward aggregate increasing returns of the chemicals and plastics industries relative to the computer electronics or medical equipment (part of Misc. sector) industries.

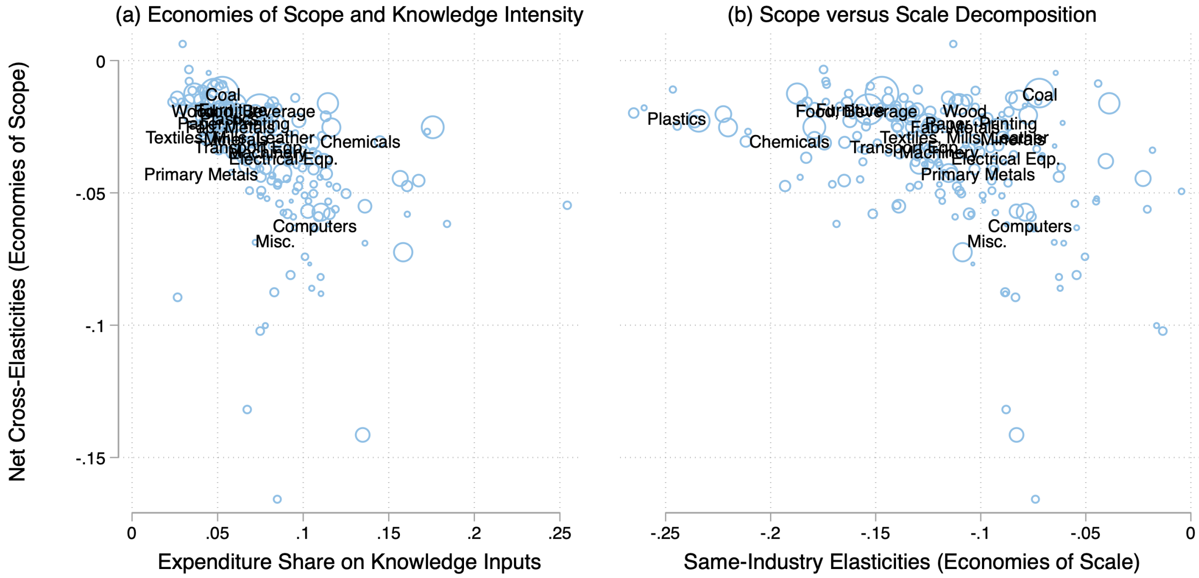
4.3 Joint Production in the US-China Trade War

Finally, I demonstrate that the endogenous responses of producer prices to market size in my model are quantitatively important in light of a real shock to US manufacturing. I analyze the impact of bilateral import tariffs applied since the ongoing US-China "trade war".

I compare the predictions of my model against an alternative supply-side assumption where firms operate linear, nonjoint, constant-returns production functions. Under joint production, US domestic producer prices respond to market size changes due to economies of scale and scope. US tariffs on imports from China protect US firms from competition and expand their market access, while Chinese tariffs on imports from the US restrict US firms and reduce their market access. In comparison, under linear production, domestic producer prices do not change absent other general equilibrium forces.

I find that the difference in the producer price response between these two models are large in light of actual tariff changes applied during the US-China trade war. I average HS product-level tariffs from Fajgelbaum et al. (2019) to the level of the 206 industries used in my paper (using, respectively, US HS10 imports and HS8 exports to China as weights). I re-calibrate the model's macro parameters to exactly match aggregates in 2017, before the onset of the trade war. I then

Figure 6: Estimates of Economies of Scope, Scale, and Knowledge Intensity by Industry



Notes: This graph plots on the y-axis, for a demand shock in each given industry, the net contribution of economies of scope to the aggregate scale elasticity. The x-axis in panel (a) plots each shocked industry's knowledge input expenditures as a share of output. The x-axis in panel (b) plots, for a demand shock in each given industry, the contribution of within-industry economies of scale to the aggregate scale elasticity. The size of each blue circle is proportional to the industry's gross output, and a list of top and bottom industries on the y-axis can be found in Appendix Table D.9. Overlaid in black text are additional scatterplots of the same y and x statistics but aggregated at the level of broad sectors.

solve for counterfactual changes to equilibrium prices (both US consumption price indices and producer price indices by industry) in response to these tariffs, holding all else equal. I develop and apply an "exact hat" system of equations (see Appendix D.6) to solve for changes in industry price indices in response to any set of exogenous shocks under the equilibrium in Definition 1.

Table 6 compares the impact of US-China tariffs on various economy-wide aggregates. I first compute the impact of unilateral US tariffs on imports from China, before computing the full impact after retaliatory tariffs by China on imports from the US. In the first two columns, under linear production, changes in market size have no effect on producer prices (recall that wages are pinned down by a non-manufacturing sector and there is no entry and exit of firms). In column (1), the only effect of US tariffs is to raise consumer prices by 0.76 percent, while expanding manufacturing output by 1.5 percent. In column (2), retaliatory tariffs have no additional impact on the US CPI but of course reduces US manufacturing output (by 0.6 percent), so that on net output increases by only 0.9 percent.

In the last two columns of Table 6, I find that the price impacts of tariffs are substantially different under joint production. Given the sizable estimates of economies of scale and scope, US producer prices fall by 0.4 percent as unilateral import protection expands US firms' market size.

Table 6: Impact of the US-China Trade War on US Manufacturing

	Linear (CRS) Production		Joint Production	
Import tariffs by:	US only	US+China	US only	US+China
Change (%) in U.S. Manuf. Sector Outcome				
CPI	0.76	0.76	0.50	0.61
PPI	0	0	-0.37	-0.17
Imports from China	-37.23	-37.23	-38.23	-38.09
Imports from RoW	6.29	6.29	4.91	5.52
Exports to China	0	-36.76	1.59	-36.82
Exports to RoW	0	0	2.01	0.63
Output	1.53	0.92	2.46	1.34
Manufacturing Sector Trade Deficit	-12.32	-7.39	-19.72	-10.75
US Tariff Revenues as share of initial manuf. output	0.57	0.57	0.55	0.55

Notes: This table presents estimates of the impact of US-China bilateral import tariffs on the US manufacturing sector under two different models calibrated to match the same US industry-level aggregates in 2017. The first two columns display results under linear production, where firms operate under constant returns to scale and no economies of scope. The last two columns display results under joint production. I first compute the impact of unilateral US tariffs on imports from China and then the full impact after Chinese tariffs on imports from the US. Data on tariffs at the HS-level are taken from from Fajgelbaum et al. (2019). See Definition 1 for a characterization of the equilibrium and Appendix D.6 for the exact hat system of equations used to solve for model responses after the shock.

Each industry-level import tariff causes prices of US goods to fall not only in that same industry but also in other, knowledge-proximate industries. Altogether these producer price declines offset about one third of the CPI impact under linear production, so that the consumption price index rises by only 0.5 percent. Other margins of the US economy also improve compared to the case of linear production. As US producer prices fall, manufacturing output and exports rise, the deficit shrinks, and import substitution towards the rest of the world is less pronounced.

However, while joint production mitigates the harms of domestic import protection, it also amplifies the harms of foreign import protection. In the last column, retaliatory tariffs by China restrict the foreign market access of US firms and push up producer prices, offsetting a majority of the aggregate producer price decline. US producer prices decrease by only 0.17 percent compared to 0.37 percent under unilateral tariffs, leading to an overall CPI increase of 0.61 percent.

Despite this offsetting effect in the aggregate, retaliation widens the distribution of producer outcomes across industries. This occurs because the industries facing import protection after US tariffs are different from those facing restricted market access after Chinese tariffs. Producer prices rise by more than one percent in optical instruments, pulp, computers, broadcast and wireless communications equipment and small electrical appliances. Producer prices fall by more than three percent in lighting fixtures, furniture, textiles, and printing ink. Figure D.2 visualizes this distribution across industries and across broad manufacturing sectors. Tariffs can therefore alter a nation's comparative advantage—not only in directly affected industries, but also in other industries linked under joint production.

While the general equilibrium assumptions used in these counterfactuals are stark, it is straightforward to extend the model to feature endogenous factor price changes or input-output linkages (see the Appendix D.5 and Table D.8 for one such extension). These additional details introduce further interactions but all preserve the intuition conveyed by the quantitative results thus far. Economies of scope generate large, negative cross-elasticities of price with respect to output among knowledge-proximate industries. In the aggregate, joint production represents a novel and economically sizable channel through which shocks propagate across industries.

Conclusion

Much of the literature in trade and macroeconomics assumes that firms operate independently across industries. I provide evidence that this assumption is inconsistent with the behavior of US manufacturing firms. A demand shock in one industry of a firm increases its sales in another industry the more that the two industries share knowledge inputs. These findings suggest that in future empirical work, the impact of industry-level shocks should be considered jointly rather than in isolation.

To explain and quantify such interdependence, I develop a model of joint production where inputs can be shared across a firm's industries. Two elasticities associated with shared inputs—scalability and rivalry—determine the direction and magnitude of cost interdependence in production. Whereas solving for a firm's profit-maximizing decisions under interdependence is typically a hard computational problem, I provide a stochastic micro-foundation for input-sharing that convexifies the problem of the firm and yields analytical expressions for the firm's extensive and intensive margin output in each industry. I estimate that knowledge inputs stand out in relation to other potentially shared inputs in terms of their scalability and non-rivalry. Outputs among knowledge-proximate industries are complements.

Joint production within the firm generates a new and quantitatively important dimension of cross-industry linkages in the aggregate. Firms derive economies of scope from the scalability and rivalry of shared knowledge inputs. On average a demand shock that raises output by 10 percent would lower prices in other industries by 0.4 percent. This accounts for almost one quarter of conventional values of aggregate increasing returns in US manufacturing. Endogenous price responses under joint production suggest that trade policy and market size are determinants of comparative advantage across countries. Moreover, the concentration of economies of scope among knowledge-proximate industries highlight sensitive industry clusters that could particularly benefit from unilateral import protection as well as be harmed by retaliatory tariffs. These results provide grounds for further research on optimal trade and industrial policy.

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APPENDICES (For Online Publication)

A Data Appendix

A.1 Data Construction and Details

Firms, Plants, and Products. I assemble data from the Economic Censuses (EC), the Longitudinal Business Database (LBD), and the Longitudinal Firm Trade Transactions Database (LFTTD) from 1997 to 2012. The Censuses are conducted quinquennially in years ending with '2' and '7'. Data on product shipments made by establishments come from the product trailer (PT) files which are attached to the Census of Manufactures (CMF). These trailer files contain responses of establishments that are sent a CMF 'Long Form'. The long form is sent to all establishments belonging to multi-establishment firms as well as a sample of single-establishment firms. The long form elicits shipments made by the establishment at a disaggregated level (varying from 6 to 10 digit NAICS).[36]

Using firm identifiers in the LBD, I match establishments to their parent firms and aggregate industry-level shipments to the level of the firm. The firm identifier in the LBD comes from information the Census collects from the Company Organization Survey and from tax identifier and plant identifier information in the Business Register. An establishment is a physical location where business activity occurs. The firm is defined (by the Census) as the highest level entity that controls more than 50% of each of the establishments assigned to the firm. I drop establishments that are administrative records (for which sales data is imputed).

External Sales. The CMF contains data on the shipments of a plant made to other plants within the same firm. However, this data is not broken down at the product-line level. For plants that produce in multiple industries, I apportion this inter-plant shipment data into industry-level intra-firm shipments using shares taken from the plant's total sales across industries. I then define the external sales of a firm in each industry as its total sales in that industry minus its intra-firm shipments. I drop external sales computed in this way in any industries of the firm that (i) account for less than 0.5% of firm-wide external shipments and (ii) are never the main produced industry of any plant the firm owns. This is conservative and allows product shipments in very small industries of the firm to be entirely intra-firm. This also prevents the spurious adding / dropping of products simply because of changes to the PT forms over the years.

Firm Trade Data. I use two sources. First, the LFTTD contains the value of all import and export transactions, by trading country and by HS10 product, that each firm entity (a set of EIN tax codes) is a counter-party to. Second, the CMF also contains data on plant-level shipments that are ultimately destined for export markets (whether directly or indirectly through an intermediary). If the plant is a multi-industry plant, I apportion this plant-level shipment across the plant's industries using product trailer product shipment shares. I use both LFTTD and CMF data on exports to construct the export demand shock, detailed below. Data on firm exports and imports reported in Table 1 come from the LFTTD.

Country-level Trade Data. I use data from BACI and Comtrade (bilateral country-level trade flows at the HS6 level) to generate the five-year growth rates in imports of a destination n in product h used in the analysis, $\Delta \log IMP_{nht}$.

Knowledge Inputs. I use BEA data from 1997 to collect input expenditure data by industry. Table

[36] This procedure is likely to underestimate the significance of multi-product activity in the US economy for two reasons. First, the long-form elicits questions about product sales over a pre-specified list of products (specific to the plant's classified industry). Although there is space for the firm to report shipments in products not covered by that pre-specified list, in practice firms rarely do. Second, the long-forms do not cover all single-establishment firms in the economy. A single-establishment firm could be selling in multiple industries but would not report the breakdown of its sales over these industries unless it was sent a long-form.

Table A.1: Definition of Knowledge Inputs and their Use in Manufacturing in 1997

	Input Expenditures as Share (%) of Gross Output by Manufacturing Industries			
Code	Input industry description	Mean	25th pctile	75th pctile
550000	Management of companies and enterprises	3.54	2.60	4.94
541700	Scientific research and development services	0.62	0.25	0.96
541300	Architectural, engineering, and related services†	0.62	0.31	0.96
5419A0	All other professional, scientific, and technical services	0.61	0.61	0.63
541511	Custom computer programming services†	0.58	0.21	0.86
541800	Advertising, public relations, and related services	0.48	0.14	0.62
541610	Management consulting services	0.28	0.28	0.30
541100	Legal services	0.28	0.09	0.30
541200	Accounting, tax prep., bookkeeping, & payroll services	0.15	0.08	0.21
541400	Specialized design services	0.09	0.01	0.02
541512	Computer systems design services†	0.07	0.02	0.06
54151A	Other computer related services	0.04	0.02	0.04
5416A0	Environmental and other technical consulting services	0.04	0.01	0.02
541940	Veterinary services	0.00	0.00	0.00
541920	Photographic services	0.00	0.00	0.00
533000	Lessors of nonfinancial intangible assets	0.69	0.06	0.34
5111A0	Wired telecommunications carriers	0.34	0.17	0.37
511200	Software publishers†	0.33	0.05	0.19
518200	Data processing, hosting, and related services	0.20	0.17	0.26
512100	Motion picture and video industries	0.03	0.00	0.03
512200	Sound recording industries	0.00	0.00	0.00
KLG	All Knowledge Inputs	9.01	6.38	11.48

Notes: Mean refers to the weighted average across all 206 BEAX manufacturing industries, with industry gross output as weights. 25th and 75th pctile refers to expenditure shares of the corresponding percentiles (unweighted) across the 206 manufacturing industries. Codes in the first column refer to BEAX codes that are hand-developed; they roughly correspond to codes available in BEA I/O tables but are aggregated to ensure consistency over time.
Source: BEA Input-Output & Capital Flow Tables, 1997.
† Indicates industries where data on capitalized investments from the capital flow tables are used to compute expenditures. Capitalized investments make up only 0.64% of gross manufacturing output.

A.1 lists industries from BEA input-output and capital flow tables that I classify as supplying knowledge inputs. These fall under NAICS sectors 55, 54, 533, and 51. Although results are robust to including finance, insurance, real estate, and other rental leasing (NAICS 52, 531, and 532) under the definition of knowledge, I separate out these financial inputs from the definition of knowledge because of the potentially different way financial inputs affect businesses.

In the three columns of Table A.1, I compute aggregate expenditures by manufacturing firms on these input industries. The input-output tables record only expenses on inputs whose accounting value fully depreciates within one year. Given the arbitrary depreciation rates of many intangible assets and idiosyncratic rules around which inputs are expensed versus capitalized, I incorporate data from the capital flow tables on capitalized investments made by firms in manufacturing industries on knowledge input industries (for example, a shoemaker investing in software capital). I count both capitalized investments and expensed investments as knowledge input expenditures. In practice, capitalized expenditures on intangibles in 1997 are so small (0.64% of output) that it makes no difference to the results in the paper if I exclude data from the capital flow tables. Most knowledge input expenditures circa 1997 (like R&D) were still expensed under national accounting rules. I do not use data on knowledge input expenditures after 1997 because of subsequent changes to accounting rules that generate a lot of time variation in the data series, and because the capital flow tables are no longer published.

The largest category of knowledge input expenditures is NAICS 55, 'Management of companies and enterprises', at 3.54% of gross output. To my understanding, this reflects the BEA's best estimates of the value of professional services (the categories under NAICS 54) produced internally by the firm's headquarters establishments for use by the firm's other manufacturing plants. By comparison, expenses over the remaining delineated professional services industries (NAICS 54) are outsourced.

Other Inputs. Similar to the above, I use similar BEA input-output and capital flow table data to construct expenditures (as a share of gross output) by each manufacturing industry j on other types of inputs m (roughly a NAICS 5-digit industry). These shares inform the value of β_{jm} in Section 1 in the paper.

Industry Definition. I define industries j and inputs m at the level of BEAX (roughly a 5-digit NAICS industry). I hand-construct BEAX as a unified industry nomenclature that is time-invariant over the period 1997 and 2012 and the most disaggregated industry classification that is concordable on a 1:m basis with HS, NAICS, and BEA industry codes in each year. There are 206 BEAX industries in manufacturing. I use the HS-NAICS concordance in US Census Bureau data provided by Schott (2008) and Pierce and Schott (2012) to convert import and export HS codes (at the 10-digit and 6-digit levels) in each year to NAICS. I use the concordances provided by US Census Bureau and BEA to go between NAICS codes and BEA codes in each year. I use an iterative algorithm to aggregate over m:m splits over years and in each cross section so that in any given year, each NAICS and HS code is entirely contained within a BEAX code.

A.2 Export Demand Shocks and Export Intensity

I leverage both the LFTTD and CMF sources of data on firm-industry exports to construct demand shocks, $\Delta \log S_{fjt}$. First, among LFTTD data, I compute export shares of each industry of each firm across destinations n and HS6 products h. I exclude destination-product markets whenever the firm's exports in those markets exceed 10% of the market's imports from the rest of the world. I use these shares as $s_{fjnh,t-1}$ in equation (2). Next, I use data from the CMF on export shipments to compute export intensity, $s^*_{fj,t-1}$. If a firm reports no exports in an industry from among its manufacturing plants that produce in that industry, it is likely that its exports in the customs data is an instance of carry-along trade, made by the firm's wholesale / retail arm. In this case customs-data-derived demand shocks would be uninformative: they are as likely to affect this firm as they are to affect any other exporter in the industry. Export intensity from the CMF thus helps to discipline the export demand shocks derived from the LFTTD. I also set export intensity to zero for instances where carry-along trade of the firm (customs exports less census exports) in an industry exceeds its total external shipments in the CMF. After purging these edge cases, I am left with two measures of export intensity: (i) census exports divided by census sales in an industry, and (ii) customs exports divided by census sales in an industry. I take the average of these two measures as my measure of $s^*_{fj,t-1}$.

A.3 Regression Analysis

A.3.1 Summary Statistics

Table A.2 displays summary statistics on common variables that appear in regression Table 2. The regression sample consists of all continuing firm-industries (across 5-year periods) of firms that have at least one industry with a non-zero export demand shock. For example, suppose a firm f produces in industries A

Table A.2: Summary Statistics on Key Regression Sample

Statistics by firm-industry:	Variable	Mean	Std. Dev.
Change in sales	$\Delta \log X_{fjt}$	0.15	0.99
Has export demand shock?	-	0.68	0.47
Export intensity	$s^*_{fj,t-1}$	0.06	0.10
Same-industry demand shock	$\Delta \log S_{fjt}$	0.028	0.082
Other-industry demand shocks			
(i) Average effect	$\Delta \log S^{OTHER}_{fjt}$	0.025	0.063
(ii) × knowledge input-proximity	$\Delta \log S^{OTHER \times KLG}_{fjt}$	0.002	0.007
Initial Period Sales (millions)	$X_{fj,t-1}$	165	1225
Initial Period Employment	-	522	2245
Other Statistics			Value
Number of manuf. firms from 1997-2002			5000
Number of manuf. firms from 2002-2007			4700
Share of U.S. manuf. sales accounted for by sample			0.51
Share of U.S. manuf. employment accounted for by sample			0.37

Notes: This table reports statistics on the sample of multi-industry firms and their continuing industries from the regressions in Table 2. Number of firms are rounded for disclosure avoidance. Included as observations are any firm-industries with continuing sales over a 5-year period, and belonging to a firm with at least one exporting industry in the initial period.

and B in 1997 but only produces in A in 2002. As long as the firm received a demand shock in either industry A or B in 1997, I include the firm in the sample (where it takes up a single observation). However, if the firm had switched to producing industries C and D in 2002, there is no intensive margin overlap and this firm would not be included in my sample.

A.3.2 Export Demand Shock Relevance

I verify that demand shocks are indeed able to shift firm sales in the same industry by running the following regression for the sub-sample of firm-industries that have non-zero same-industry export demand shocks:

$$\Delta \log X_{fjt} = \alpha \Delta \log S_{fjt} + Controls_{jt}(s^*_{fj,t-1}) + FE_{jt} + \epsilon_{fjt},$$

where $Controls_{jt}(s^*_{fj,t-1})$ refers to various ways of controlling for the export intensity scaling variable to ensure that the estimated impact of the export demand shock is not driven by firms with different export intensities being on different growth trends. Results are presented in Table A.3. Across all three columns (that vary in terms of the control for export intensity used), the coefficient on the shock variable is positive and ranges from 0.32 to 0.59. The impact of the demand shock is higher without controlling for export intensity (column 1), consistent with selection on export intensity.

In undisclosed results I also run a placebo test where for each firm-industry fj, I compute $\Delta \log S^{placebo}_{fjt} = s^*_{fj,t-1} \frac{\Delta \log S_{ijt}}{s^*_{ij,t-1}}$ where, for each fj, i references a randomly selected firm from the set of firms with non-zero demand shocks in j. The placebo tests return false positives in column (1) but not columns (2) and (3). This suggests that linear controls for export intensity control adequately for selection on export intensity.

Table A.3: Relevance of Export Demand Shocks for Predicting Change in Sales

Change in sales, $\Delta \log X_{fjt}$	(1)	(2)	(3)	(4)
Same-industry demand shock	0.59	0.36	0.34	0.32
$\Delta \log S_{fjt}$	(0.10)	(0.12)	(0.11)	(0.11)
Industry-year-FE	✓	✓	✓	✓
$s^*_{fj,t-1} \times$ year-FE		✓	✓	
$s^*_{fj,t-1} \times$ Industry-year-FE				✓
Control for pre-period sales, $\log X_{fj,t-1}$			✓	✓
Observations	14,500	14,500	14,500	14,500
R^2	0.08	0.08	0.12	0.15

Notes: This table displays responses of firm-industry sales to same-industry demand shocks, in 5-year differences over the period 1997-2007. Standard errors in parentheses are clustered at the firm level. Number of observations are rounded for disclosure avoidance. The control $s^*_{fj,t-1}$ is the firm's export intensity (exports over sales) in industry j in the initial census year.

A.3.3 Heterogeneity of Cross-Industry Impacts by Input-Proximity

Table A.4 displays the regression table counterpart to coefficients shown in Figure 1. Each row of the regression refers to a specification that where demand shocks in other industries of the firm are interacted with proximity to j with respect to a specific category of inputs. The numbers next to the description in parentheses display the NAICS subroot (1, 2 or 3 digits) of the input category. Taxes, government sector inputs, and the two types of value-added (labor and gross operating surplus) are specific BEA categories that have no corresponding numeric NAICS code. The first three rows of the table break out the knowledge category interaction (column (3) of Table 2 and the first row of Figure 1) into finer constituent subcategories $\mathcal{M}$: the leasing of intangibles (NAICS 533), headquarter services (NAICS 55), and professional services and information (NAICS 51, 54) and show that cross-industry impacts increase with proximity with respect to each constituent subcategory.

Table A.4: Cross-industry Impact of Demand Shocks: Heterogeneity by Input-Proximity

Change in sales, $\Delta \log X_{fjt}$	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)
Same-industry demand shock $\Delta \log S_{fjt}$	0.48***	0.45***	0.46***	0.46***	0.46***	0.45***	0.45***	0.45***	0.45***	0.46***	0.45***	0.45***	0.45***	0.45***
Other-industry demand shocks														
(i) Average effect $\Delta \log S_{fjt}^{OTHER}$	-0.26**	-0.68**	-0.49**	-0.82*	-0.18	-0.45*	-0.12	-0.15	-0.29	-0.00	-0.07	0.25	-0.05	0.19
(ii) $\times$ proximity by use of inputs in $\mathcal{M}^{BLK}$: $\Delta \log S_{fjt}^{OTHER \times BLK}$														
$\times$ Sub-categories of knowledge inputs:														
Leasing of Intangibles (533)	27.73***													
Headquarter Services (55)		15.82**												
Prof. Services & Information (54, 51)			8.262*											
$\times$ Other input categories:														
Finance, Insurance, & Real Estate (52, 531)				40.26										
Leasing of Tangibles (532)					24.92									
Transportation, Wholesale, & Retail (4)						14.94*								
Taxes and Government							3.603							
Utilities and Construction (2)								3.413						
Gross Operating Surplus									1.766					
Labor Value-Added										-0.325				
Agriculture (1)											-0.715			
Manufacturing (3)												-0.889		
Administrative Services (56)													-4.305	
All Other Services (6, 7, 8, 9)														-20.8
Industry-year-FE	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
Observations	21,500	21,500	21,500	21,500	21,500	21,500	21,500	21,500	21,500	21,500	21,500	21,500	21,500	21,500
R^2	0.06	0.06	0.06	0.06	0.06	0.06	0.06	0.06	0.06	0.06	0.06	0.06	0.06	0.06

Notes: This table reproduces the specification in column (3) of Table 2 but with other-industry demand shocks interacted with proximity in the use of various categories of inputs (given by the rows of the table). Standard errors are clustered at the firm level (omitted for brevity), with asterisks indicating p-values below 0.1 (*), 0.05 (**), and 0.01 (***) respectively. Number of observations are rounded for disclosure avoidance.

Table A.5: Same and Cross-Industry Impacts of Demand Shocks: Additional Specifications

Change in Sales, $\Delta \log X_{fjt}$	(1)	(2)	(3)	(4)
Same-industry demand shock			0.47	0.47
$\Delta \log S_{fjt}$			(0.09)	(0.09)
Other-industry demand shocks				
(i) Average effect	-0.74			-0.48
$\Delta \log S_{fjt}^{OTHER}$	(0.24)			(0.39)
(ii) $\times$ knowledge input-proximity	7.51	6.54	7.02	8.14
$\Delta \log S_{fjt}^{OTHER \times KLG}$	(2.22)	(2.08)	(2.11)	(2.22)
(iii) $\times$ remaining input-proximity		-0.75	-0.86	-0.44
$\Delta \log S_{fjt}^{OTHER \times REM}$		(0.26)	(0.26)	(0.44)
Industry-year-FE	✓	✓	✓	✓
Observations	21,500	21,500	21,500	21,500
R^2	0.12	0.05	0.06	0.06

Notes: This table displays additional specifications using the same sample of firms as regression Table 2. Standard errors in parentheses are clustered at the firm level. Number of observations are rounded for disclosure avoidance.

Table A.5 shows that cross-industry elasticities of sales with respect to demand shocks are robust to alternative specifications of the main regression equation (1). Column (1) finds that the impact of other-industry demand shocks are robust to dropping controls for same-industry demand shocks, providing reassurance that same-industry demand shocks have independent variation with respect to other-industry demand shocks in the data. In columns (2)-(4), instead of using average other-industry demand shocks $\Delta \log S_{fjt}^{OTHER}$, I focus only on the input-sharing mechanism and separate out knowledge inputs from all remaining inputs. I denote the remaining set of inputs by $\mathcal{M}_f^{REM}$ and construct $\Delta \log S_{fjt}^{OTHER \times REM}$ using the same equation (5) as $\Delta \log S_{fjt}^{OTHER \times KLG}$. Column (2) shows that they pull in opposite directions within the firm. Cross-industry impacts increase with the sharing of knowledge inputs, and decrease with the sharing of remaining inputs. Column (3) adds the same-industry shock back to the regression, and column (4) includes both $\Delta \log S_{fjt}^{OTHER \times REM}$ and $\Delta \log S_{fjt}^{OTHER}$. Across specifications (2)-(4), the effect of other-industry demand shocks interacted with knowledge input proximity is always positive and statistically significant.

A.3.4 Impact of Demand Shocks at the Firm-level

Weights η_{fkt} used in the firm-level regression equation (6) are given by:

$$\eta_{fkt} = \frac{\beta_{k,y} X_{fkt}}{\sum_{k'} \beta_{k',y} X_{fk't}},$$

where X_{fkt} is firm sales in industry k and $\beta_{k,y}$ is defined depending on the outcome of interest y:

(i) Purchased professional services: $\beta_{k,y} = \beta_{k,PROF}$, professional services expenses as a share of gross output in industry k,

(ii) Sales: $\beta_{k,y} = 1$ (so η are simply sales shares),

(iii) Capex: $\beta_{k,y} = \beta_{k,CAP}$, gross operating surplus as a share of gross output in industry k,

(iv) Payroll: $\beta_{k,y} = \beta_{k,LAB}$, labor value added as a share of gross output in industry k.

Data on purchased professional services at the firm level come from aggregating responses of plants of the firm to the following questions in the Annual Survey of Manufactures (ASM): expenses on legal, accounting, management, communication, advertising, and computer software and data processing services. Firms that do not have plants respond to these questions in the ASM and firms that more than 5% of sales outside of the manufacturing sector are dropped for this particular regression. Data on firm-wide capex come from summing up plant-level capital expenditures, and data on payroll come from summing up plant-level production worker payroll. Both variables come from the CMF.

A.3.5 Threats to Identification

Related to the discussion on threats to identification in Section 1.4, I directly test and reject the hypothesis that import growth patterns across industries within a destination are positively correlated among knowledge-intensive manufacturing industries. I aggregate imports of each destination to the industry level, $IMP^{\sim US}_{nk,t-1}$ for $k \in \mathcal{J}$, and interact industry-level import growth with knowledge proximity corresponding to the intra-firm equation (5) used in the main firm-industry regressions:

$$\Delta \log IMP^{\sim US,OTHER\times KLG}_{njt} \equiv \sum_{k\neq j}\sum_{m\in\mathcal{M}^{KLG}} \beta_{jm}\left(\frac{\beta_{km} IMP^{\sim US}_{nk,t-1}}{\sum_{k\neq j}\beta_{km} IMP^{\sim US}_{nk,t-1}}\right)\Delta \log IMP^{\sim US}_{nkt}.$$

For each given industry in a destination, I compute the change in import demand in other industries of that destination:

$$\Delta \log IMP^{\sim US,OTHER}_{njt} \equiv \sum_{k\neq j}\left(\frac{X_k}{\sum_{k'\neq j} X_{k'}}\right)\Delta \log IMP^{\sim US}_{nkt},$$

I then run the following regression, at the level of destination-industries, over the same time period (in 5-year differences):

$$\Delta \log IMP^{\sim US}_{njt} = \psi \Delta \log IMP^{\sim US,OTHER}_{njt} + \psi^{KLG}\Delta \log IMP^{\sim US,OTHER\times KLG}_{njt} + FE_{jt} + FE_{nt} + \epsilon_{njt}.$$

I do not find that ψ^{KLG} is positive, either with or without destination-year fixed effects.

A.3.6 Vertical Explanations

There are four general reasons a demand shock in industry k may increase sales in industry j within the firm: (i) j supplies k, (ii) k supplies j, (iii) k, j use similar inputs, and (iv) k, j are demand-complementary and have similar buyers. My focus is on mechanism (iii). The discussion in Section 1.4 rules out (iv), demand-complementarity. I also rule out the first two, vertical mechanisms:

(i) This is unlikely to explain the main regression results, which show *external* sales of the firm in industry j changing. However, it could still be the case that external sales growth is driven by productivity effects (i.e. increasing returns to scale) induced by increased intra-firm shipments of j to supply k. I test this hypothesis by using growth in industry j inter-plant (intra-firm) shipments as an outcome variable in the same specification as column (3) of Table 2. I find, however, that inter-plant shipments do not respond to demand shocks in k (even when limiting the regression sample to the tiny fraction of fj observations that have positive inter-plant shipments).

(ii) For this to occur there must first be an increase in internal shipments in the shocked industry k. Then the story would be that increased quality of shipments (as measured by increased internal sales) drives productivity growth in industry j. I use growth in inter-plant (intra-firm) shipments as an outcome variable across the "same-industry" specifications in Table A.3. I find that they do not respond (even when limiting the regression sample to observations of firms with positive inter-plant shipments).

A.3.7 Deflating

The main regressions specifications all include industry-year fixed effects, which absorb changes in industry price deflators over time. Nevertheless, deflating could still make a difference in terms of the relative sizes of export shares and expenditure shares used in weights in the pre-period. I find that all the reduced-form results are virtually unchanged when the following variables are deflated with industry-level price deflators from the NBER-CES manufacturing database: demand shocks (import growth at destinations), outcomes (external shipments of a firm-industry), as well as 'initial-period' variables, for example the proximity weights behind $\Delta \log S_{fjt}^{OTHER \times KLG}$.

B Theory Appendix

B.1 A micro-foundation for input non-rivalry

I provide more intuition for equation (14), which specifies that the expected profit contribution of a given idea is a θ_m-norm of the firm's vector of industry profitability shifters. The lower is θ_m, the more variable are ideas generated by that input, and the more the firm benefits in expectation from a given idea. This occurs because the firm selects the best host for an idea from among all potential industries, and the maximum out of a set of $i.i.d.$ draws is increasing with the variance of each draw.

Combine the assumption of additive separability (equation 10) with the expression for firm-industry profits in equation (11) to derive the expected change in firm gross profits from an additional idea i:

$$\frac{\Delta_{fm}}{Z} \equiv \mathbb{E}\left[\max_j \tilde{\alpha}_{mj} B_j \xi_{fj}\, \phi_{fmi,j}\right],$$

where $\phi_{fmi,j}$ is an independent random draw from a Fréchet distribution. The firm chooses the industry j in which the idea i (conditional on the match-specific values of $\phi_{fmi,j}$ in different industries) generates the highest increase in profits, $\tilde{\alpha}_{mj} B_j \xi_{fj}\, \phi_{fmi,j}$. The remainder of this proof simply relies on properties of the Fréchet distribution popularized by Eaton and Kortum (2002). The profit contribution can be re-expressed as:

$$\frac{\Delta_{fm}}{Z} = \mathbb{E}\left[\max_j \tilde{\phi}_{fmi,j}\right],$$

where $\tilde{\phi}_{fmi,j}$ is an independent random draw from a modified Fréchet distribution that absorbs the multiplicative shifters:

$$Pr(\tilde{\phi}_{fmi,j} \leq x) = e^{-\left(\tilde{\alpha}_{mj} B_j \xi_{fj}\right)^{\theta_m} x^{-\theta_m}}, \qquad \forall j \in \mathcal{J},$$

and it follows that

$$\Delta_{fm} = \left(\sum_j \left(Z \tilde{\alpha}_{mj} B_j \xi_{fj}\right)^{\theta_m}\right)^{\frac{1}{\theta_m}} \Gamma(1 - 1/\theta_m) = \left(\sum_j \delta_{fmj}^{\theta_m}\right)^{\frac{1}{\theta_m}}.$$

where Γ is the gamma function and $\delta_{fmj} \equiv \xi_{fj} \alpha_{mj} B_j Z$.

B.2 Proof of Lemma 1: The Firm's Solution

In stage II, the firm decides its expenditures on industry-specific inputs given its accumulated knowledge, $\{\varphi_{fj}\}_{j\in\mathcal{J}}$. This problem is separable by industry. Under monopolistic competition, the solution for the firm's gross profits (sales less production input expenses) and sales is given by equation (11).

At the beginning of stage I, the firm decides its expenditures on shared inputs. Throughout stage I, the firm receives a stream of ideas indexed by $i = 1, ..., A_{fm}$ for each type of shared input m and adapts each idea i to a given industry j (denoted $\mathbf{1}_{fmi,j} = 1$). Given the additive separability assumption in equation (10), expected firm net profits Π_f can be written as:

$$\mathbb{E}[\Pi_f] = \max_{\{\iota_{fm}\}_{m\in\mathcal{M}}} \mathbb{E}\left[\sum_j \sum_m B_j \xi_{fj} \tilde{\alpha}_{mj} \sum_i^{A_{fm}} \phi_{fmi,j} \mathbf{1}_{fmi,j}\right] - \sum_m w \iota_{fm}.$$

The first half of the expression denotes the expected gross profits of the firm. The expectation is taken over (i) the Poisson-distributed number of ideas A_{fm}, (ii) their match-specific values $\phi_{fmi,j}$, and (iii) the industries in which they are adapted $\mathbf{1}_{fmi,j}$. The second half of the expression relates to the unit costs of shared inputs used to generate ideas, which I normalize at w. Any differences in unit prices across types of shared inputs can be absorbed by differences across m in technology parameters α_{mj}.

Given the linearity of this problem and the independence of the Poisson and Fréchet distributions, the adaptation of each given idea i is independent of past and future ideas, and independent of the total number of arrivals of ideas. We can solve for the three sources of stochasticity separately. First, the adaptation decision $\mathbf{1}_{fmi,j}$ has the following expectational properties inherited from Fréchet (Section B.1):

$$\mathbb{E}[\mathbf{1}_{fmi,j}] = Pr(j = \arg\max_{k\in\mathcal{J}} \tilde{\phi}_{fmi,k}) = \frac{\delta_{fmj}^{\theta_m}}{\Delta_{fm}^{\theta_m}} \equiv \mu_{fmj},$$

where μ_{fmj} are industry adaptation probabilities (from equation 13). Second, the expected change in profits from a given idea adapted to the best industry is already solved in Appendix B.1 as:

$$\mathbb{E}[\tilde{\alpha}_{mj} B_j \xi_{fj} \phi_{fmi,j} \mid \mathbf{1}_{fmi,j} = 1] = \mathbb{E}\left[\max_j \tilde{\alpha}_{mj} B_j \xi_{fj}\, \phi_{fmi,j}\right] = \frac{\Delta_{fm}}{Z}.$$

Finally, A_{fm} is distributed independently with Poisson mean $Z\left(\frac{\rho_m}{\rho_m-1}\iota_{fm}\right)^{\frac{\rho_m-1}{\rho_m}}$. Putting the three pieces together, expected firm net profits $\mathbb{E}[\Pi_f]$ can be re-written as

$$\begin{aligned}\mathbb{E}[\Pi_f] &= \max_{\{\iota_{fm}\}_{m\in\mathcal{M}}} \sum_m \sum_j \mathbb{E}[A_{fm} \mid \iota_{fm}]\mathbb{E}[\tilde{\alpha}_{mj} B_j \xi_{fj} \phi_{fmi,j} \mid \mathbf{1}_{fmi,j} = 1]\mathbb{E}[\mathbf{1}_{fmi,j}] - \sum_m w \iota_{fm}.\\ &= \max_{\{\iota_{fm}\}_{m\in\mathcal{M}}} \sum_m \left(\frac{\rho_m}{\rho_m - 1}\iota_{fm}\right)^{\frac{\rho_m-1}{\rho_m}} \Delta_{fm} - \sum_m w \iota_{fm}.\end{aligned}$$

This is a convex optimization problem separable across shared input types m, with optimal inputs given by:

$$\iota_{fm} = \frac{\rho_m - 1}{\rho_m} \Delta_{fm}^{\rho_m} w^{-\rho_m}, \qquad \forall m,$$

and thus net profits are equal to

$$\mathbb{E}[\Pi_f] = \sum_m \Delta_{fm}^{\rho_m} w^{1-\rho_m} - \sum_m \frac{\rho_m - 1}{\rho_m} \Delta_{fm}^{\rho_m} w^{1-\rho_m} = \sum_m \frac{1}{\rho_m} \Delta_{fm}^{\rho_m} w^{1-\rho_m}.$$

Likewise, expected gross profits in a single industry j are given by

$$\mathbb{E}[\pi_{fj}] = \sum_m \mu_{fmj} \Delta_{fm}^{\rho_m} w^{1-\rho_m}.$$

The probability that a firm is active in industry j, denoted $\chi_{fj} = 1$, is one minus the probability that no ideas (of any type) is adapted to that industry. Since adaptation probabilities are independent across ideas, and the total arrival rate of ideas of any type m is a Poisson process with rate A_{fm}, the arrival of *adapted ideas* in j is also a Poisson process, with rate $\mu_{fmj} A_{fm}$. The probability of industry entry is thus one minus the probability that there are no arrivals from the joint Poisson processes over all shared input types $m \in \mathcal{M}$:

$$Pr(\chi_{fj} = 1) = 1 - \exp\left(\sum_m \mu_{fmj} A_{fm}\right) = 1 - \exp\left(-Z \sum_m \delta_{fmj}^{\theta_m} \Delta_{fm}^{\rho_m - 1 - \theta_m} w^{1-\rho_m}\right),$$

and is independent across industries due to Poissonization. Similarly, an inactive firm is a firm with no ideas arrive at all. The probability that a firm is active is thus (also endogenous to its inputs used and to profitability shifters) and given by one minus the probability that no ideas arrive:

$$Pr(\chi_f = 1) = 1 - \exp\left(\sum_m A_{fm}\right) = 1 - \exp\left(-Z \sum_m \Delta_{fm}^{\rho_m - 1} w^{1-\rho_m}\right).$$

B.3 Proof of Proposition 1: Cross-Industry Elasticities within the Firm

Log-differentiating equation (15) with respect to shifters of firm profitability in industries k, holding factor prices w constant, yields

$$\mathrm{d}\log \mathbb{E}[X_{fj}] = \mathrm{d}\log \mathbb{E}[\pi_{fj}] = \sum_m \lambda_{fjm} \left(\theta_m \mathbf{1}_{k=j} \,\mathrm{d}\log\left(\xi_{fk} B_k\right) + (\rho_m - \theta_m) \sum_k \mu_{fmk} \,\mathrm{d}\log\left(\xi_{fk} B_k\right)\right),$$

where μ_{fmj} are industry adaptation shares given in equation (13), and λ_{fjm} denote input utilization shares: the share of gross profits of industry j attributable to ideas from input type m (relative to m'):

$$\lambda_{fjm} \equiv \frac{\mu_{fmj} \Delta_{fm}^{\rho_m} w^{1-\rho_m}}{\sum_{m'} \mu_{fm'j} \Delta_{fm'}^{\rho_{m'}} w^{1-\rho_{m'}}}.$$

B.4 Connecting Firm-level Elasticities in the Model and Reduced-Form

The firm-level cross-industry elasticity from Proposition 1 combines responses on both intensive and extensive margins ($\mathbb{E}[X_{fj}]$ includes the non-trivial probability of zero sales). But for sufficiently large firms (high in ξ_f), virtually all of the adjustment loads on the intensive margin. The intuition is that the largest firms choose an initial level of expenditures on shared inputs high enough that the likelihood of cross-industry shocks affecting the extensive margin vanishes. For example, a demand shock for General Electric's MRI machines might affect GE's intensive margin sales of jet engines but is unlikely to affect whether the company is active at all in the jet engine business. When a large business expects hundreds of ideas / tasks to be allocated to an industry, a ten percent change in the number of ideas doesn't change the extensive margin probability by much. When a smaller business expects just a few ideas / tasks to be allocated, a ten percent change in the number of ideas has a larger impact on the extensive margin. With a high enough arrival rate, the expectation operator becomes exact due to the law of large numbers. (This large firm limit corresponds to the framework pioneered in Tintelnot (2016) and Antràs et al. (2017), whereby outcomes are smoothed across a continuum within the firm instead of being granular.) The following Lemma clarifies this point and motivates the focus of the reduced-form regressions on the intensive margin (given that the regression sample comprises large firms):

Lemma 2 (Intensive Margin Cross-Elasticities in Large Firms) *Cross-industry elasticities between j and k characterized by Proposition 1 load completely onto the intensive margin as ξ_{fj} and ξ_{fk} become arbitrarily high:*

$$\lim_{\min(\xi_{fj},\xi_{fk})\to\infty} \frac{\mathrm{d}\log \mathbb{E}[X_{fj}]}{\mathrm{d}\log \xi_{fk}B_k} = \frac{\mathrm{d}\log \mathbb{E}[X_{fj}|X_{fj}>0]}{\mathrm{d}\log \xi_{fk}B_k}.$$

As a corollary, the share of the cross-industry elasticity in Proposition 1 explained by the extensive margin ranges from 1 (for the lowest ξ firms) to 0 (for the highest ξ firms).

Proof. Decompose the expected gross sales into intensive margin and extensive margins:

$$\log \mathbb{E}[X_{fj}] = \log \mathbb{E}[X_{fj}|X_{fj}>0] + \log Pr(X_{fj}>0).$$

Differentiate the extensive margin:

$$\frac{\mathrm{d}\log Pr(X_{fj}>0)}{\mathrm{d}\log \xi_{fk}B_k} = \frac{\exp(-\Sigma_{fj})\Sigma_{fj}}{1-\exp(-\Sigma_{fj})}\sum_m s_{mj}\left(\theta_m \mathbf{1}_{k=j} + (\rho_m-\theta_m)\mu_{fmk}\right),$$

where s_{mj} are weights bounded between 0 and 1:

$$s_{mj} \equiv \frac{Z\mu_{fmj}\Delta_{fm}^{\rho_m-1}w^{1-\rho_m}}{\Sigma_{fj}},$$

and $\Sigma_{fj} \equiv Z\sum_m \mu_{fmj}\Delta_{fm}^{\rho_m-1}w^{1-\rho_m}$. Because the term $s_{mj}\left(\theta_m \mathbf{1}_{k=j} + (\rho_m-\theta_m)\mu_{fmk}\right)$ in the derivative of the extensive margin is bounded (weighted average of elasticities),

$$\lim_{\min(\xi_{fj},\xi_{fk})\to\infty} \frac{\mathrm{d}\log Pr(X_{fj}>0)}{\mathrm{d}\log \xi_{fk}B_k} = \lim_{\Sigma_{fj}\to\infty}\frac{\exp(-\Sigma_{fj})\Sigma_{fj}}{1-\exp(-\Sigma_{fj})} = 0,$$

where the last equality makes use of L'hopital's rule. ■

C Estimation Appendix

C.1 Identification of Macro Variables

Conditional on micro parameters $\Theta, \boldsymbol{v}$, I identify macro variables—technology coefficients $\boldsymbol{\alpha}$, industry profitability, $\boldsymbol{B}_t$, and the average arrival rate Z_t—by relating the aggregate predictions of the model to their counterparts in the data. I do so in a block-recursive manner.

First, I solve the second line of equation (21) separately for each of the three types of shared knowledge inputs $m \in \mathcal{M}^{KLG}$. For each type of knowledge input m, given data on expenditures $\{M_{mj}\}_{j\in\mathcal{J}}$ in the base period ($t = 1$), I invert a separate system of $|\mathcal{J}|$ equations for $|\mathcal{J}|$ model variables $\{\alpha_{mj}B_{j,t=1}Z_t\}_{j\in\mathcal{J}}$ (with the three terms grouped together).

For each industry j, the mean of α_{mj} across m is isomorphic to a constant term in $B_{j,t=1}$. Thus, I am free to normalize the technology coefficient of the residual shared input $\alpha_{RES,j} = 1$. As a second step, I subtract knowledge expenditures (the second line of equation 21 for all $m \in \mathcal{M}^{KLG}$) from gross profits (the first line of equation 21) to yield (for $t = 1$):

$$\frac{\pi_j - \sum_{m\in\mathcal{M}^{KLG}} \frac{\rho_{KLG}}{\rho_{KLG}-1} M_{mj}}{N} = \int \delta_{f,RES,j}^{\theta_{RES}} \Delta_{f,RES}^{\rho_{RES}-\theta_{RES}} \, \mathrm{d}G(\boldsymbol{\xi}), \quad \forall j \in \mathcal{J},$$

where the left-hand-side variables (π_j, M_{mj}) is data contained in BEA input-output tables, and the right-hand side contains a $|\mathcal{J}|$ vector of unknowns $\{B_{j,t=1}Z\}_{j\in\mathcal{J}}$ (since $\alpha_{RES,j} = 1$).[37] This represents the forth system of $|\mathcal{J}|$ equations for $|\mathcal{J}|$ model variables that I invert. (The other three being each of the three types of knowledge inputs, described in the first step).

Third, given values of $\{B_{j,t=1}Z_{t=1}\}_{j\in\mathcal{J}}$ from step 2, and $\{\alpha_{mj}B_{j,t=1}Z_{t=1}\}_{j\in\mathcal{J},m\in\mathcal{M}^{KLG}}$ from step 1, I can directly back out technology coefficients $\{\alpha_{mj}\}_{j\in\mathcal{J},m\in\mathcal{M}^{KLG}}$. I hold $\boldsymbol{\alpha}$ constant over all three time periods due to lack of expenditure data on knowledge inputs in subsequent years.

Forth, I use the expression for gross output $\boldsymbol{X}_t$ in equation (21) in years $t = 2, 3$ to find future-period industry profitability $\{B_{j,t=2}Z_{t=2}, B_{j,t=3}Z_{t=3}\}_{j\in\mathcal{J}}$. In each year, given values of α_{mj}, I can invert a system of $|\mathcal{J}|$ equations for $|\mathcal{J}|$ model variables $\{B_{jt}Z_t\}_{j\in\mathcal{J}}$.

Fifth, given the full set of $\{B_{jt}Z_t\}_{j\in\mathcal{J},t=1,2,3}$ (from steps 2 and 4) and technology coefficients $\boldsymbol{\alpha}$, I solve for Z_t such that the closed-form expression for the share of single-industry firms in the model matches 0.8 in the data:

$$\frac{\int \sum_j \left(Pr(\chi_{fjt} = 1) \prod_{k\neq j}(1 - Pr(\chi_{fkt} = 1)) \right) \mathrm{d}G(\boldsymbol{\xi})}{\int Pr(\chi_{ft} = 1)\, \mathrm{d}G(\boldsymbol{\xi})} = 0.8. \tag{27}$$

where entry probabilities by industry (χ_{fjt}) and firm-wide (χ_{ft}) are given in Appendix B.2.

Finally, from values of $\{B_{jt}Z_t\}_{j\in\mathcal{J},t=1,2,3}$ and Z_t, I can directly back out $\boldsymbol{B}_t$.

C.2 Identification and Inference of Scalability and Rivalry

Notationally, many functions described below depend on macro variables (i.e. $\boldsymbol{B}_t, \boldsymbol{\alpha}, Z_t$), which I suppress into a time subscript t for ease of exposition.

[37]Note that no expenditure data on the residual capital input category is needed. The residual category is set up to also absorb payments to latent factors (e.g., venture capital and sweat equity). This equation imposes a non-negativity restriction which manifests as a lower bound on the value of ρ_{KLG} according to the model:

$$\begin{aligned} \pi_j > \sum_m \frac{\rho_{KLG}}{\rho_{KLG}-1} M_{mj} &\iff (1-\varsigma_j) > \frac{\rho_{KLG}}{\rho_{KLG}-1} \sum_{m\in KLG} \beta_{jm}, \qquad \forall j \\ &\iff \frac{\rho_{KLG}-1}{\rho_{KLG}} > \max_j \frac{\beta_{j,KLG}}{1-\varsigma_j} = \max_j \frac{\beta_{j,KLG}}{\beta_{j,KLG}+\beta_{j,RES}}, \end{aligned}$$

for BEA expenditure shares β_{jm}. In the data, this restriction corresponds roughly to imposing that $\rho_{KLG} > 3$.

Proof of Proposition 2. I show that at true parameter values $\Theta, \boldsymbol{v}$, the following $J \times J$ structural moment conditions hold true for any pair of industries j, k:

$$\mathbf{E}_f \left[\Delta\epsilon_{fjt} \; \Delta \log S_{fkt} \;\middle|\; \chi_{f,t-1} \right] = 0, \qquad \forall t = \{2, 3\}. \tag{28}$$

where $\Delta\epsilon_{fjt}$ is a structural residual defined as:

$$\Delta\epsilon_{fjt} \equiv \left(X_{fjt} - X_{fj,t-1} \right) - \left(\mathbb{E}_t[X_{fjt} \,|\, \boldsymbol{\xi}_{f,t-1}, \boldsymbol{\chi}_{f,t-1}, \Delta \log \boldsymbol{S}_{ft}] - \mathbb{E}_{t-1}[X_{fj,t-1} \,|\, \boldsymbol{\xi}_{f,t-1}, \boldsymbol{\chi}_{f,t-1}, \Delta \log \boldsymbol{S}_{ft}] \right).$$

By the law of iterated expectations, the moment condition for any pair of industries j, k in any year $t = \{2, 3\}$ can be written as

$$\mathbf{E}_{\xi_{f,t-1}, \Delta \log S_{ft}} \left[\Delta \log S_{fkt} \; \mathbf{E}_f \left[\Delta\epsilon_{fjt} \;\middle|\; \chi_{f,t-1}, \xi_{f,t-1}, \Delta \log S_{ft} \right] \;\middle|\; \chi_{f,t-1} \right],$$

so it suffices to show that the inner conditional expectation is zero. The inner conditional expectation is a sum of the four terms making up $\Delta\epsilon_{fjt}$. The first and third terms cancel out:

$$\mathbf{E}_f \left[X_{fjt} \;\middle|\; \chi_{f,t-1}, \xi_{f,t-1}, \Delta \log S_{ft} \right] = \mathbb{E} \left[X_{fjt} \;\middle|\; \xi_{f,t} \right],$$

and the second and forth terms cancel out since Assumption 4 (conditional independence) implies that $\Delta \log \boldsymbol{S}_{ft}$ is independent of outcomes $X_{fj,t-1}$ conditional on the industry presence $\boldsymbol{\chi}$ and unobserved profitability shifters $\boldsymbol{\xi}_{f,t-1}$, so:

$$\mathbf{E}_f \left[X_{fj,t-1} \;\middle|\; \chi_{f,t-1}, \xi_{f,t-1}, \Delta \log \boldsymbol{S}_{ft} \right] = \mathbb{E} \left[X_{fj,t-1} \;\middle|\; \chi_{f,t-1}, \xi_{f,t-1} \right].$$

Inference. Next, I construct sample analogs of the moment conditions in equation (28). Since the moment conditions are valid conditional on $\boldsymbol{\chi}_{t-1}$, I am free to limit observations to the set of firms that are active in each pair of industries j, k in year $t-1$. I label this set of firms by $\mathcal{F}^D_{jk,t-1}$. I break out the linear terms inside the structural residual into two parts. The first part is pure data—involving the interaction of realized sales growth $\boldsymbol{X}_{ft} - \boldsymbol{X}_{f,t-1}$ and demand shocks:

$$\Xi^o_{jkt} \equiv \frac{1}{|\mathcal{F}^D_{jk,t-1}|} \sum_{f \in \mathcal{F}^D_{jk,t-1}} \Delta X_{fjt} \; \Delta \log S_{fkt}, \quad \forall j, k, \; \forall t = \{2, 3\}.$$

The second part of the moment conditions involve the model-based counterpart, given by

$$\mathbf{E}_f \left[\left(\mathbb{E}[X_{fjt} \,|\, \boldsymbol{\chi}_{f,t-1}, \boldsymbol{\xi}_{f,t-1}, \Delta \log \boldsymbol{S}_{ft}] - \mathbb{E}[X_{fj,t-1} | \boldsymbol{\chi}_{f,t-1}, \boldsymbol{\xi}_{f,t-1}, \Delta \log \boldsymbol{S}_{ft}] \right) \Delta \log S_{fkt} \;\middle|\; \boldsymbol{\chi}_{f,t-1} \right].$$

Note that one cannot compute this by integrating over the unconditional distribution $G(\xi)$ because the moments condition on the firm's extensive margin $\boldsymbol{\chi}_{f,t-1}$, which informs the likely values of $\boldsymbol{\xi}$. I respect this potential correlation between demand shocks, fundamental profitability, and the firm's initial-period extensive margin by integrating over the *conditional* distribution $Pr(\boldsymbol{\xi} \,|\, \boldsymbol{\chi}_{f,t-1}, \Delta \log \boldsymbol{S}_{ft})$. I express this likelihood analytically using Bayes' rule and the model's closed-form solutions for the extensive margin probability of entry. I derive this result in a series of steps. First, in step (i), I define a closed-form analytical object g_{jk} as a function of three terms: demand shocks $\Delta \log \boldsymbol{S}_{ft}$ and extensive margin presence $\boldsymbol{\chi}_{f,t-1}$ which are observable in the data, as well as unobservable profitability shifters $\xi_{f,t-1}$:

$$g_{jk}(\xi_{f,t-1}, \Delta \log \mathbf{S}_{ft}, \chi_{f,t-1}) \equiv \\ \mathbf{E}_f \left[\left(\mathbb{E}[X_{fjt} \mid \xi_{f,t}, \chi_{f,t-1}] - \mathbb{E}[X_{fj,t-1} \mid \xi_{f,t-1}, \chi_{f,t-1}] \right) \Delta \log S_{fkt} \;\middle|\; \xi_{f,t-1}, \Delta \log \mathbf{S}_{ft}, \chi_{f,t-1} \right].$$

using the fact that under Assumption 3 (relevance), $\xi_{f,t}$ can be computed from $(\xi_{f,t-1}, \Delta \log \mathbf{S}_{ft})$, so $\mathbb{E}[X_{fjt} | \chi_{f,t-1}, \xi_{f,t-1}, \Delta \log \mathbf{S}_{ft}] = \mathbb{E}[X_{fjt} \mid \xi_{f,t}, \chi_{f,t-1}]$ and the fact that under Assumption 4 (conditional independence), demand shocks are as good as randomly assigned conditional on initial-period extensive margin, so $\mathbb{E}[X_{fj,t-1} | \chi_{f,t-1}, \xi_{f,t-1}, \Delta \log \mathbf{S}_{ft}] = \mathbb{E}[X_{fj,t-1} | \chi_{f,t-1}, \xi_{f,t-1}]$. Over the next series of steps I manipulate the second part of the moment condition line by line as follows:

$$\begin{aligned}
&\mathbf{E}_f \left[\left(\mathbb{E}[X_{fjt} \mid \chi_{f,t-1}, \xi_{f,t-1}, \Delta \log \mathbf{S}_{ft}] - \mathbb{E}[X_{fj,t-1} \mid \chi_{f,t-1}, \xi_{f,t-1}, \Delta \log \mathbf{S}_{ft}] \right) \Delta \log S_{fkt} \;\middle|\; \chi_{f,t-1} \right] \\
&= \mathbf{E}_{\Delta \log \mathbf{S}_{ft}, \xi_{f,t-1}} \left[g_{jk}(\xi_{f,t-1}, \Delta \log \mathbf{S}_{ft}, \chi_{f,t-1}) \;\middle|\; \chi_{f,t-1} \right] \\
&= \mathbf{E}_{\Delta \log \mathbf{S}_{ft}} \left[\int_\xi g_{jk}(\xi, \Delta \log \mathbf{S}_{ft}, \chi_{f,t-1}) \; Pr(\xi \mid \chi_{f,t-1}) \, \mathrm{d}\xi \;\middle|\; \chi_{f,t-1} \right] \\
&= \mathbf{E}_{\Delta \log \mathbf{S}_{ft}} \left[\int_\xi g_{jk}(\xi, \Delta \log \mathbf{S}_{ft}, \chi_{f,t-1}) \; \frac{Pr(\chi_{f,t-1} \mid \xi)}{\int_\xi Pr(\chi_{f,t-1} \mid \xi) \, \mathrm{d}G(\xi)} \, \mathrm{d}G(\xi) \;\middle|\; \chi_{f,t-1} \right] \\
&= \mathbf{E}_{\Delta \log \mathbf{S}_{ft}} \left[\int_\xi g_{jk}(\xi, \Delta \log \mathbf{S}_{ft}, \chi_{f,t-1}) \; \frac{\prod_{j \in \mathcal{J}} Pr(\chi_{fj,t-1} \mid \xi)}{\int_\xi \prod_{j \in \mathcal{J}} Pr(\chi_{fj,t-1} \mid \xi) \, \mathrm{d}G(\xi)} \, \mathrm{d}G(\xi) \;\middle|\; \chi_{f,t-1} \right],
\end{aligned}$$

where step (ii) applies the law of iterated expectations and replaces the inner expectation term with g_{jk}, step (iii) breaks up the expectation over the joint probability distribution of $\Delta \log \mathbf{S}_{ft}, \xi_{f,t-1}$ in terms of a conditional $Pr(\xi \mid \Delta \log \mathbf{S}_{ft}, \chi_{f,t-1}) = Pr(\xi \mid \chi_{f,t-1})$ (given the conditional independence Assumption 4) and a marginal $Pr(\Delta \log \mathbf{S}_{ft} \mid \chi_{f,t-1})$, left with the expectation operator $\mathbf{E}_{\Delta \log \mathbf{S}_{ft}}$. Step (iv) applies Bayes' rule to transform $Pr(\xi \mid \chi_{f,t-1})$ into known analytical extensive margin probabilities. Finally, step (v) exploits known properties of the Poisson arrival process where the probability of industry entry in $j \in \mathcal{J}$ is independent of any $j' \in \mathcal{J}$ conditional on shifters ξ.

I construct the sample analog of the last line above using a sample $\mathcal{F}^S$ of simulated firms with profitability shifters ξ_i drawn from distribution $G(\xi_i)$ under Assumption 3:

$$\Xi^m_{jkt} \equiv \frac{1}{|\mathcal{F}^D_{jk,t-1}|} \sum_{f \in \mathcal{F}^D_{jk,t-1}} \sum_{i \in \mathcal{F}^S} \omega_{i,t-1}(\chi_{f,t-1}) \; \Delta \hat{X}_{jt}(\chi_{f,t-1}, \xi_i, \Delta \log \mathbf{S}_{ft}) \; \Delta \log S_{fkt},$$

where (i) $\Delta \hat{X}_{jt}$ is model-implied expected sales growth of a firm conditional on prior-period extensive margin $\chi_{f,t-1}$, fundamental profitability $\xi_{f,t-1} = \xi_i$ and demand shocks $\Delta \log \mathbf{S}_{ft}$ that inform next-period profitability ξ'_i (under Assumption 3):

$$\Delta \hat{X}_{jt}(\chi_{f,t-1}, \xi_i, \Delta \log \mathbf{S}_{ft}) \equiv \mathbb{E}[X_{fjt} \mid \xi'_i, \chi_{f,t-1}] - \mathbb{E}[X_{fj,t-1} \mid \xi_i, \chi_{f,t-1}],$$

and (ii) Bayes probability weights $\omega_{i,t-1}$ reflect the probability that a firm f in the data with extensive margin $\chi_{f,t-1}$ has shifters equal to ξ_i of simulated firm i relative to that of other simulated firms $i' \in \mathcal{F}^S$:

$$\omega_{i,t-1}(\chi_{f,t-1}) \equiv \frac{\prod_{j \in \mathcal{J}} Pr(\chi_{ij,t-1} = \chi_{fj,t-1} \mid \xi_i)}{\sum_{i' \in \mathcal{F}^S} \prod_{j \in \mathcal{J}} Pr(\chi_{i'j,t-1} = \chi_{fj,t-1} \mid \xi_{i'})}.$$

Table C.6: Distribution of Outcomes by Firm Scope in the Data and Model, 1997

	Share of Firms (%)		Share of Sales (%)	
Number of Industries	Data	Model	Data	Model
1	80.99	80.15	26.13	25.00
2	13.01	13.54	10.80	15.74
3	3.32	3.29	7.34	9.43
4	1.33	1.07	5.51	6.33
5	0.61	0.44	4.08	5.48
6	0.28	0.23	2.89	5.76
7	0.14	0.15	3.54	6.71
8	0.08	0.10	3.63	4.05
9 +	0.25	1.03	36.05	21.49

Notes: The distribution of outcomes by firm scope, in the data and in the model (with the six estimated parameters, Θ, v_0, v_1). Sales of firms with 9 or more industries could not be simulated via brute force due to memory issues when simulating the discrete Poisson process. Instead, it is backed out from the fact that the share of sales by firms with one industry was set to equal 25% in the estimation.

Lastly, putting the data and model halves of the sample analog together, and invoking the law of large numbers, $m_{jkt} \equiv \Xi^o_{jkt} - \Xi^m_{jkt}$ approaches the moment condition in Proposition 2:

$$\lim_{|\mathcal{F}^D_{jk,t-1}|\to\infty} \lim_{|\mathcal{F}^S|\to\infty} \Xi^o_{jkt} - \Xi^m_{jkt} = \mathbf{E}_f \left[\Delta\epsilon_{fjt}\ \Delta \log S_{fkt} \;\middle|\; \chi_{f,t-1}\right] = 0.$$

At true parameter values Θ, as the data and simulation samples become large, $|\mathcal{F}^D|, |\mathcal{F}^S| \to \infty$, the sample analog $m_{jkt} = 0$ for any j, k and $t \in \{2, 3\}$.

C.3 Nested Fixed Point Estimation Algorithm

I combine a search over both micro and macro parameters of the model. Estimation proceeds over five steps:

1. Simulate a set of 2000 firms $i \in \mathcal{F}^S$ with fixed draws of ζ_{ij}, ζ_i from standard normal distributions. I use stratified sampling to over-weigh firms with higher ζ_i.
2. Guess a starting $\hat{\Theta}$, $\hat{v}_0$, $\hat{v}_1$, then repeat Steps 3-5 until convergence.
3. Compute $\{\xi_i\}_{i\in\mathcal{F}^S}$ given $\hat{v}_0$, $\hat{v}_1$ from Assumption 3 and baseline draws ζ_{ij}, ζ_i.
4. Use $\{\xi_i\}_{i\in\mathcal{F}^S}$ and $\hat{\Theta}$ to compute $\boldsymbol{\alpha}, \boldsymbol{B}_t, Z_t$ via equation (21) and Table 4.
5. Compute the sample moment conditions in Proposition 2, stack the moments according to the four groups as described above, and use a bounded Nelder-Mead simplex search algorithm to adjust the guess of $\hat{\Theta}$, $\hat{v}_0$, $\hat{v}_1$ given the change in the objective value.

C.4 External Validity

Table C.6 displays the distribution of firms and sales over firm scope behind Figure 3. Unlike estimation, which requires only simulated values of firm profitability shifters ξ_i, these outcomes are computed by simulating the actual, granular, outcomes of firms in the model.

D Quantitative Appendix

D.1 General Equilibrium Definition

I introduce some more notation used to characterize the open economy equilibrium under Definition 1.

There is a single factor of production, labor, that can frictionlessly reallocate across industries, stages of production, and from manufacturing to non-manufacturing. Household preferences are Cobb-Douglas with shares $\beta_{F,NM}, \{\beta_{F,j}\}_j$ on non-manufacturing (NM) and manufacturing industries j respectively. The US is the domestic economy (denoted u) trading with foreign countries denoted $d \in \mathcal{D}^F$. Let $\overline{Y}_{d,j}$ denote the exogenous market size faced by US firms in each industry j in a foreign destination $d \in \mathcal{D}^F$, and suppose that all firms are common exporters.[38] Let $\overline{PX}_{dj}$ represent exogenous indices of price competitiveness in foreign market d by all non-US firms, and let $\overline{PM}_{dj}$ represent exogenous indices of price competitiveness in the US market by foreign firms from d, so that:

$$\overline{PX}_{dj} \equiv \int_{f \in \mathcal{D}^F} p_{fdj}^{1-\sigma_j} \, \mathrm{d}f, \qquad \overline{PM}_{dj} \equiv \int_{f \in \mathcal{D}^F} p_{fuj}^{1-\sigma_j} \, \mathrm{d}f,$$

where p_{fdj} is the price of a good sold by foreign firm f in destination d. For example, an increase in $\overline{PM}_{CHN,j}$ indicates that prices of Chinese goods in the US have been lowered (become more competitive).

The equilibrium described in Definition 1 keeps wages w fixed in response to manufacturing-sector shocks, preventing wage changes from contaminating cross-industry impacts. I assume that the foreign residual demand curve for US exports in non-manufacturing is perfectly elastic, so wages are pinned down by world prices of the non-manufacturing good and all adjustment loads on US net exports of non-manufacturing goods, denoted D. Under balanced total trade, D is therefore equal to the (endogenous) US manufacturing trade deficit.

The definition of equilibrium allows for the possibility of an input-output production structure, where β_{kj} is the share of gross output of industry k expensed on stage-II production inputs from industry j. The quantitative results in the body of the paper are obtained with these input-output coefficients set to zero. In this Appendix I explore robustness of the paper's results to including input-output linkages.

Definition 1 (General Equilibrium) *Let PD_j denote domestic price competitiveness in an industry $j \in \mathcal{J}$:*

$$PD_j \equiv \mathcal{P}^{1-\sigma_j} \equiv N \int \mathbb{E}\left[p_{fj}^{1-\sigma_j}\right] \mathrm{d}G(\boldsymbol{\xi}).$$

Let $w = 1$ be the numeraire. Given total labor L, a mass of firms N, exogenous foreign price competitiveness abroad and at home, $\{\overline{PX}_{dj}, \overline{PM}_{dj}\}_{j \in \mathcal{J}, d \in \mathcal{D}^F}$, foreign expenditures $\{\overline{Y}_{dj}\}_{j \in \mathcal{J}, d \in \mathcal{D}^F}$, and other parameters of the model, general equilibrium is described by the tuple $\{D, \eta_M, \boldsymbol{PD}\}$ of net exports of non-manufacturing goods, manufacturing labor share, and industry price competitiveness such that the following equilibrium conditions and related definitions hold:

[38] This common-exporter assumption is less extreme that one might think in the context of a stochastic model. The equilibrium does not require all firms to *actually* export; it suffices that firms have ex-ante *expectations* of exporting. One micro-foundation, for example, would be if each idea adapted in industry j increases domestic output and also has a probability of being used at the same time for export market output. A firm then enters into exporting if and only if it has a non-zero amount of ideas adapted for export markets. Despite this common probability of exporting *per instance of an idea*, empirically, larger firms would be more likely to export because they have more ideas arrive on average, and therefore a higher chance that at least one idea is adaptable for export markets.

(i) Total industry expenditures in the US is given by

$$Y_j = \sum_{k \in \mathcal{J}} \beta_{kj} X_k + \beta_{F,j} wL, \quad \forall j \in \mathcal{J},$$

where X_k stands for US industry k gross output, β_{kj} is the share of gross output of industry k expensed on inputs from industry j, and $\beta_{F,j}$ is the share of final consumption spent on industry j.

(ii) Goods market clearing yields a system of $|\mathcal{J}|$ equations in $|\mathcal{J}|$ industry profitability levels $\boldsymbol{B}$: output produced over all firms has to equal total domestic industry output, which has to equal output consumed at home plus output exported to foreign markets $d \in \mathcal{D}^F$:

$$\begin{aligned} X_j &= N \int \mathbb{E}[X_{fj}\,; \boldsymbol{B}]\, \mathrm{d}G(\boldsymbol{\xi}) \\ &= Y_j \frac{PD_j}{PD_j + \sum_{d \in \mathcal{D}^F} \overline{PM}_{dj}} + \sum_{d \in \mathcal{D}^F} \overline{Y}_{dj} \frac{PD_j}{PD_j + \overline{PX}_{dj}}, \quad \forall j \in \mathcal{J}, \end{aligned} \tag{29}$$

(iii) Domestic competitiveness $\boldsymbol{PD}$ can be related to industry profitability $\boldsymbol{B}$ by combining equation (29) with an open-economy version of equation (12):

$$B_j = (1 - \varsigma_j) \left(\frac{c_j}{\varsigma_j} \right)^{\frac{\varsigma_j}{\varsigma_j - 1}} \left(\frac{X_j}{PD_j} \right)^{\frac{1}{\sigma_j (1 - \varsigma_j)}}, \quad \forall j \in \mathcal{J}, \tag{30}$$

where c_j is the unit price index of a Cobb-Douglas bundle of stage-II industry-specific inputs:

$$c_j \equiv w^{\tilde{\beta}_{jl}} \prod_{k \in J} P_k^{\tilde{\beta}_{jk}},$$

where $\tilde{\beta}_{jl}, \tilde{\beta}_{jk}$ denote expenditures of industry j on labor value-added l or input $k \in J$ as a share of total expenditures on production inputs (not to be confused with $\beta_{jk} = \tilde{\beta}_{jk} \varsigma_j$, which are expenditure shares over gross output), and the domestic consumption price index P_j (for both final and intermediate consumption) is:

$$P_j^{1-\sigma_j} \equiv PD_j + \sum_{d \in \mathcal{D}^F} \overline{PM}_{dj}. \tag{31}$$

(iv) Balance in overall trade requires that the consumption value of manufacturing imports (less any tariffs T collected) equal manufacturing exports plus net exports in the non-manufacturing sector, denoted D:

$$\sum_{j \in \mathcal{J}} Y_j \frac{\sum_{d \in \mathcal{D}^F} \overline{PM}_{dj}}{PD_j + \sum_{d \in \mathcal{D}^F} \overline{PM}_{dj}} - T = D + \sum_{j \in \mathcal{J}} \sum_{d \in \mathcal{D}^F} \overline{Y}_{dj} \frac{PD_j}{PD_j + \overline{PX}_{dj}}. \tag{32}$$

(v) The residual non-manufacturing sector is produced with constant returns to scale using labor under perfect competition. Domestic value-added and output in the residual sector is given by

$$(1 - \eta_M) wL = D + \Pi + T + \beta_{F,NM} wL,$$

where Π is net profits in the manufacturing sector given by equation (17).

(vi) Manufacturing sector payroll is the sum of payments to labor used in stages I and II:

$$\eta_M wL = \sum_{k \in \mathcal{J}} \left(1 - \sum_{j \in \mathcal{J}} \beta_{kj}\right) X_k - \Pi.$$

D.2 Aggregate Economies of Scale and Scope

I supplant Proposition 3 in the main text with Lemma 3, a more general version that allows for arbitrary input-output linkages (use of inputs in stage-II) across manufacturing industries. Proposition 3 is a special case of Lemma 3 when input-output coefficients are set to zero.

Definition of Shocks. I group together different types of exogenous shocks into two terms: (i) changes to market size faced by US producers, $\mathrm{d}\log \boldsymbol{S}$, and (ii) changes to prices of foreign goods in the US, $\mathrm{d}\log \boldsymbol{PS}$ (while this is also a market size shifter, I single it out here because import prices are a cost shifter in the supply-side equation):

$$\mathrm{d}\log S_j \equiv \lambda^X_{uj}\lambda^X_{j,F}\,\mathrm{d}\log L + \sum_{d \in \mathcal{D}^F} \lambda^X_{dj}\,\mathrm{d}\log \overline{Y}_{dj} - \sum_{d \in \mathcal{D}^F} \lambda^X_{dj}(1 - \lambda^M_{dj})\,\mathrm{d}\log \overline{PX}_{dj}, \tag{33}$$

$$\mathrm{d}\log PS_j \equiv - \sum_{d \in \mathcal{D}^F} \lambda^{UM}_{dj}\,\mathrm{d}\log \overline{PM}_j, \qquad \forall j \in \mathcal{J}$$

where λ^M_{dj} is the share of country d's industry j consumption on US firms, λ^X_{dj} is the share of US firms' industry j sales exported to d (and λ^X_{uj} is the share sold domestically in u), and λ^{UM}_{dj} is the share of US industry j consumption on firms from country d.

Lemma 3 (Aggregate Consequences of Joint Production w/ Input-Output Linkages) *In the open economy equilibrium in Definition 1, domestic producer price indices $\mathrm{d}\log \boldsymbol{\mathcal{P}}$ and output $\mathrm{d}\log \boldsymbol{X}$ respond to exogenous shocks to market size $\mathrm{d}\log \boldsymbol{S}$ (defined in equation 33) according to:*

$$\mathrm{d}\log \boldsymbol{\mathcal{P}} = diag\left(\frac{1}{\sigma - 1}\right)\left(\mathbb{I} - \boldsymbol{\Omega}^S diag(\boldsymbol{\lambda}^M_u) + \boldsymbol{\Psi}\left(\mathbb{I} - \boldsymbol{\Omega}^D\right)^{-1} diag(\boldsymbol{\lambda}^{cpt})\right)^{-1} \times \boldsymbol{\Psi}\left(\mathbb{I} - \boldsymbol{\Omega}^D\right)^{-1} \mathrm{d}\log \boldsymbol{S}, \tag{34}$$

$$\mathrm{d}\log \boldsymbol{X} = \left(\mathbb{I} - \boldsymbol{\Omega}^D\right)^{-1}\left[diag(\boldsymbol{\lambda}^{cpt}(1-\sigma))\,\mathrm{d}\log \boldsymbol{PD} + \mathrm{d}\log \boldsymbol{S}\right], \tag{35}$$

where (i) $\mathbb{I}$ is the identity matrix, (ii) $\boldsymbol{\Psi}$ is a macro joint production matrix containing inverse cross-industry supply-side elasticities $\boldsymbol{\Upsilon}$ for the 'average' firm:

$$\begin{aligned}
[\boldsymbol{\Psi}]_{jk} &\equiv \sigma_j(1-\varsigma_j)[\boldsymbol{\Upsilon}^{-1}]_{jk} - \mathbf{1}_{j=k}, \\
[\boldsymbol{\Upsilon}]_{jk} &\equiv \sum_{m \in \mathcal{M}} (\rho_m - \theta_m)\,\bar{\lambda}_{jm}\,\bar{\mu}_{jmk} + \mathbf{1}_{j=k} \sum_{m \in \mathcal{M}} \theta_m\,\bar{\lambda}_{jm}, \qquad \forall j,k \in \mathcal{J},
\end{aligned} \tag{36}$$

where industry allocation shares $\bar{\mu}_{jmk}$ indicate the average propensity for type-m ideas to be allocated to industry k (relative to other industries k') among firms that produce in j, and input utilization shares $\bar{\lambda}_{jm}$ indicate the average profit-contribution to industry j of shared input m (relative to other shared inputs m'):

$$\bar{\mu}_{jmk} \equiv \int \frac{\mathbb{E}[X_{fj}]\lambda_{fjm}}{\int \mathbb{E}[X_{fj}]\lambda_{fjm}\,\mathrm{d}G(\boldsymbol{\xi})}\mu_{fmk}\,\mathrm{d}G(\boldsymbol{\xi}), \qquad \bar{\lambda}_{jm} \equiv \int \frac{\mathbb{E}[X_{fj}]}{\int \mathbb{E}[X_{fj}]\,\mathrm{d}G(\boldsymbol{\xi})}\lambda_{fjm}\,\mathrm{d}G(\boldsymbol{\xi}),$$

(iii) $\mathbf{\Omega}^S$, $\mathbf{\Omega}^D$ are matrices containing input-output coefficients β_{jk}: the share of industry j gross output expensed on production inputs from industry k, and $\lambda^X_{j,F}$, the share of final use among all expenditures on industry j:

$$[\mathbf{\Omega}^S]_{jk} \equiv \beta_{jk}\frac{\sigma_j}{\sigma_k - 1}, \qquad [\mathbf{\Omega}^D]_{jk} \equiv \lambda^X_{uj}(1-\lambda^X_{j,F})\frac{\beta_{kj}X_k}{\sum_{k'\in\mathcal{J}}\beta_{k'j}X_{k'}}, \tag{37}$$

and (iv) $\boldsymbol{\lambda}^{cpt}$ reflects the potential for US firms to gain market share from foreign competitors:

$$\lambda^{cpt}_j \equiv \sum_{d\in\{u,\mathcal{D}^F\}} \lambda^X_{dj}(1-\lambda^M_{dj}) \qquad \forall j \in \mathcal{J}. \tag{38}$$

Proof of Lemma 3 (and Proposition 3). I log-differentiate the system of equations in Definition 1 to express endogenous equilibrium variables (domestic price competitiveness, sales, etc) as a function of changes in exogenous variables (changes to domestic scale L, foreign demand $\overline{\boldsymbol{Y}}_d$, and foreign price competitiveness $\boldsymbol{PX}, \boldsymbol{PM}$).

It is convenient to solve for the equilibrium impact on endogenous variables through their effect on domestic producer price competitiveness $\boldsymbol{PD}$ (an inverse price term introduced in Definition 1). Equation (29) is a market clearing condition that equates supply with demand. The first line describes the supply-side relationship between domestic producer price competitiveness $\boldsymbol{PD}$ and market output $\boldsymbol{X}$ such that firm production incentives are sustained under monopolistic competition. The second line describes a downward-sloping industry demand-curve: the higher is domestic price competitiveness PD_j (the lower are producer prices), the greater is the value of market output. In autarky, this demand curve would be unit-elastic because both final demand and intermediate demand is Cobb-Douglas. In the open economy setup assumed here, demand is more than unit-elastic due to an additional foreign-market-share-stealing effect. I separately differentiate the demand and supply sides (as functions of endogenous $\boldsymbol{PD}$ and exogenous shocks) before putting the two together in equilibrium (and solving for endogenous $\boldsymbol{PD}$).

Demand-side. Log-differentiating the second line of equation (29) yields the following demand-side equilibrium relationship between sales $\boldsymbol{X}$ and domestic competitiveness $\boldsymbol{PD}$, in matrix algebra:

$$\mathrm{d}\log\boldsymbol{X} = \left(\mathbb{I} - \mathbf{\Omega}^D\right)^{-1}\left(\mathrm{diag}(\boldsymbol{\lambda}^{cpt})\,\mathrm{d}\log\boldsymbol{PD} + \mathrm{d}\log\boldsymbol{S} + \mathrm{diag}(\boldsymbol{\lambda}^X_u)\,\mathrm{d}\log\boldsymbol{PS}\right), \tag{39}$$

where λ^{cpt}_j is given by equation (38) and $\mathbf{\Omega}^D$ by equation (37).

Note that when the home country is in autarky, $\boldsymbol{\lambda}^{cpt} = 0$, and there is no demand-side adjustment of industry output with respect to prices (given the unit-elastic demand curve).

Supply-side. I next turn to the supply-side relationship between market size (industry profitability) and prices. Log-differentiating the first line of equation (29) yields (after switching the order of summation across inputs, industries, and firms):

$$\boldsymbol{\Upsilon}^{-1}\,\mathrm{d}\log\boldsymbol{X} = \mathrm{d}\log\boldsymbol{B},$$

where $\boldsymbol{\Upsilon}$ is the aggregate matrix of supply-side elasticities given by equation (36). I futher solve out for $\mathrm{d}\log\boldsymbol{B}$ in this expression. I log-differentiate the expression for industry profitability B_j in equation (30), open up the production input cost index c_j to reflect intermediate input purchases from manufacturing industries to yield,

$$\mathrm{d}\log B_j = \frac{\varsigma_j}{\varsigma_j - 1}\sum_{k\in\mathcal{J}}\tilde{\beta}_{jk}\,\mathrm{d}\log P_k + \frac{1}{\sigma_j(1-\varsigma_j)}(\mathrm{d}\log X_j - \mathrm{d}\log PD_j), \tag{40}$$

and using equation (31) to replace P_k with PD_k and exogenous foreign cost shocks $\overline{PM}_{dk}$ and combining the previous two equations yields

$$\boldsymbol{\Psi}\,\mathrm{d}\log\boldsymbol{X} = -\left(\mathbb{I} - \boldsymbol{\Omega}^S\ \mathrm{diag}\left(\lambda_u^X\right)\right)\mathrm{d}\log\boldsymbol{PD} - \boldsymbol{\Omega}^S\,\mathrm{d}\log\boldsymbol{PS}, \tag{41}$$

where $\boldsymbol{\Psi}$ is an inverse matrix of supply elasticities given by equation (36) and $\boldsymbol{\Omega}^S$ by equation (37).

Equations (41) and (39) represent two systems of equations in two vectors of unknowns ($\mathrm{d}\log\boldsymbol{X}$ and $\mathrm{d}\log\boldsymbol{PD}$)—aggregate industry-level demand and supply curves. I combine them to solve for the change in domestic price competitiveness $\mathrm{d}\log\boldsymbol{PD}$ as a function of arbitrary external shocks collected in $\mathrm{d}\log\boldsymbol{PS}$ and $\mathrm{d}\log\boldsymbol{S}$:

$$\begin{aligned}\mathrm{d}\log\boldsymbol{PD} = -\left(\mathbb{I} - \boldsymbol{\Omega}^S\mathrm{diag}(\lambda_u^M) + \boldsymbol{\Psi}\left(\mathbb{I} - \boldsymbol{\Omega}^D\right)^{-1}\ \mathrm{diag}(\lambda^{cpt})\right)^{-1} \times \\ \times\left(\boldsymbol{\Psi}\left(\mathbb{I} - \boldsymbol{\Omega}^D\right)^{-1}\ \mathrm{d}\log\boldsymbol{S} + \left(\boldsymbol{\Omega}^S + \boldsymbol{\Psi}\left(\mathbb{I} - \boldsymbol{\Omega}^D\right)^{-1}\mathrm{diag}(\lambda_u^X)\right)\mathrm{d}\log\boldsymbol{PS}\right),\end{aligned} \tag{42}$$

and likewise, the change in equilibrium output $\mathrm{d}\log\boldsymbol{X}$ can be solved for using equation (39) and the equation for $\mathrm{d}\log\boldsymbol{PD}$ above.

Lemma 3 follows the fact that the PPI is defined as $(1-\sigma_j)\,\mathrm{d}\log\mathcal{P}_j = \mathrm{d}\log PD_j$ and from setting $\mathrm{d}\log PS = 0$ in the above expression (so that the only exogenous shock is to market size). Proposition 3 is a special case of Lemma 3 when $\boldsymbol{\Omega}^S = \boldsymbol{\Omega}^D = 0$, i.e., there is no input-output structure in stage II of production.

D.3 Other Results in General Equilibrium

Proposition 3 describes how domestic producer prices $\mathrm{d}\log\boldsymbol{\mathcal{P}}$ respond to exogenous shifters of market size $\mathrm{d}\log\boldsymbol{S}$. This relationship depends on both demand-side and supply-side elasticities. I focus on the relationship between $\mathrm{d}\log\boldsymbol{\mathcal{P}}$ and $\mathrm{d}\log\boldsymbol{S}$ because it is useful for directly evaluating the impact of a range of counterfactual shocks. For example, market size shifters $\mathrm{d}\log\boldsymbol{S}$ include not only demand shocks such as changes in the labor force (a conventional scale shock), but also other shocks that shift the residual demand curve of the firm in an open economy, such as changes in foreign competitiveness.

I present two additional Corollaries of Proposition 3 that are of interest. For the sake of brevity I focus on the economy under autarky (and, in the case of Corollary 1, without input-output linkages), although the open-economy and input-output versions are straightforward to derive.

Corollary 1 focuses on the supply-side relationship and characterizes aggregate price-elasticities of supply. Suppose that there is an industry-wide composite good, Q_j, defined as a homothetic CES aggregator over individual quality-adjusted quantities q_{fj} provided by monopolistically competitive firms (who operate joint production functions as described in our model):

$$Q_j \equiv \left(N\int q_{fj}^{\frac{\sigma_j-1}{\sigma_j}}\ \mathrm{d}G(\xi)\right)^{\frac{\sigma_j}{\sigma_j-1}}, \quad \forall j \in \mathcal{J}.$$

The price index dual to this aggregator is the domestic PPI, $\mathcal{P}_j$ in each industry j.

With this representation I define aggregate economies of scale and scope in terms of own and cross-industry elasticities of prices $\boldsymbol{\mathcal{P}}$ with respect to composite quantities $\boldsymbol{Q}$. Locally, there are industry-level economies of scale if own-price elasticities are negative, and pairwise economies of scope (cost-complementarities) if cross-price elasticities are negative between j, k. To derive these partial-equilibrium supply-side elasticities I take equation (41) and replace $\mathrm{d}\log \boldsymbol{X} = \mathrm{d}\log \boldsymbol{Q} + \mathrm{d}\log \boldsymbol{\mathcal{P}}$. Rearranging terms yields the following result.

Corollary 1 (Aggregate Price Elasticities of Supply) *Under autarky without input-output linkages, the partial-equilibrium supply-side elasticity of prices to composite quantities is given by*

$$\mathrm{d}\log \boldsymbol{\mathcal{P}} = -\left(\boldsymbol{\Psi} - diag(\boldsymbol{\sigma} - 1)\right)^{-1} \boldsymbol{\Psi}\, \mathrm{d}\log \boldsymbol{Q}.$$

In general, off-diagonals in the joint production matrix $\boldsymbol{\Psi}$ generate non-zero cross-price elasticities. Note, however, that because $\boldsymbol{\Psi}$ appears twice and contain the inverse cross-industry matrix of sales responses to demand shocks for the average firm, $\boldsymbol{\Upsilon}$, the sign of pairwise industry responses within the firm (as measured in Section 1) is neither sufficient nor necessary for inferring economies of scope. In general equilibrium percolation effects across all industries need to be considered.

Under the special case of nonjoint production (i.e., $\rho_m = \theta_m$ for all shared inputs m), the off-diagonals of $\boldsymbol{\Psi}$ are zero and we recover certain well-known cases. When there are constant returns to scale in stage II production $\gamma_j = 1$, it is easy to check that as $\rho_m = \rho \to \infty$, we reach the limit where $\mathrm{d}\log \mathcal{P}_j = -\frac{1}{\sigma_j}\,\mathrm{d}\log Q_j$, so that (replacing Q_j with X_j/P_j) industry-level returns to scale reaches its maximum, $\mathrm{d}\log \mathcal{P}_j = -\frac{1}{\sigma_j - 1}\,\mathrm{d}\log X_j$. On the other hand as $\rho_m = \rho \to 1$, there are overall constant returns to scale over both stages I and II and so $\mathrm{d}\log \mathcal{P}_j = 0\,\mathrm{d}\log Q_j$.

Next, in Corollary 2 I show that productivity (TFP) shocks operate differently from demand shocks in a monopolistically competitive environment. Define TFP shocks as industry-wide shifts to the $\tilde{\xi}_{fj}$ terms in the production function (equation 1). Under the special case of autarky and Cobb-Douglas demand, Corollary 2 shows that joint production parameters do not affect the propagation of industry-wide cost shocks in the economy. Intuitively, firms are driven by profit incentives and industry-level changes in the cost structure do not affect profits at all in monopolistically competitive equilibrium. Cost savings are passed-through fully to the consumer, and due to unit-elastic industry-level demand there is no adjustment in industry-level expenditures. In the absence of input-output linkages (when $\boldsymbol{\Omega}^S = 0$) industry-level TFP shocks are contained within the industry of origin and do not propagate.

Corollary 2 (Propagation of Cost Shocks under Joint Production) *Under autarky, the general equilibrium elasticity of prices with respect to profitability shocks* $\mathrm{d}\log \tilde{\boldsymbol{\xi}}$ *in the firm's physical production function (in equation 1) is given by*

$$\mathrm{d}\log \boldsymbol{\mathcal{P}} = diag\left(\frac{1}{\boldsymbol{\sigma} - 1}\right)\left(\mathbb{I} - \boldsymbol{\Omega}^S\right)^{-1} diag\left(\boldsymbol{\sigma} - 1\right)\,\mathrm{d}\log \tilde{\boldsymbol{\xi}}.$$

Proof. The proof operates in similar fashion to the supply-side part of the proof of Lemma 3. Starting, again, with the supply-side equation (29) but this time accounting for industry-level changes in $\tilde{\xi}_{fj}$ yields

$$\boldsymbol{\Upsilon}^{-1}\,\mathrm{d}\log \boldsymbol{X} = \mathrm{d}\log \boldsymbol{B} + \mathrm{diag}\left(\frac{\boldsymbol{\sigma} - 1}{\boldsymbol{\sigma}(1 - \varsigma)}\right)\mathrm{d}\log \tilde{\boldsymbol{\xi}}.$$

Substituting in for $\mathrm{d}\log \boldsymbol{B}$ using equation (40) and rearranging terms and noting that $\mathrm{d}\log \boldsymbol{X} = 0$ on the demand side in autarky (unit-elastic industry demand curve) yields the result. ∎

A final remark is that Corollary 2 is close to the result in Hulten (1978) with the exception of wedges σ_j created by monopolistic competition (in the outer sandwich diagonal matrices and also $\sigma_j/(\sigma_k - 1)$ in the input-output matrix Ω). In the limit as $\sigma_j = \sigma \to \infty$, Hulten's theorem holds for evaluating the impact of industry-level TFP shocks. I leave the evaluation of the impact of firm-level TFP shocks to future work.

D.4 Calibration to US Manufacturing Sector

Notation: $\mathcal{D}^F = \{c, r\}$ refer to China and the rest-of-the-world composite respectively.

Data in the Initial Equilibrium. These equilibrium definitions allow me to impute consumption expenditure shares β_j, the manufacturing deficit D, all price competitiveness indices, and foreign expenditures $\overline{Y}_{c,j}, \overline{Y}_{r,j}$ given US and world trade and industry level data in 2017. I use the following publicly available data in 2017:

1. Data on gross output by manufacturing industry, X_j come from the BEA in 2017.
2. I hold the number of total manufacturing firms, N, fixed, at 318,000, the total number of firms (including inactive and administrative records) in the 1997 Census of Manufactures.
3. Data on β_{jk} and ς_k come from the 2012 BEA I/O tables (the 2017 tables are not yet available).[39]
4. Trade data in 2017 on US imports and exports by country and industry (after mapping HS10 to BEAX) come from the US Census Bureau (made available by Schott (2008)).[40]
5. World trade data in 2017 by industry and country come from BACI Comtrade.

Variables in the Model. Using the trade data, I compute λ^X_{dj} as the share of US firms' total sales in industry j going to destination $d \in \{u, r, c\}$, and λ^M_{dj} as the share of consumption in destination $d \in \{u, r, c\}$'s in industry j on goods sold by the US. I express all the ratios of price competitiveness in Definition 1 as functions of these observable trade shares. I compute industry gross expenditures as $Y_j = \frac{X_j \lambda^X_{uj}}{\lambda^M_{uj}}$.

Using the estimated micro parameters, I repeat the same macro inversion steps as in the structural estimation to estimate macro variables $\boldsymbol{\alpha}, \boldsymbol{B}, Z$ in 2017. I use 1997 expenditure shares on knowledge inputs categories $m \in \mathcal{M}$ by each industry j combined with 2017 output data to impute expenses on knowledge inputs M_{jm} used in the inversion. With these macro variables on hand I compute net profits in the manufacturing sector Π integrating equation (17) over $G(\boldsymbol{\xi})$.

I compute the manufacturing deficit as the difference between total consumption and total output: $D = \sum_{j \in \mathcal{J}} Y_j - \sum_{j \in \mathcal{J}} X_j$. I normalize the wage w to 1 by choosing an appropriate unit in which to measure efficiency-adjusted labor, so that

$$L = GDP - \Pi,$$

where GDP is 19.4 trillion in 2017. The share of consumption on non-manufacturing is then given by:

$$1 - \beta_{F,NM} = \frac{\sum_{j \in \mathcal{J}} \left(Y_j - \sum_k \beta_{kj} X_k \right)}{L}.$$

I compute final consumption shares in manufacturing, $\beta_{F,k}$, as:

$$\beta_{F,k} = \frac{Y_k - \sum_j \beta_{kj} X_k}{L}.$$

Foreign demand in the model is given by $\overline{Y}_{r,j} \lambda^M_{rj} = EX_{urj}$ where EX_{urj} is US exports to destination r in industry j. An identical expression pins down $\overline{Y}_{c,j}$.

[39] There are a few industries where implied input-output use shares are so large that final use is predicted to be negative. I adjust input-output shares downward by a proportional factor for that industry until final use is at least 2% of gross consumption.

[40] There are a few industries where US exports is higher than measures of gross output in BEA data. I harmonize the two data sources by adjusting gross output, X_j, to be at least 1% higher than gross exports.

D.5 Quantifying the Impact of Joint Production

I use the calibrated model and Lemma 3 to quantify the impact of small shocks on equilibrium producer price indices $\boldsymbol{\mathcal{P}}$ and output $\boldsymbol{X}$ across industries. Changes in export demand in each industry j can be represented as $\mathrm{d}\log S_j = \sum_{d\in\mathcal{D}^F} \lambda^X_{dj} \mathrm{d}\log \overline{Y}_{dj}$. I compute the impact of a uniform one percent change in export market demand (so $\mathrm{d}\log \overline{Y}_{dj} = 0.01\ \forall d \in \mathcal{D}^F, \forall j$ and thus $\mathrm{d}\log S_j = 0.01(1-\lambda^X_{uj})$) on prices as:

$$\mathrm{d}\log \boldsymbol{\mathcal{P}} = \Xi\, \mathrm{d}\log \boldsymbol{S},$$

where Ξ is a transmission matrix defined below:

$$\Xi = \mathrm{diag}\left(\frac{1}{\sigma-1}\right)\left(\mathbb{I} - \boldsymbol{\Omega}^S \mathrm{diag}(\boldsymbol{\lambda}^M_u) + \boldsymbol{\Psi}\left(\mathbb{I}-\boldsymbol{\Omega}^D\right)^{-1}\ \mathrm{diag}(\boldsymbol{\lambda}^{cpt})\right)^{-1} \boldsymbol{\Psi}\left(\mathbb{I}-\boldsymbol{\Omega}^D\right)^{-1}. \tag{43}$$

To compute the effect on industry gross output $\mathrm{d}\log \boldsymbol{X}$, I solve out for the demand and supply equations (41) and (39) to express $\mathrm{d}\log \boldsymbol{X}$ also in terms of exogenous shocks $\mathrm{d}\log \boldsymbol{S}$.

I define the aggregate scale elasticity as the change in the overall manufacturing PPI with respect to total manufacturing output in response to a specified set of industry demand shocks $\mathrm{d}\log \boldsymbol{S}$. Overall manufacturing sector aggregates (PPI, and output) are simple the sales-share-weighted ($\boldsymbol{\lambda}^X$) average over industries j. Note that since industries differ in economies of scale and scope, the overall scale elasticity depends on compositional make-up of different industries in the economy as well as the levels of shocks received in each industry $\mathrm{d}\log S_j$. Different combinations of shocks would induce different "scale elasticities'.

Under the benchmark scenario (no input-output linkages), cross-price elasticities (off-diagonal elements of the transmission matrix Ξ) are due solely to economies of scope within the firm (i.e., from non-zero off-diagonal elements of $\boldsymbol{\Psi}$). This enables the following decomposition:

$$\begin{aligned}\left.\frac{\mathrm{d}\log PPI_{manuf}}{\mathrm{d}\log X_{manuf}}\right|_{\mathrm{d}\log \boldsymbol{S}} &\equiv \left.\frac{\sum_{j\in\mathcal{J}} \lambda^X_j \mathrm{d}\log \mathcal{P}_j}{\sum_{j\in\mathcal{J}} \lambda^X_j \mathrm{d}\log X_j}\right|_{\mathrm{d}\log \boldsymbol{S}} \\ &= \underbrace{\left.\frac{(\boldsymbol{\lambda}^X)' \times \Xi^{DIAG} \times \mathrm{d}\log \boldsymbol{S}}{\mathrm{d}\log X_{manuf}}\right|_{\mathrm{d}\log \boldsymbol{S}}}_{\text{economies of scale}_k} + \underbrace{\left.\frac{(\boldsymbol{\lambda}^X)' \times \Xi^{OFF-DIAG} \times \mathrm{d}\log \boldsymbol{S}}{\mathrm{d}\log X_{manuf}}\right|_{\mathrm{d}\log \boldsymbol{S}}}_{\text{economies of scope}_k},\end{aligned} \tag{44}$$

where Ξ^{DIAG} and $\Xi^{OFF-DIAG}$ are main and off-diagonal elements of the matrix Ξ, respectively:

$$\Xi = \Xi^{DIAG} + \Xi^{OFF-DIAG}.$$

When there are input-output linkages, the above decomposition no longer works since demand shocks can trigger same-industry economies of scale in upstream industries, and these price changes can in turn affect costs of other downstream industries. These cross-industry effects are embedded in Ξ and can occur even when the joint production matrix $\boldsymbol{\Psi}$ is populated with only main-diagonals as long as input-output matrices, Ω^D, Ω^S are non-zero. I therefore attribute the role of economies of scope to the *additional* price change generated by off-diagonal elements of joint production $\boldsymbol{\Psi}$, relative to a case where the transmission matrix is computed only with main-diagonals of $\mathbf{\Psi}$.

$$\left.\frac{\mathrm{d}\log PPI_{manuf}}{\mathrm{d}\log X_{manuf}}\right|_{\mathrm{d}\log \boldsymbol{S}}^{(\mathrm{IO})} \equiv \left.\frac{\sum_{j\in\mathcal{J}} \lambda_j^X \,\mathrm{d}\log \mathcal{P}_j}{\sum_{j\in\mathcal{J}} \lambda_j^X \,\mathrm{d}\log X_j}\right|_{\mathrm{d}\log \boldsymbol{S}}^{(\mathrm{IO})}$$
$$= \underbrace{\left.\frac{(\boldsymbol{\lambda}^X)' \times \boldsymbol{\Xi}^{(\mathrm{IO,\,no\,scope})} \times \mathrm{d}\log \boldsymbol{S}}{\mathrm{d}\log X_{manuf}}\right|_{\mathrm{d}\log \boldsymbol{S}}^{(\mathrm{IO})}}_{\text{economies of scale}_k} + \underbrace{\left.\frac{(\boldsymbol{\lambda}^X)' \times (\boldsymbol{\Xi}^{\mathrm{IO,\,scope}} - \boldsymbol{\Xi}^{\mathrm{IO,\,no\,scope}}) \times \mathrm{d}\log \boldsymbol{S}}{\mathrm{d}\log X_{manuf}}\right|_{\mathrm{d}\log \boldsymbol{S}}^{(\mathrm{IO})}}_{\text{economies of scope}_k}, \tag{45}$$

where $\boldsymbol{\Xi}^{(\mathrm{IO,\,no\,scope})}$ and $\boldsymbol{\Xi}^{(\mathrm{IO,\,scope})}$ are computed from equation (43) using main diagonal elements of joint production matrix $\mathbf{\Psi}^{DIAG}$ (which shuts down economies of scope) and all elements of $\mathbf{\Psi}$, respectively. And finally, $\mathrm{d}\log \boldsymbol{X}$ is computed under the full IO linkage and joint production transmission matrix by solving out for the demand and supply equations (41) and (39) to express $\mathrm{d}\log \boldsymbol{X}$ also in terms of exogenous shocks $\mathrm{d}\log \boldsymbol{S}$.

Calibration of Demand Elasticities σ. The transmission matrix $\boldsymbol{\Xi}$ depends on values of demand elasticities $\boldsymbol{\sigma}$. I calibrate σ_j to exactly match estimates of sector-level increasing returns from Bartelme et al. (2019). I first generate a concordance to map each of my 206 BEAX industry codes j to a two-digit manufacturing sector in Table 1 of Bartelme et al. (2019), denoted by s. I assume σ_j is the same across all industries j within a sector s but potentially different across sectors s. I then solve for the value of σ_s that satisfy, for each BCDR sector s:

$$\frac{\mathrm{d}\log PPI_s}{\mathrm{d}\log X_s} = -\gamma_s^{BCDR} = \left.\frac{\sum_{j\in s} \lambda_j^X \sum_{k\in s} \Xi_{jk}\,\mathrm{d}\log S_k}{\sum_{j\in s} \lambda_j^X \,\mathrm{d}\log X_j}\right|_{\mathrm{d}\log \boldsymbol{S}}, \qquad \forall s,$$

where γ_s^{BCDR} are estimates of scale elasticities in Table 1 of Bartelme et al. (2019) and Ξ_{jk} corresponds to jk element of the transmission matrix under the no input-output linkage benchmark. The estimates correspond to using the industry-level export demand shocks $\mathrm{d}\log \boldsymbol{S}$ as instruments in a sector-by-sector s regression of the change in equilibrium producer prices on the change in equilibrium output.[41] Table D.7 shows the calibrated estimates of σ_j by across BCDR (3-digit) sectors.

I consider two alternative calibration strategies for $\boldsymbol{\sigma}$. First, I consider different common values of $\sigma_j = \sigma$ across all industries. Setting σ to a common value across industries shows that the asymmetric cross-industry linkages obtained are not dependent on the demand elasticities being different across industries. Second, I allow σ_j to vary across industries j by assuming that profit shares (gross operating profits) in each industry are equal to $\frac{1}{\sigma_j}$ (as would be true in a case with constant returns to scale, monopolistically competitive firms, and sunk entry costs paid in some pre-period). While this assumption is ad-hoc (and not consistent with the model), the values of σ_j nevertheless serve as a useful benchmark as they are used in other papers.

Decomposition of the Aggregate Scale Elasticity. Table D.8 computes the impact of export demand shocks on the US manufacturing PPI and output and computes the fraction of the response attributable to

[41]Note that this procedure does (correctly) attribute the cross-industry impacts *within* a sector to sectoral economies of scale. The only PPI impacts that would be missed in BCDR occur across sectors s.

Table D.7: Calibrated Demand Elasticities σ_j

BCDR sector s	BEAX industries $j \in s$ (sector roots)	BCDR scale elasticity	σ under Joint Production
Food, Beverage, Tobacco	311, 312	0.16	5.7
Textiles	313, 314, 316	0.12	6.6
Wood Products	321	0.11	7.0
Paper Products	322, 323	0.11	6.5
Coke Petroleum	324	0.07	10.7
Chemicals	325	0.20	4.5
Rubber and Plastics	326	0.25	4.1
Mineral Products	327	0.10	6.1
Basic Metals	331	0.11	5.7
Fabricated Metals	332	0.13	5.8
Mach and Equipment	333	0.13	6.0
Computers	334	0.09	6.0
Electrical Machinery	335	0.09	7.4
Transport	336	0.15	6.1
All Other	337, 339	0.13	5.7

Notes: This table depicts the values of σ_j assigned to each BCDR sector in my baseline quantitative results. For example, industry 311930 and 311221 are both assigned $\sigma_j = 5.7$ as they belong to the same BCDR sector Food, Beverage, and Tobacco. These values of σ_j are calibrated so my joint production model yields the exact same within-sector increasing returns to scale as Bartelme et al. (2019). Under joint production own-sector scale elasticities are comprised of both within-industry and across-industry price declines among all industries that fall within each given BCDR sector.

economies of scope (cross-industry elasticities). The first column corresponds to the results in Figure 5 under the benchmark assumptions of no input-output linkages and when $\boldsymbol{\sigma}$ is calibrated to match estimates of sector-level increasing-returns to scale as BCDR. The next two columns reproduces these aggregate changes in the PPI and output under the two alternative calibration strategies for $\boldsymbol{\sigma}$.

The six rows of Table D.8 correspond to various components of equation (44): the first row computes the change in overall manufacturing PPI, the numerator $\mathrm{d}\log PPI_{manuf}$, and the second and third rows break this out into the change in prices attributable to main-diagonals of the transmission matrix (economies of scope) versus off-diagonals (economies of scope), the two respective numerators in the second line of equation (44). The fourth row computes the overall change in manufacturing output, the denominator $\mathrm{d}\log X_{manuf}$. The fifth row computes the scale elasticity as the change in overall prices divided by the change in overall output. Finally, the sixth row isolates the contribution of economies of scope to the scale elasticity, measured as the change in the PPI attributable to economies of scope divided by the change in overall output (the second fraction in the second line of equation (44).

The different calibrations of $\boldsymbol{\sigma}$ do not alter the main quantitative message that cross-industry price impacts due to joint production are large. Economies of scope cause aggregate prices to fall with output with an elasticity of about -0.04. Across the three calibration strategies, economies of scope account for between 20 and 22 percent of aggregate increasing returns. Figure D.1 further shows that this result is not sensitive to using any common value of σ between 3 and 10. Interestingly, whereas the contribution of economies of scale does depend on σ (due to profit motives under monopolistic competition), the contribution of economies of scope is steady at between -0.03 and -0.05.

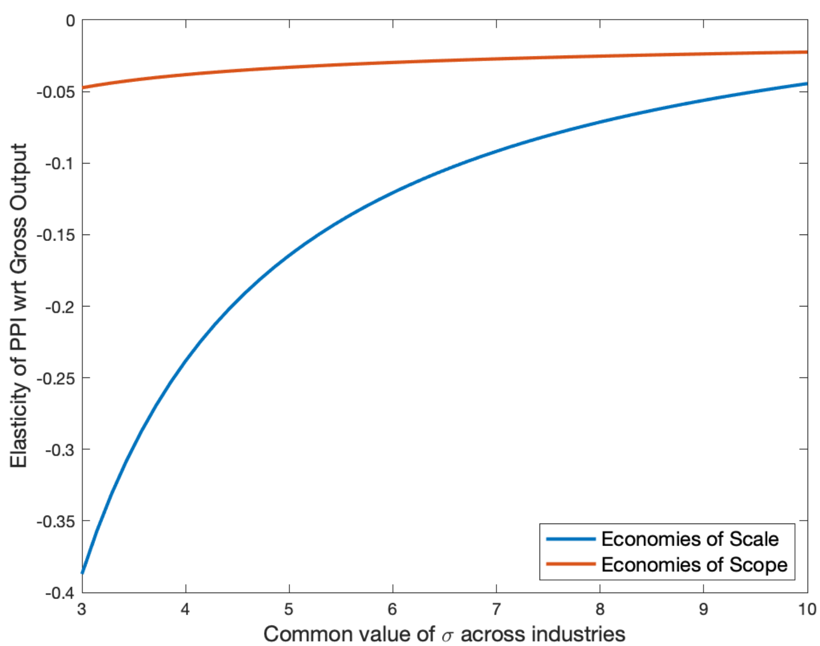
The next set of three columns in Table D.8 show how these results are affected by input-output linkages in stage-II of production. A demand shock in one industry now generates higher demand for its upstream input-supplying industries. Because of industry-level economies of scale, prices change in both directly-affected and upstream industries, invoking larger changes in aggregate output as well as prices. Both input-output linkages and economies of scope are mechanisms that transmit demand shocks to other

Table D.8: Effect of a 1% Increase in Foreign Demand on US Manufacturing PPI and Output

	No input-output (benchmark)			Input output linkages		
Calibration strategy for $\boldsymbol{\sigma}$:	BCDR	$\sigma = 5$	Profit	BCDR	$\sigma = 5$	Profit
Total Change in PPI (%)	-0.059	-0.077	-0.077	-1.198	-1.718	-0.527
- due to economies of scale	-0.046	-0.062	-0.062	-0.606	-0.820	-0.317
- due to economies of scope	-0.013	-0.015	-0.015	-0.593	-0.897	-0.210
Total Change in Output (%)	0.363	0.383	0.370	3.662	4.692	2.005
Aggregate scale elasticity	-0.163	-0.202	-0.207	-0.327	-0.366	-0.263
- due to economies of scope	-0.036	-0.040	-0.040	-0.162	-0.191	-0.105

Notes: This table depicts the change in manufacturing PPI and output in the US due to a proportional 1% foreign demand shock across all industries. The sub-rows decompose the total impact into those accruing due to economies of scale versus scope. The last two rows computes the aggregate scale elasticity (the change in PPI per change in output) and the amount attributable to economies of scope (see equations 44 and 45 for the decomposition). The six columns depict results obtained under three different calibrations of the demand elasticity $\boldsymbol{\sigma}$ under two assumptions on the input-output structure of the economy. 'BCDR' refers to the benchmark strategy of calibrating $\boldsymbol{\sigma}$ so that sector-level returns to scale match those in Bartelme et al. (2019). '$\sigma = 5$' refers to using a common value of 5 for all demand elasticities σ_j. 'Profit' refers to picking σ_j so that CES-implied profit shares match those in the national accounts. Over the rows of each column I compute the change in PPI and output due to a proportional 1% foreign demand shock (across all industries).

Figure D.1: Decomposition of the aggregate scale elasticity under alternative common values of σ



Notes: This graph reports the sensitivity of the decomposition in Figure 5 and Appendix Table reftbl:spillovers to alternative common values of σ across industries.

industries, and it is hard to do a decomposition given how they interact. In the second row of the table, I follow equation (45) and isolate the role of industry-level economies of scale by shutting down off-diagonal elements of the joint production matrix. In the next row, I compute the role of economies of scope as the difference between the overall PPI change and this halfway-scenario with only economies of scale. In a world with input-output linkages, more than half of the aggregate scale elasticity can be attributed to the presence of economies of scope.

Finally, Table D.9 presents more detailed industry-level estimates behind Figure 6, when foreign demand shocks occur industry-by-industry rather than manufacturing sector-wide. A shock to any industry j induces an aggregate scale elasticity—a change in manufacturing prices divided by manufacturing output. The first two columns decompose the aggregate scale elasticity into the contribution of scale (same-industry price changes) and scope (cross-industry price changes), respectively. The third column shows the industry's knowledge intensity—its expenditures on inputs from the knowledge sector as a share of gross output. Table D.9 summarizes this information for a few select industries: the top twenty and bottom twenty industries in terms of their contribution to aggregate economies of scope.[42]

D.6 Counterfactuals: The Impact of Large Shocks

I evaluate the impact of new ad-valorem tariffs imposed by the US on imports from China, denoted by τ_{cuj}, as well as retaliatory tariffs imposed by China on imports from the US, denoted by τ_{ucj}, given the economy in Definition 1. To avoid introducing new notation (τ), I first express tariffs in terms of equivalent shocks to exogenous price and demand variables under Definition 1:

1. (US Tariffs): The change in Chinese price competitiveness in the US is $\hat{\overline{PM}}_{cj} = \tau_{cuj}^{1-\sigma_j}$.
2. (US Tariffs): Government revenues are given by

$$T' = \sum_j \frac{\tau_{cuj} - 1}{\tau_{cuj}} \tau_{cuj}^{1-\sigma_j} \lambda_{cj}^{UM} \hat{P}_j^{\sigma_j - 1}.$$

3. (Chinese Tariffs): The change in US price competitiveness in China can be modeled as $\hat{\overline{PX}}_{cj} = \tau_{ucj}^{\sigma_j - 1}$.
4. (Chinese Tariffs): US firms' take-home revenues fall to $\frac{1}{\tau_{ucj}}$ of tax-inclusive sales. This can be reflected by a change in $\hat{\overline{Y}}_{cj} = \tau_{ucj}^{-1}$.

I assume that pre-existing US tariffs on Chinese imports are zero. If they are non-zero, the new tariffs change infra-marginal tariff revenues and a slight modification to equilibrium conditions is required. I also assume that all tariff revenues are spent on the Chinese non-manufacturing sector and thus do not go towards increasing market demand in the US (Y_j) or China ($\overline{Y}_{c,j}$).

For any set of shocks to the model's exogenous variables (to e.g., $\hat{\overline{PM}}_{cj}$, $\hat{\overline{PX}}_{cj}$, $\overline{Y}_{c,j}$), the system of equations in Definition 1 can be solved for changes to endogenous variables $\hat{PD}_j$ and $\hat{D}$ using exact hat algebra.[43]

[42]Note that the top industries listed do not indicate the industries where demand shocks generate the highest total PPI change, nor industries where the total elasticity of PPI to gross output is highest. The top five industries by total aggregate returns to scale (greatest elasticity in the overall PPI with respect to change in output) are aircraft manufacturing, petroleum refineries, other motor vehicle parts, light truck and utility vehicles, and broadcast and wireless communications equipment. These results as well as the full ranking of industries are available upon request.

[43]Recall that under the assumption of perfectly elastic labor across manufacturing and non-manufacturing and perfectly elastic foreign demand for non-manufacturing, wages are fixed as long as the US remains a net exporter in

Table D.9: List of top and bottom industries by level of economies of scope

Industry	Description	(1)	(2)	(3)
	Statistics by column: (1) Knowledge intensity, (2) Economies of scale, (3) Economies of scope			
Top Twenty Industries:				
334300	Audio and video equipment manufacturing	0.08	-0.07	-0.17
334118	Computer terminals and other computer peripheral equipment manufacturing	0.13	-0.08	-0.14
339910	Jewelry and silverware manufacturing	0.07	-0.09	-0.13
33461X	Manufacturing and reproducing magnetic and optical media	0.07	-0.01	-0.10
334514	Totalizing fluid meter and counting device manufacturing	0.08	-0.02	-0.10
33141X	Nonferrous Metal (except Aluminum) Smelting and Refining	0.03	-0.08	-0.09
336991	Motorcycle, bicycle, and parts manufacturing	0.11	-0.09	-0.09
333242	Semiconductor machinery manufacturing	0.08	-0.09	-0.09
33641A	Propulsion units and parts for space vehicles and guided missiles	0.11	-0.06	-0.09
33451B	Watch, clock, and other measuring and controlling device manufacturing	0.11	-0.06	-0.08
334510	Electromedical and electrotherapeutic apparatus manufacturing	0.09	-0.05	-0.08
336992	Military armored vehicle, tank, and tank component manufacturing	0.10	-0.10	-0.08
334516	Analytical laboratory instrument manufacturing	0.10	-0.05	-0.07
334111	Electronic computer manufacturing	0.16	-0.11	-0.07
339115	Ophthalmic goods manufacturing	0.14	-0.06	-0.07
334517	Irradiation apparatus manufacturing	0.07	-0.06	-0.07
33521X	Small electrical appliance manufacturing	0.11	-0.07	-0.06
335221	Household cooking appliance manufacturing	0.09	-0.05	-0.06
335999	All other miscellaneous electrical equipment and component manufacturing	0.11	-0.08	-0.06
339114	Dental equipment and supplies manufacturing	0.11	-0.09	-0.06
Bottom Twenty Industries:				
3212XX	Veneer, plywood, and engineered wood product manufacturing	0.03	-0.11	-0.01
33131B	Aluminum product manufacturing from purchased aluminum	0.04	-0.14	-0.01
3211XX	Sawmills and wood preservation	0.03	-0.12	-0.01
335314	Relay and industrial control manufacturing	0.10	-0.14	-0.01
312120	Breweries	0.06	-0.09	-0.01
31161A	Animal (except poultry) slaughtering, rendering, and processing	0.04	-0.19	-0.01
337110	Wood kitchen cabinet and countertop manufacturing	0.05	-0.16	-0.01
336111	Light truck and utility vehicle manufacturing	0.05	-0.15	-0.01
324110	Petroleum refineries	0.05	-0.07	-0.01
311920	Coffee and tea manufacturing	0.04	-0.14	-0.01
331313	Alumina refining and primary aluminum production	0.04	-0.10	-0.01
314110	Carpet and rug mills	0.04	-0.13	-0.01
326160	Plastics bottle manufacturing	0.04	-0.25	-0.01
311514	Dry, condensed, and evaporated dairy product manufacturing	0.05	-0.15	-0.01
337215	Showcase, partition, shelving, and locker manufacturing	0.05	-0.17	-0.01
312140	Distilleries	0.05	-0.04	-0.01
311221	Wet corn milling	0.03	-0.17	-0.01
327992	Ground or treated mineral and earth manufacturing	0.04	-0.06	0.00
33142X	Copper rolling, drawing, extruding and alloying	0.03	-0.17	0.00
311930	Flavoring syrup and concentrate manufacturing	0.03	-0.11	0.01

Notes: This table presents estimates of knowledge intensity and the strength of economies of scale and scope induced by demand shocks to select industries (top and bottom twenty in terms of the amount of economies of scope—column (3)). Column (1) displays the industry's knowledge intensity (expenditures on knowledge inputs as a share of gross output). Column (2) computes the same-industry component of the manufacturing PPI change divided by the total change in manufacturing output from a demand shock to that industry. Column (3) computes the cross-industry component of the manufacturing PPI change divided by the total change in manufacturing output from a demand shock to that industry. These effects are computed using equation (24); for these twenty select industries, column (1) corresponds to the x-axis in Figure 6a, column (2) corresponds to the x-axis in Figure 6b, and column (3) corresponds to y-axis values in the scatterplot in Figure 6.

Specifically, for any guess of $\hat{PD}_j$ and $\hat{D}$, I can compute

$$\hat{B}_j = \hat{c}_j^{\frac{\varsigma_j}{\varsigma_j - 1}} \left(\frac{\hat{X}_j}{\hat{PD}_j} \right)^{\frac{1}{\sigma_j(1-\varsigma_j)}},$$

where $\hat{c}_j$ is given by

$$\hat{c}_j = \prod_{k \in J} \hat{P}_k^{\tilde{\beta}_{jk}},$$

and $\hat{P}_j$ is the change in the domestic consumption price index given by

$$\hat{P}_j^{1-\sigma_j} = \hat{PD}_j \lambda_{uj}^{UM} + \hat{\overline{PM}}_{cj} \lambda_{cj}^{UM} + \hat{\overline{PM}}_{rj} \lambda_{rj}^{UM},$$

and $\hat{X}_j$ is given by

$$\hat{X}_j X_j = Y_j' \hat{PD}_j \lambda_{u,j}^{M} \hat{P}_j^{\sigma_j - 1} + \overline{Y}_{c,j} \hat{\overline{Y}}_{c,j} \hat{PD}_j \lambda_{cj}^{M} \hat{P}_{chn,j}^{\sigma_j - 1} + \overline{Y}_{r,j} \hat{\overline{Y}}_{r,j} \hat{PD}_j \lambda_{rj}^{M} \hat{P}_{row,j}^{\sigma_j - 1},$$

and $\hat{P}_{row,j}$ is the change in the rest-of-world consumption price index given by

$$\hat{P}_{row,j}^{1-\sigma_j} = \hat{PD}_j \lambda_{rj}^{M} + \hat{\overline{PX}}_{rj} (1 - \lambda_{rj}^{M}),$$

$\hat{P}_{chn,j}$ is the change in the consumption price index in China given by

$$\hat{P}_{chn,j}^{1-\sigma_j} = \hat{PD}_j \lambda_{cj}^{M} + \hat{\overline{PX}}_{cj} (1 - \lambda_{cj}^{M}),$$

and finally the new vector of gross expenditures Y_j' can be inverted from

$$Y_j' = \sum_k \beta_{kj} \left(\hat{X}_k X_k \right) + \beta_{F,j} L \hat{L},$$

where T' is tariff revenues defined above.

To evaluate the guess I use a system of $|\mathcal{J}|$ equations equal to deviations between industry sales as computed above, X_j', and the implied industry sales (by solving the firm's problem) given by equation (29) under the new B_j'. I also use the trade balance condition:

$$\sum_j Y_j' = \hat{D} D + \sum_j \hat{X}_j X_j,$$

to pin down $\hat{D}$. I find that standard gradient based optimization algorithms work very well for convergence in this system of equations.

Effects of the US-China Trade War. I consider two counterfactual scenarios and report the impact of these shocks in Table 6. First, I consider unilateral US import tariffs on Chinese goods. Next, I consider the net effect after retaliatory tariffs imposed by China on US goods. Tariffs are industry-level τ_{ucj} and τ_{cuj} obtained from aggregating HS-line data from Fajgelbaum et al. (2019) to the industry level using trade values as weights. I solve for the change in endogenous variables $\hat{PD}_j$ and $\hat{D}$ using the system of equations

non-manufacturing.

above. Table 6 computes the change in various aggregate variables of interest:

1. The change in the US manufacturing CPI is

$$\prod_j \hat{P}_j^{\beta_{F,j}}$$

2. The change in the US manufacturing PPI is

$$\prod_j \hat{PD}_j^{\frac{\lambda_j^X}{1-\sigma_j}},$$

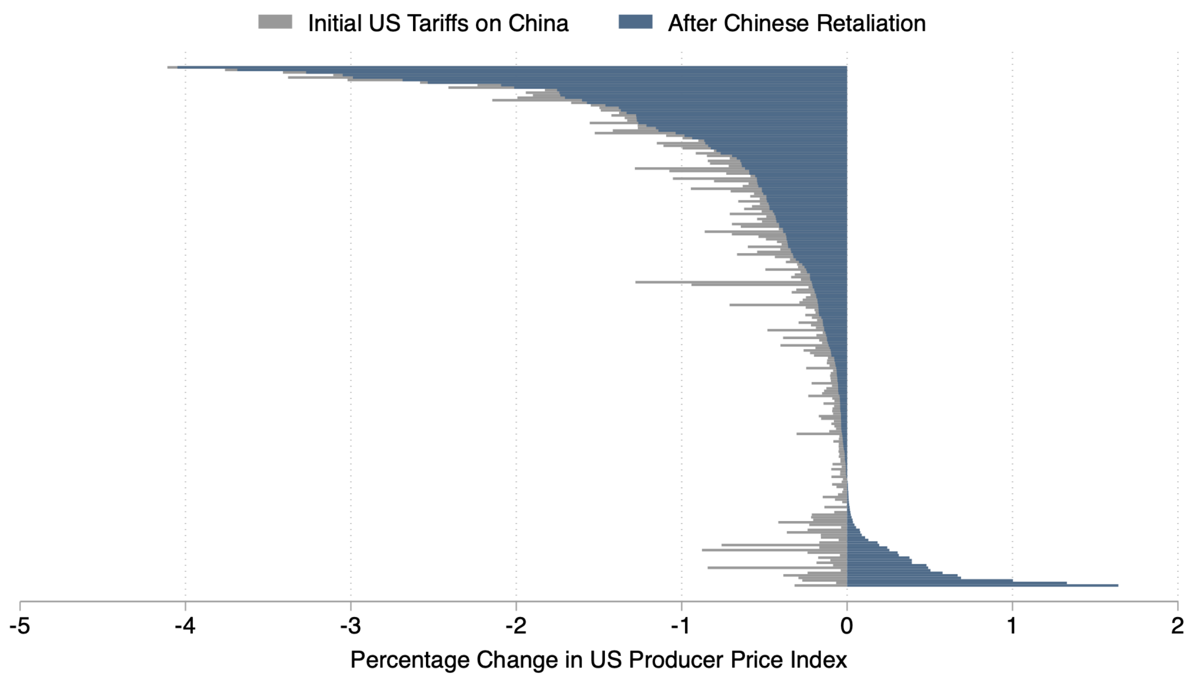
3. Expressions for the change in imports, US output and US exports in each industry, tariff revenues and the deficit can also be computed directly given the equations above.

In Table 6 I also re-compute these impacts under a different linear production assumption, which corresponds to setting $\hat{PD} = 1$ in the above system of equations. Since wages are pinned down and there are no input-output linkages under this benchmark scenario, producer prices do not adjust to tariffs.

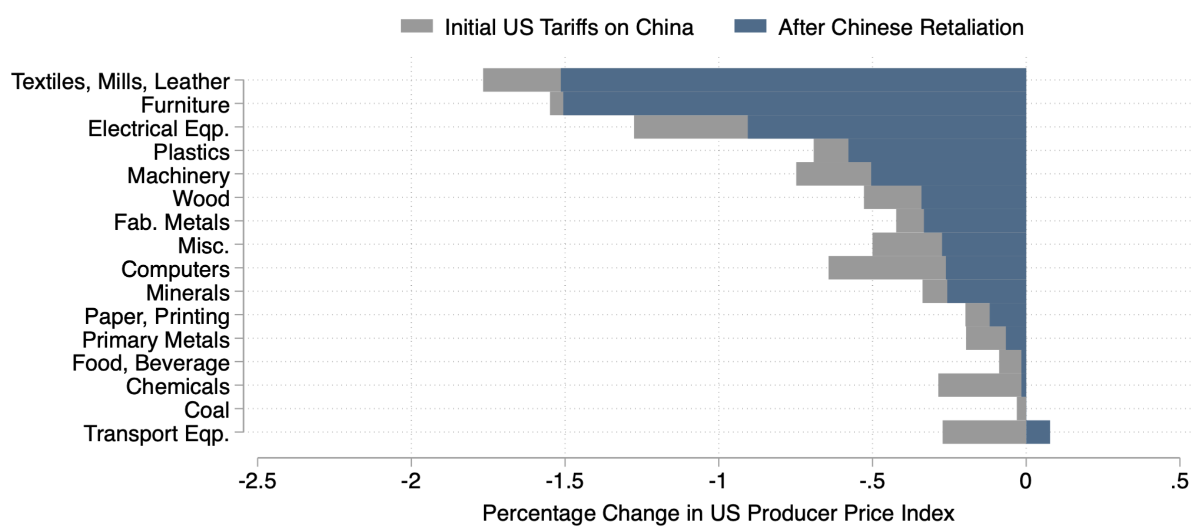
Figure D.2 graphs the model's predicted producer-price changes by industry (in panel a) and by broad sector (in panel b), constituting the disaggregated numbers that make up the aggregate PPI change reported in Table 6 (under joint production). The graph sorts industries and sectors in order of their predicted total price change net of Chinese retaliation (the navy bar). Also shown is the intermediate counterfactual scenario (in gray) where only US tariffs on Chinese imports are considered.

Figure D.2: Impact of the US-China Trade War on US Manufacturing PPI

(a) By Industry



(b) By Sector



Notes: This table presents estimates of the impact of US-China bilateral import tariffs on US manufacturing PPI under the joint production model calibrated to match US industry-level aggregates in 2017. Shown here are the model's industry-level predictions aggregated (using sales-weights) to the level of NAICS 3-digit manufacturing sub-sectors. The gray bar illustrates the impact of unilateral US tariffs on imports from China. The navy bar illustrates the full impact after Chinese tariffs on imports from the US. See Definition 1 for a characterization of the equilibrium and Appendix D.6 for the exact hat system of equations used to solve for model responses after the shock.