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FINANCING COSTS AND DEVELOPMENT

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Abstract

Most aggregate theories of financial frictions model credit available at a cost of financing equal to the savings rate but rationed. However, using a comprehensive loan-level credit registry, we document both high levels and high dispersion in default-adjusted credit spreads to Brazilian firms. We develop a quantitative dynamic general equilibrium model in which spreads arise from intermediation costs and market power. Calibrating to the Brazilian data, we show that, for equivalent levels of external financing, spreads have profound impacts on aggregate development—indeed more so than credit rationing does—and spreads yield firm dynamics that are consistent with observed patterns.

Keywords: Financial frictions, Credit spreads, Aggregate misallocation

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1 Introduction

The credit market features a sizable gap between lending and deposit rates, and these spreads are larger in poorer countries. According to the International Financial Statistics, the average interest rate spread is approximately 0.7 percent in Japan, 3 percent in the United States, 10 percent in Uruguay and 40 percent in Brazil, and these spreads can vary considerably across borrowers.¹ Such empirical work examines small subsets of the credit market, however, so that the relevance of spreads for the macroeconomy is less clear. Moreover, the quantitative literature assessing the role of financial frictions on development has focused mainly on credit rationing at a fixed zero-spread interest rate. This paper addresses the role of financing spreads on economic development and firm dynamics, showing their importance both empirically and quantitatively.

Empirically, we focus on Brazil because of the availability of high-quality data that are especially useful for our purposes. We use the Brazilian credit registry, a confidential loan-level dataset covering all credit operations in Brazil from January 2006 to December 2016 and containing information on loan characteristics and interest rates. We merge these data with Brazil's linked employer-employee administrative dataset to examine how interest rates and loan size vary with firm characteristics. Even after controlling for a host of firm and loan characteristics, loan interest rates (and indeed *ex post* or default-adjusted interest rates) are strikingly high, vary substantially across loan type, and vary with firm size and age. In particular, young and small firms pay higher interest rates. For instance, on average a firm with three employees pays an *ex post* spread above 75 percentage points, as does the average new firm. Firms that are 10 years old pay spreads roughly 10 percentage points lower, while firms with 100 employee on average pay spreads roughly 20 percentage points less. These lower average spreads still exceed 65 and 55 percentage points, respectively.

Quantitatively, we introduce financing spreads into a standard model of credit-constrained entrepreneurs and demonstrate their important impact on entrepreneurship, firm dynamics, and economic development, even relative to hard quantity constraints. We start with a continuous-time general equilibrium model with heterogeneous agents and occupational choice. Agents are heterogeneous in their stochastic managerial ability and, at any instant, choose whether to be a worker or an entrepreneur. Following the existing literature, entrepreneurs can acquire capital from intermediaries but face a quantity limit, which can distort a firm's decision away from its optimal level of capital.

We innovate by introducing price-driven distortions in the form of interest rate spreads on externally financed capital. Like quantity constraints, spreads in interest rates distort capital, but

¹Micro empirical studies report a high variability in the interest rate charged by lenders for similar loan transactions within the same economy. See Banerjee (2003), Banerjee and Duflo (2010), for example. Gilchrist, Sim, and Zakrajsek (2013) provide similar evidence for the United States.

they also distort retained income conditional on capital. These interest rate spreads stem from two sources: intermediation costs that decline with productivity and assets and intermediary market power. The productivity-dependent intermediation costs generates fixed variation across firms, while the asset-dependent costs yield life cycle variation as firms grow. Intermediary market power arises from Nash bargaining between intermediaries and firms. This bargaining creates a dependence of interest rates on ability and wealth through firms' outside option and the surplus of the transaction. With market power, *ceteris paribus*, high-productivity firms generate larger surplus and therefore pay *higher* interest rates. Also, distinct from intermediation costs, intermediary market power distorts financing costs without necessarily distorting the quantity. Taken together, financial frictions help jointly determine the loan size and the interest rate.

We calibrate the model to match key characteristics of the Brazilian economy, including standard macro aggregates as well as firm characteristics and credit market moments based on our micro-level datasets. Both unweighted and credit-weighted average spreads are important moments because they discipline not only the costs of credit that firms face but also the extent to which firms borrow at high financing costs. Specifically, the fact that large loans are associated with low interest rates implies little presence of market power driving spreads. Since market power leads to higher interest rates for larger loans, they do not distort loan sizes but simply transfer rents to the lender. Instead, the calibration attributes most of the high spreads to intermediation costs.

In quantifying the aggregate impacts, the calibrated financial frictions lower output per capita by 39% relative to a frictionless credit benchmark. Wages fall by 32%. Both lower TFP and lower capital usage play key roles in driving these aggregate results. TFP is 28% lower and capital is 41% lower relative to the frictionless credit benchmark.

Counterfactual simulations reveal that spreads coming from direct intermediation costs drive the vast majority of impacts. First, the calibration implies that direct quantity constraints play a minor role in Brazil. So spreads account for essentially the full aggregate impacts, with the vast majority of losses in our benchmark calibration stemming from the high overall level of spreads. Second, intermediation costs rather than market power are the dominant spread frictions, especially those that vary by productivity rather than assets. (In principle, spreads that are higher for low-asset firms can be extremely harmful, with more severe impacts on the credit market and output, but not in our calibrated economy.) Finally, the sources of frictions interact with one another, so eliminating one friction has smaller impacts in the presence of others.

To further assess the impacts of spreads, we compare our calibration to an alternative model with only a quantity constraint, similar to the existing literature (e.g., Buera, Kaboski, and Shin, 2011). When both models are calibrated to match the ratio of external finance to GDP, the model with spreads yields larger losses (from the perfect credit benchmark) on all dimen-

sions: roughly 60% higher losses in output and capital and 75% higher losses in TFP. Thus incorporating spreads into models appears quantitatively important, not only for decomposing the source of frictions but for aggregates as well. These conclusions are all robust to an alternative calibration with more moderate levels and dispersion in spreads.

To evaluate the model mechanisms, we study how financial frictions affect the dynamics in spreads, firm growth, and misallocation. Both spread and firm growth patterns in the Monte Carlo simulations follow those observed in the data. Both decline and do so more rapidly in early years. Misallocation patterns over the life cycle are related to growth and spreads. In particular, under the benchmark model of spreads, firms begin constrained, with high and disperse marginal products of capital, but since they grow and spreads fall quickly, the marginal product of capital falls and converges across firms over the life cycle. Misallocation is therefore strongest for young firms.

Related Literature. This paper is related to a large literature on the quantitative effects of financial frictions on entrepreneurship and economic development (e.g., Jeong and Townsend, 2007; Antunes, Cavalcanti, and Villamil, 2008; Banerjee and Moll, 2010; Buera, Kaboski, and Shin, 2011; Buera and Shin, 2013; Erosa, 2001; Greenwood, Sanchez, and Wang, 2010, 2013; Moll, 2014; Moll, Townsend, and Zhorin, 2017; Itskhoki and Moll, 2019). We differ from these papers in two ways. First, most papers in this literature have a single interest rate. When there is a spread, it does not vary within the same economy. Second, these papers typically consider a collateral constraint as the only financial friction.² An exception on both fronts is Greenwood, Sanchez, and Wang (2010), which shows how monitoring technology can generate dispersion in interest rates. However, they abstract from self-financing, a key element of our model and analysis.³

Micro studies have noted variation in interest rates and their potential for misallocation (e.g., Banerjee and Duflo, 2005, 2010). Some macro papers have looked to measure the extent and quantify the aggregate impact of misallocation among an existing set of firms (e.g., Hsieh and Klenow, 2009; Sraer and Thesmar, 2018; David and Venkateswaran, 2019). Most closely related is the work of Bai, Lu, and Tian (2018) and Gilchrist, Sim, and Zakrajsek (2013). Bai, Lu, and Tian (2018) impute interest rates as the ratio of interest payments to debt for large Chinese firms and relate interest rates to leverage to infer fixed costs. Gilchrist, Sim, and Zakrajsek (2013) directly observe interest rates using contracts of publicly traded bonds in U.S. manufacturing. We complement this work with evidence on spreads for the universe of loans to formal firms, not just large or publicly traded firms where frictions are almost surely less severe, and we do so for a developing economy. We show that the details of contracts

²An exception to the single friction is Moll, Townsend, and Zhorin (2017) which considers an economy with both collateral constraints and moral hazard frictions, but there is regional variation where each agent only faces one of the two frictions and the economy has a single prevailing cost of capital.

³Besley, Burchardi, and Ghatak (2017) study a model with spread variation, but the model is static and variation is purely driven by risk.

matter for understanding spreads. Moreover, in modeling entry, we account for losses along the extensive margin, which have been shown to be important (e.g., Buera, Kaboski, and Shin, 2011), and endogenous selection into borrowing, which we show to be important.

We follow other researchers in examining firm dynamics in response to credit constraints both theoretically and empirically (e.g., Albuquerque and Hopenhayn, 2004; Clementi and Hopenhayn, 2006; Arellano, Bai, and Zhang, 2012; Gopinath, Kalemli-Özcan, Karabarbounis, and Villegas-Sanchez, 2017). We innovate by linking the details of financial contracts to firm dynamics, however, and we use both spreads and firm growth to evaluate our key mechanism. Perhaps closest to us is the work of Kochen (2022), who documents and evaluates spread variation over the life cycle, but in higher-income countries with much milder spreads. He uses Orbis balance sheet data rather than loan-level data.

We also contribute to a large literature distinguishing the form and causes of financial frictions. Important examples that look at the macro implications of these distinctions include Paulson et al. (2006); Karaivanov and Townsend (2014) and Dabla-Norris et al. (2021). A related but distinct literature in financial economics addresses the challenges of explaining credit spreads in the United States. Risk is a primary consideration (e.g., David et al., 2022) in both spreads and misallocation. The credit-spread puzzle, that default alone cannot explain US corporate spreads, is well established (e.g., Elton et al., 2001; Collin-Dufresne et al., 2001; Huang and Huang, 2012; Gilchrist et al., 2013). We find a similar but larger puzzle in spreads on Brazilian bank loans.⁴ Another factor receiving increasing attention is market power. Villa (2022) is similar to us in evaluating the role of intermediary market power in generating spreads. His work is focused on market power, however, whereas market power is one of multiple spread-inducing mechanisms in our model.

Finally, we contribute to the literature on macro development by using detailed loan-level data for the universe of credit operations in Brazil. These data have been used in different contexts: financial inclusion and inequality (Fonseca and Matray, 2022), bank competition and the cost of credit (Joaquim, van Doornik, and Ornelas, 2020), financial flows and structural transformation (Bustos, Garber, and Ponticelli, 2020), credit supply shocks and firm growth (Bazzi, Muendler, Oliveira, and Rauch, 2020), among others. This body of works lends confidence to the high quality of the data we use. We contribute by applying these data to the study of the macroeconomic effects of financial frictions in developing countries.

⁴Arellano et al. (2019) use default rates to match the smaller but still substantial dispersion in U.S. spreads.

2 Empirical Analysis

This section provides a description of our data and documents empirical evidence of the high level of and dispersion in financing costs. These patterns provide motivation, inform our modeling strategies, and ultimately discipline our quantitative approach.

2.1 Data

Our data on firm credit are based on underlying loan-level data from the Brazilian Public Credit Registry (SCR - *Sistema de Informações de Crédito*), which covers all bank loans to formal firms.⁵ This dataset contains information on all formal loans granted from January 2006 until December 2016. For any bank-to-firm loan during the period of analysis, it identifies the lender, borrower, size of the loan, interest rate on the loan, loan maturity, default status, and whether or not it was at a subsidized or “earmarked” interest rate.⁶ These underlying data allow us to construct information on the borrower-lender relationship, such as the length of a firm-bank relationship. However, this loan-level database is confidential and managed by the Central Bank of Brazil, and the underlying loan-level data are aggregated to construct credit flows and stocks at the level of firm-bank-loan type-year combination. Similarly, we use loan-weighted averages to construct average spreads, maturity, non-performing loans, and other measures at these levels. Spreads are the difference between the weighted average of a firm’s outstanding loans rates and the one-year interbank deposit rate.⁷ It is this constructed firm-bank-loan type-year level dataset, which is still confidential, that we access and base our analysis on.

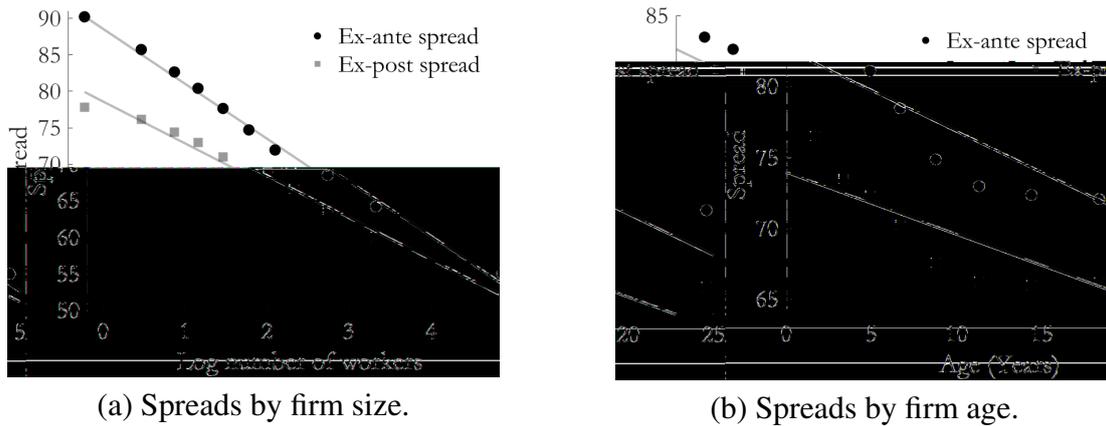
The second dataset that we use in our empirical analysis is RAIS (*Relação Anual de Informações Sociais*), a matched employer-employee administrative dataset covering all formal firms in Brazil. This is a mandatory annual survey maintained by the Ministry of Labor. RAIS provides information on firms, such as industry and location, and information on employees, which we use to construct firm-level measures of employment and labor compensation. It is also possible to identify the date of entry and exit of firms. With this dataset, we can capture important firm dynamics for all formal firms in Brazil. Using the unique firm tax identifier, we merge the SCR and RAIS datasets. Although our rich data and coverage are a huge advantage, it is also important to appreciate what data we do not have. We have no data on capital or output, so we are unable to construct measures of productivity, and we rely on labor measures as metrics of firm size.

⁵For more detailed information on the datasets, see Online Appendix A.

⁶The Brazilian Development Bank (BNDES) is the main financing institution for productive investment in the country, and it offers subsidized interest rates for long-term investments.

⁷Loan maturity is typically short, with a mean of 14 months and a median of 4. Hence, we found similar patterns using only new loans. The short maturity also explains why current default status is quite close to measures of ultimate default status, and we use the former in constructing default rates.

Figure 1: Ex Ante and Ex Post Spreads



Notes: Ex ante spreads are based on contracted interest rates. Ex post spreads are calculated by setting the interest rate to -100% for loans in default.

2.2 Empirics

Here we document a high level and dispersion of spreads above and beyond what is required to compensate for default risks. We show that these spreads vary considerably across firms, with much of the variation explained by the type of credit that firms access. Observable firm characteristics also explain important variation. In particular, spreads decrease considerably with firm size and age. These characterizations motivate our model in Section 3.⁸

Figure 1 illustrates these claims. The left panel plots *ex ante* and *ex post* spreads against firm size (number of employees), while the right panel plots the same spreads against firm age. *Ex ante* spreads are based on contracted interest rates. *Ex post* spreads are calculated by setting the interest rate to -100% for loans in default. Such an assumption is conservative in the sense that it assumes default occurs immediately and fully. Here each dot represents a decile average along the x-axis of the data, while the black lines indicate linear fits to the data. These panels make four important points that will motivate our analysis. First, spreads are high, reaching as high as 90 percentage points for the lowest firm size decile. Second, *ex ante* spreads fall considerably with firm size, roughly 35 percentage points across the 10 deciles. Third, default alone explains neither the high levels nor the variation with firm size. *Ex post* spreads are still roughly 78 percentage points for the lowest decile of firm size and still fall roughly 28 percentage points across the firm size deciles. Fourth, we see similar patterns with firm age, although the decline in average spread is less linear. Average spreads fall roughly 11 percentage points over the first 10 years of a firm’s age and decline only modestly after that.

Table 1 analyzes these patterns from a multivariate perspective using Ordinary Least Squares (OLS) with the percentage point spread as a dependent variable and a progressively larger set

⁸Basic summary statistics on the interest rate spreads, which are high and variable, are provided in Table A1 in Online Appendix A.

of explanatory variables as we move across columns.⁹

Columns 1 and 2 regress the spread on observable measures and proxies of risk. Column 1 uses the observable measures of maturity, maturity squared, and dummies for firm non-performing loans in the past, currently, or in the future.¹⁰ Spreads are considerably higher when loans are non-performing, but these risk measures together explain only 24 percent of the variation. Column 2 adds state-industry-time fixed effects, which might also conceivably proxy for risk. The amount of additional variation explained is about 2 percent. Roughly one-quarter of the variation can be explained by risk.

Column 3 adds the inverse loan size as an explanatory variable, which would explain spread variation if it were compensating for a fixed cost of loan issuance, as in Bai et al. (2018). The significant positive coefficient indicates that smaller loans indeed pay higher spreads, but both the coefficient and R^2 indicate that this is quantitatively negligible. We therefore abstract from fixed cost of issuance in our model.

The remaining columns display regressions with factors that we do model. In particular, Column 4 adds the log of firm size, which yields a sizable coefficient reflecting the negative relationship in Figure 1. Column 5 adds firm age and firm-age squared, which captures the somewhat weaker but still important non-linear relationship with firm age in Figure 1. While the coefficients indicate quantitatively important relationships, comparing R^2 across Column 5 (0.272) and Column 3 (0.260) indicates that the overall fraction explained by these two observable characteristics is only one percentage point.

Column 6 accounts for explicit known reasons for variation in spreads by adding dummies for when a loan comes from the Brazilian development bank (BNDES), the type of loan, and whether a loan is earmarked as subsidized. The $R^2 = 0.587$, so these sources explain an additional roughly 30% of the variation.

Column 7 adds additional observables that might proxy for market power in the lender-borrower relationship. These variables include the number of banks from which a firm borrows, the number of months of the firm-bank relationship, and bank-specific fixed effects. Spreads are decreasing in the former two. The $R^2 = 0.649$ indicates that that market power might explain an additional six percent of the variation.

⁹These regressions are robust to several variants, including using the log of one plus the spread as the dependent variable. Credit-weighted regressions also yield similar patterns.

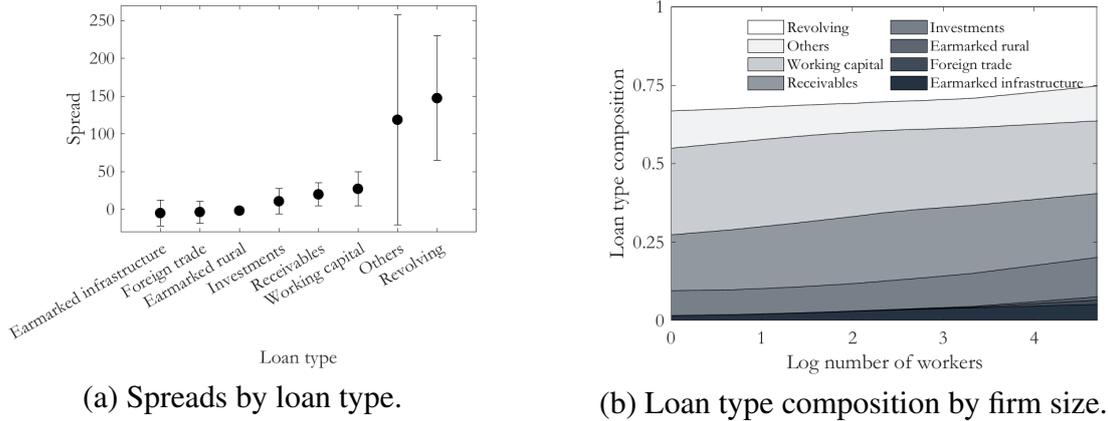
¹⁰In principle, future non-performing loans could be impacted by the spread itself, but regressions that drop this explanatory variable yield similar results. Including higher order leads and lags does not appreciably impact our conclusions, which again may relate to maturity being typically short-term.

Table 1: Spreads and Firm and Loan Characteristics

VARIABLES	(1) Spread	(2) Spread	(3) Spread	(4) Spread	(5) Spread	(6) Spread	(7) Spread	(8) Spread
Maturity	-3.232*** (0.0788)	-3.248*** (0.0800)	-3.248*** (0.0800)	-3.174*** (0.0803)	-3.174*** (0.0803)	-1.385*** (0.0450)	-1.269*** (0.0447)	-1.166*** (0.0461)
Maturity squared	0.00910*** (0.000172)	0.00916*** (0.000175)	0.00916*** (0.000175)	0.00897*** (0.000176)	0.00897*** (0.000176)	0.00484*** (0.000101)	0.00465*** (0.000103)	0.00450*** (0.000109)
NPL	45.49*** (1.507)	52.43*** (1.088)	52.42*** (1.088)	51.27*** (1.090)	51.45*** (1.099)	53.66*** (0.541)	58.27*** (0.491)	60.69*** (0.502)
Lag of NPL	47.76*** (1.196)	43.58*** (1.163)	43.56*** (1.163)	43.39*** (1.172)	43.14*** (1.145)	41.10*** (0.601)	44.90*** (0.483)	52.87*** (0.440)
Lead of NPL	54.12*** (1.164)	54.43*** (1.102)	54.41*** (1.102)	55.10*** (1.118)	55.03*** (1.112)	61.57*** (0.523)	68.14*** (0.428)	67.95*** (0.526)
Inverse loan size			0.0978*** (0.00852)	0.0925*** (0.00887)	0.0928*** (0.00889)	0.0340*** (0.00649)	0.0403*** (0.00602)	0.0350*** (0.00617)
Log firm size				-7.472*** (0.151)	-7.309*** (0.152)	-5.398*** (0.135)	-6.346*** (0.188)	0.135*** (0.0467)
Firm age					-0.129*** (0.0339)	-0.284*** (0.0158)	-0.246*** (0.0196)	0.937*** (0.0241)
Firm age squared					0.000927*** (0.000272)	0.00207*** (0.000125)	0.00172*** (0.000161)	-0.00713*** (0.000186)
Number of banks							-1.148*** (0.0672)	0.0551* (0.0325)
Firm-Bank relat.							-0.0121*** (0.00174)	-0.00972*** (0.00160)
Constant	103.8*** (1.965)	103.8*** (0.712)	103.8*** (0.712)	118.6*** (0.866)	119.8*** (0.927)	69.14*** (0.779)	74.39*** (0.978)	36.47*** (0.776)
Observations	7,174,930	7,174,917	7,174,917	6,926,846	6,926,846	6,926,846	6,926,789	6,621,098
R-squared	0.239	0.260	0.260	0.272	0.272	0.587	0.649	0.791
Year FE	YES	NO	NO	NO	NO	NO	NO	NO
IndustryxYearxState	NO	YES	YES	YES	YES	YES	YES	YES
BNDES	NO	NO	NO	NO	NO	YES	YES	YES
Loantype	NO	NO	NO	NO	NO	YES	YES	YES
Earmarked	NO	NO	NO	NO	NO	YES	YES	YES
BankFE	NO	NO	NO	NO	NO	NO	YES	YES
FirmFE	NO	NO	NO	NO	NO	NO	NO	YES

Notes: Robust standard errors in parentheses; *** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$.

Figure 2: Loan Types



(a) Spreads by loan type.

(b) Loan type composition by firm size.

Finally, Column 8 adds firm fixed effects as well. Together, these variables explain roughly 80% of the variation. The remaining orthogonal variation is year-to-year variations in interest rates within firm-loan-type-lender relationships not explained by risk, loan size, observed firm-characteristics, or market power.

We next look at some of the additional forces behind the variation in spreads, especially the variation with age and size that we later model. Panel (a) of Figure 2 plots the average spread by loan type from smallest to largest. The bands reflect the standard deviations of these spreads in the data. Clearly, loan type captures a substantial amount of variation. The first three categories have negative interest spreads and small variation in spreads overall. These categories reflect earmarked loans for infrastructure, foreign trade loans, and earmarked rural loans. The next three loans have positive spreads ranging from an average of 15 to 30 percentage points, and they have only somewhat higher variation, as the bands display. These loan types include investment, receivables, and working capital. The last two loan types, which show markedly higher interest rate spreads, are for “other” types of credit (e.g., revolving credit cards, microcredit, and others) and loans off of revolving lines of credit, respectively. Note that, for revolving-type lines of credit, only rolled over debt that actually accrued interest appear in our data. The interest rate spread for both groups averages well over 100 percentage points. They also show much higher levels of variance, especially and understandably in the “other” category because it combines loans of various types. Assuming that firms borrow from the lowest-cost source of credit available, an important source of both the average and marginal cost of capital is therefore credit *availability*. Those who can borrow for earmarked purposes pay consistently lower spreads, while those forced to borrow using revolving forms like credit cards or lines of credit pay much higher and more variable spreads. The figure (together with the regression table) also underscores the importance of using our disaggregated data, which include loan-level variables like spreads, loan type, etc.

We have shown high average *ex post* credit spreads, high variation in these *ex post* spreads across firms, and we have also demonstrated three important predictors of spreads: size, age,

and loan type. A natural question is whether loan type covaries with size. Panel (b) of Figure 2 demonstrates that indeed it does by showing the composition of credit over various loan types and how it varies with firm size. Here we have binned loan size and aggregated the composition of loans across firms. Earmarked and foreign trade loans at the bottom are a small fraction of overall credit, but they are substantially more available to larger firms. Investment loans are also lower interest loans and they constitute a more sizable share of the overall portfolio of firm credit, but they are also more important for larger firms. Lastly, we see that two sources of credit decrease with firm size: working capital and revolving lines of credit. The former makes up almost exactly for the increase in investment loans, but constitute a more expensive loan, while the latter makes up almost exactly for the increase in the lowest cost of loans. This compositional change accounts for about a 14 percentage point decrease in the cost of credit across the plotted size distribution, an important share but still less than half of the roughly 35 percentage point drop in the left panel of Figure 1. An analogous plot for firm age allows for less clear interpretation. As in the right panel of Figure 1, most of the change in composition occurs in the early years. Earmarked credit is less important for very young firms, however, so too are revolving credit lines. Both become more important with age. The middle three categories (investment, receivables, and working capital loans) are all more important for very young firms. Quantitatively, these composition changes alone (at average spread values for these categories) account for very little of the spread vs. age relationship.

Are these spreads quantitatively important? A back-of-the-envelope estimate can be obtained using a direct approach based on Hsieh and Klenow (2009) to measure the effects of interest rate spreads on aggregate TFP. For that, consider an environment with a continuum of entrepreneurs with productivity z_i , each facing interest rate \tilde{r}_i . Let the production function of each entrepreneur be $y_i = z_i k_i^\alpha$, with $\alpha \in (0, 1)$. The asset held by each entrepreneur is a_i , and there is a fixed supply of capital K . Profit maximization implies

$$\pi(z_i, \tilde{r}_i) = \max_{k_i} z_i k_i^\alpha - \tilde{r}_i (k_i - a_i),$$

and $\tilde{r}_i = r$ if $k_i \leq a_i$. The first-order condition of this problem with respect to capital stock k_i implies that

$$MPK_i = \alpha z_i k_i^{\alpha-1} = \tilde{r}_i \Rightarrow k_i = \left(\frac{\alpha z_i}{\tilde{r}_i} \right)^{\frac{1}{1-\alpha}},$$

and market clearing

$$K = \int \left(\frac{z_i \alpha}{\tilde{r}_i} \right)^{\frac{1}{1-\alpha}} G(dz_i, d\tilde{r}_i).$$

Therefore,

$$TFP = \frac{Y}{K^\alpha} = \frac{\int z_i^{\frac{1}{1-\alpha}} \left(\frac{\alpha}{\tilde{r}_i} \right)^{\frac{\alpha}{1-\alpha}} G(dz_i, d\tilde{r}_i)}{\left(\int \left(\frac{z_i \alpha}{\tilde{r}_i} \right)^{\frac{1}{1-\alpha}} G(dz_i, d\tilde{r}_i) \right)^\alpha}.$$

In the economy without interest spreads, we have that

$$TFP_{eff} = \left(\int z_i^{\frac{1}{1-\alpha}} G(dz_i) \right)^{1-\alpha}.$$

When z_i and \tilde{r}_i are jointly log-normally distributed, this simplifies to:

$$\log(TFP_{loss}) = \log(TFP_{eff}) - \log(TFP) = \frac{1}{2} \frac{\alpha}{1-\alpha} Var(\tilde{r}_i).$$

Let $\alpha = 0.33$. If we use the fact that the standard deviation of the raw spread is 93.3%, $\log(TFP_{loss}) \approx 21\%$. However, only 26.4% of firms use credit. If we assume that these firms have no demand for credit given their internal resources, and therefore equalized their marginal product of capital, $\log(TFP_{loss})$ drops to just 1.5%. In principle, $\log(TFP_{loss})$ could be much greater than even 21%, however, since non-borrowers could instead be *the most* misallocated, if their lack of borrowing reflects prohibitive financing costs.

In sum, spreads are high, and there is a large dispersion in the spread rate, even after controlling for risk. Firm age, firm size, loan type/earmarking, competition, and other firm-specific forces appear to be important sources of variation. Moreover, simple calculations suggest that these may yield sizable aggregate impacts, but this depends on the marginal products of nonborrowers. These calculations also ignore the potential effect of spread offers on the distribution of productivity and realized spreads through their distortion of the entry and growth margins. All of this is motivation for needing a quantitative structural model.

Our model will abstract from risk but focus on sources that yield variation in spreads across firm size, firm age, and other firm-specific factors (which encompass loan type and earmarking as sources of variation). We therefore calibrate our model to the variation in spreads explained by these forces. That is, for quantitative purposes of our model, we strip out the variation explained by risk and the unexplained variation, but retain the variation explained by these components in Column (8) of Table 1. We use these constructed data to calibrate and test the model.

3 Model

This section develops a model of entrepreneurship decisions and firm dynamics under financial constraints that yields multiple sources of misallocation and dispersion in spreads. We discuss, in turn, the environment, static optimization, dynamic optimization, and equilibrium.

3.1 Environment

The economy is populated by a continuum of infinitely-lived heterogeneous individuals, who can choose at any time to be either a worker or an entrepreneur. Time is continuous. There is a single good that can be used for consumption or investment and is sold competitively. Entrepreneurs accumulate assets but can augment their own assets with capital from intermediaries. However, financial intermediation suffers from three potential sources of frictions: limited enforcement, intermediation costs, and lender market power.

3.1.1 Endowments

At any point in time, t , heterogeneous individuals vary by their entrepreneurial productivity, $z(t)$, and their assets, $a(t)$, and they make an occupational choice to be either a worker or an entrepreneur. Entrepreneurial productivity is drawn from an invariant Pareto distribution function $\mu(z) = \eta z^{-(\eta+1)}$ with $z \geq 1$. With Poisson arrival rate γ , individuals draw a new talent for managing from distribution $\mu(z)$. Agents accumulate or decumulate assets subject to their consumption decisions and budget constraint.

3.1.2 Preferences

Individuals derive utility from consumption, $c(t)$, and preferences are represented by:

$$E_0 \left[\int_0^{\infty} e^{-\rho t} u(c(t)) dt \right], \quad (1)$$

where ρ is the subjective discount rate, and E_0 is the expectations operator conditional on information at $t = 0$. The period utility takes the following form:

$$u(c(t)) = \frac{c(t)^{1-\sigma} - 1}{1-\sigma}, \quad \sigma > 0. \quad (2)$$

3.1.3 Technology

Entrepreneurs operate a technology that uses labor, $n(t)$, and capital, $k(t)$, to produce a single consumption good, $y(t)$:

$$y(t) = z(t) k(t)^\alpha n(t)^\theta, \quad \text{with } \theta, \alpha \in (0, 1), \quad \text{and } \alpha + \theta < 1. \quad (3)$$

Entrepreneurs incur a fixed-cost $\kappa \geq 0$ to operate at any time and can operate only one project. They hire labor and may finance capital through their own assets, by borrowing from financial

intermediaries, or a combination of the two.

3.1.4 Financial Intermediation

A continuum of financial intermediaries with unit mass offers agents an option to deposit assets at an endogenously-determined competitive rate $r(t)$ or borrow additional capital, where both the loan amount $l(t)$ and the borrowing interest rate $\tilde{r}(t)$ are subject to financial frictions. We model three sources of frictions: the typical limited enforcement that constrains loan sizes by the fraction of income stream that the intermediary recovers, and spreads on borrowing rates that come from real intermediation costs and lender market power. For simplicity, we model static, one-time loan relationships, in which an entrepreneur meets a financial intermediary and enters into negotiation to determine the amount and the interest on a loan. For ease of expression, we drop the time indexes in discussing the financial intermediation technology and static optimization. Since we solve for a stationary equilibrium, these relationships will not vary with time. The individual state vector (a, z) will be sufficient.

The real intermediation cost associated with making a loan $l = k - a$ to an entrepreneur who has collateral a and productivity z is $g(l, \tau(a, z))$. This cost is associated with collecting information about borrowers, monitoring, and enforcing credit contracts. Otherwise, borrowers could break their promise without any penalty. We denote this intermediation cost by

$$g(l, \tau(a, z)) = l\tau(a, z),$$

where

$$\tau(a, z) = \tau_0 + \frac{\tau_a}{1+a} + \frac{\tau_z}{z}, \quad \text{with } \tau_0 \geq 0, \tau_a \geq 0 \text{ and } \tau_z \geq 0.$$

We model per unit intermediation costs that are (weakly) decreasing in both productivity, z , and assets, a . The former captures a measure of cash flow and the latter captures a measure of collateral, two commonly evaluated lending criteria.¹¹

Market power comes from the bilateral nature of borrowing/lending opportunities. The lender and borrower negotiate over the interest rate \tilde{r} and loan amount l via Nash bargaining, where $\chi \in [0, 1]$ denotes the intermediaries' bargaining power.

The range of possible loans over which the lender will bargain is constrained by the limited enforcement of contracts, however. Limited enforcement means borrowers have the option to strategically default and lenders can only recover a fraction ϕ of the output produced net of labor

¹¹Bai, Lu, and Tian (2018) model intermediation costs that are decreasing in loan size. Intermediation costs that vary with loan size, while reasonable, would introduce an additional nonlinearity into the static optimization, increasing the required computation substantially. Related, although spreads do not exogenously vary in leverage, the model generates an endogenous correlation, since both intermediation costs and desired borrowing decline in assets. Leverage itself cannot be observed in our data, since we lack data on capital or assets.

costs. ϕ is therefore a measure of financial enforcement, and will lead to a quantity restriction that is common in the literature (e.g., Buera, Kaboski, and Shin, 2011). For simplicity, we follow this literature in modeling only static penalties for defaulting.

3.2 Static Optimization

Given the continuous-time set-up, occupational choice, intermediary meeting, negotiation, contracting, disbursal, and repayment all happen contemporaneously. We now solve for static quantities, including occupational choice, contract terms, factor usage, and instantaneous income.

3.2.1 Entrepreneurial Profits

Static entrepreneurial profits are subject to multiple frictions. Entrepreneurs can freely hire labor at wage w but may face different costs for external capital, \tilde{r} , and internal capital, r . Given the fixed cost, κ , the flow of income of an entrepreneur with asset a and productivity z using capital k and labor n is:

$$\pi(k, n, \tilde{r}; a, z) = zk^\alpha n^\theta - wn - \tilde{r}(k - a) - ra - \kappa. \quad (4)$$

Entrepreneurs maximize (4) given factor prices and their choice set for capital. Solving the constrained static optimization involves first solving for the entrepreneur's unconstrained capital level (denoted by $k^u(z)$) and comparing it with the entrepreneur's assets. Then, for those with assets below their unconstrained capital, we solve for the set of capital levels satisfying the limited enforcement constraint:

$$\pi(k, \tilde{r}; a, z) \geq (1 - \phi) (x(k, \tilde{r}; a, z) + \tilde{r}(k - a)) - ra - \kappa,$$

where $\pi(k, \tilde{r}; a, z)$ denotes entrepreneurial profits and $x(k, \tilde{r}; a, z) \equiv \max_{n \geq 0} \{\pi(k, n, \tilde{r}; a, z)\} + \tilde{r}(k - a) + ra + \kappa$ is output net of labor costs, both expressed as a function of capital given the optimal choice of labor. The left-hand side of the constraint is therefore income from repayment, which must exceed the income from defaulting. This income in default is expressed on the right-hand side as the retained fraction, $1 - \phi$, of the total unrepaid loans and output (net of labor costs), netting out foregone interest on unborrowed capital and the fixed cost from this retained fraction. When the expression holds with equality, it defines a hard quantity constraint on borrowing, i.e., a maximum level of capital (and implicitly a maximum loan size) that depends on the borrowing rate, $\bar{k}(\tilde{r})$.

For an entrepreneur who borrows, the loan size, l , quantity of capital, k^b , and the borrowing

rate, \tilde{r} , are the solution to the bargaining problem:

$$\max_{\bar{k} \geq k \geq a, \tilde{r}} [(\tilde{r} - r - \tau(a, z))(k - a)]^\chi [\pi(k, \tilde{r}; a, z) + \tilde{r}(k - a) - \tilde{w}(a, z)]^{1-\chi}.$$

The first term (raised to the χ power) is the intermediary's surplus (S^b) from the loan. The second term (raised to the $1 - \chi$ power) is the surplus to the entrepreneur (S^e). The expression $\tilde{w}(a, z)$ is the best outside option of the borrowing entrepreneur, either: (i) the entrepreneurial profits from operating the business with internal capital only or (ii) the wage from becoming a worker. (Formally, $\tilde{w}(a, z)$ is defined as $\max\{w, x(a, z) - ra - \kappa\}$ or, equivalently, $\max\{w, \tilde{\pi}(a, z)\}$.) The solution to this problem defines a financing spread $\tilde{r} - r$ that depends on the intermediation costs and the bargaining power of the intermediary.

When the contracted capital, k^b , satisfies a strict inequality (i.e., $k^b < \bar{k}(\tilde{r})$), one can use the first-order conditions to solve for the contract terms:

$$k^b(a, z) = \left(z \left(\frac{\alpha}{r + \tau(a, z)} \right)^{(1-\theta)} \left(\frac{\theta}{w} \right)^\theta \right)^{\frac{1}{1-\alpha-\theta}}, \quad (5)$$

and

$$\tilde{r}(a, z) = r + \tau(a, z) + \chi \left(\frac{\pi^b(a, z) - \tilde{w}(a, z)}{k^b(a, z) - a} \right). \quad (6)$$

Equation (5) corresponds to the optimal level of contracted capital for given intermediation costs, $\tau(a, z)$. Indeed, if $\tau(a, z) = 0$, this would be the unconstrained level of capital. However, in equation (6), when the intermediary has market power, i.e., $\chi > 0$, the loan interest $\tilde{r}(a, z)$, will be distorted even in the case of $\tau(a, z) = 0$. Thus, intermediary market power will not statically distort capital but will impact the profits of the entrepreneur, and will therefore dynamically impact the entrepreneur's ability to self-finance. Moreover, the borrowing interest rate will vary with assets a and productivity z , as these determine the loan size and the entrepreneur's flow surplus. We characterize elements of this dependence in the proposition below.

Proposition 1 Consider an agent (a, z) such that $a < k^u(z)$, $k < \bar{k}(\tilde{r})$ and $S^e(a, z) \geq 0$.

1. Case of $\chi = 0$:

$$\tilde{r}(a, z) = r + \tau(a, z),$$

$$\frac{\partial \tilde{r}(a, z)}{\partial z} \leq 0 \text{ and } \frac{\partial k^b(a, z)}{\partial z} \geq 0; \quad \frac{\partial \tilde{r}(a, z)}{\partial a} \leq 0 \text{ and } \frac{\partial k^b(a, z)}{\partial a} \geq 0.$$

2. Case of $\chi \in (0, 1)$ and $\tau(a, z) = \tau_0 \geq 0$: $\frac{\partial \tilde{r}(a, z)}{\partial \chi} > 0$ and $\frac{\partial k^b(a, z)}{\partial \chi} = 0$.

3. Case of $\chi = 1$ and $\tau(a, z) = \tau_0 \geq 0$:

$$\tilde{r}(a, z) = r + \tau_0 + \frac{1}{k^b(a, z) - a} (y^b(a, z) + \tau_0 a - \kappa - \tilde{w}(a, z)),$$

$\frac{\partial \tilde{r}(a, z)}{\partial z} > 0$ and $\frac{\partial k^b(a, z)}{\partial z} > 0$. In addition, if $w > \tilde{\pi}(a, z)$, then $\frac{\partial \tilde{r}(a, z)}{\partial a} > 0$ and $\frac{\partial k^b(a, z)}{\partial a} = 0$; and if $w < \tilde{\pi}(a, z)$, then $\frac{\partial \tilde{r}(a, z)}{\partial a} < 0$ and $\frac{\partial k^b(a, z)}{\partial a} = 0$.

Proof. See Online Appendix B.1 ■

Proposition 1 shows how assets, productivity, and bargaining power affect the loan interest rate and the size of the loan when the limited commitment constraint does not bind. The first case isolates the role of intermediation costs by focusing on the case with no intermediary bargaining power ($\chi = 0$): the loan interest rate equals the marginal cost of a loan, $\tilde{r}(a, z) = r + \tau(a, z)$. Following the intermediation costs, the loan interest rate varies negatively with productivity z and assets a , and the capital used by entrepreneurs is consequently increasing in assets and productivity. The variation in intermediation costs therefore leads to dispersion in both borrowing rates and the marginal productivity of capital among borrowing entrepreneurs.

The second case illustrates the direct influence of intermediate levels of market power. Market power does not influence the level of capital, since this is chosen to maximize total surplus. Instead, the loan interest rate simply increases with market power, since this is the intermediary's tool for capturing surplus. Hence, market power alone does not cause intensive margin misallocation. Nevertheless, the dynamic effects of the dispersion of loan interest rates on the distribution of assets might still influence whether or not the limited enforcement constraint binds and the extent of intermediation costs.

The third case looks at the extreme case of the intermediary having full bargaining power ($\chi = 1$). When this is the case and intermediation costs are independent of assets and productivity ($\tau(a, z) = \tau_0$), then Proposition 1 shows that the loan interest rate will still vary with the entrepreneur's type (a, z) . However, market power leads loan interest rates to be *increasing* in productivity. For a given a , a higher z implies a higher entrepreneurial surplus, which is captured through the higher loan interest rate charged by financial intermediaries. Whether the loan interest rate is increasing or decreasing in assets depends on whether the outside option is the wage or self-financed entrepreneurship. When $\tilde{\pi}(a, z) > w$, a higher level of assets increases the outside option of an entrepreneur, and so the loan interest rate decreases with assets. When $\tilde{\pi}(a, z) < w$, the outside option is independent of a . Since the loan size decreases with a , however, the loan interest rate increases to capture the same share of surplus. In this case of pure market power, we would observe dispersion in spreads but no dispersion in marginal productivity of capital.

When the enforcement constraint binds, $k = \bar{k}(\tilde{r})$, the following optimality conditions solve

for the constrained level of capital k^b and loan interest rate \tilde{r} :

$$\phi x(k, z) = \tilde{r}(k - a) \quad (7)$$

and

$$\chi S^e \left(\phi \frac{\partial x(k, z)}{\partial k} - r - \tau(a, z) \right) + (1 - \chi) S^b (1 - \phi) \frac{\partial x(k, z)}{\partial k} = 0. \quad (8)$$

Proposition 2 summarizes the main results for the case in which the enforcement constraint binds.

Proposition 2 Consider an agent (a, z) such that $a < k^u(a, z)$, the incentive-compatible constraint binds and $S^e(a, z) \geq 0$.

1. Case of $\chi = 0$:

$$\tilde{r}(a, z) = r + \tau(a, z),$$

$$\frac{\partial \tilde{r}(a, z)}{\partial z} \leq 0 \text{ and } \frac{\partial k^b(a, z)}{\partial z} > 0; \frac{\partial \tilde{r}(a, z)}{\partial a} \leq 0 \text{ and } \frac{\partial k^b(a, z)}{\partial a} > 0.$$

2. Case of $\chi \in (0, 1)$ and $\tau(a, z) = \tau_0 \geq 0$: $\frac{\partial \tilde{r}(a, z)}{\partial \chi} > 0$ and $\frac{\partial k^b(a, z)}{\partial \chi} < 0$.

3. Case of $\chi = 1$ and $\tau(a, z) = \tau_0 \geq 0$:

$$\tilde{r}(a, z) = \frac{\phi x(k^b(a, z))}{k^b(a, z) - a}.$$

If $w > \tilde{\pi}(a, z)$, then $\frac{\partial \tilde{r}(a, z)}{\partial z} > 0$ and $\frac{\partial k^b(a, z)}{\partial z} < 0$; and $\frac{\partial \tilde{r}(a, z)}{\partial a} \leq 0$ and $\frac{\partial k^b(a, z)}{\partial a} > 0$. If $w < \tilde{\pi}(a, z)$, then $\frac{\partial \tilde{r}(a, z)}{\partial z} \geq 0$ and $\frac{\partial k^b(a, z)}{\partial z} \leq 0$; and $\frac{\partial \tilde{r}(a, z)}{\partial a} \geq 0$ and $\frac{\partial k^b(a, z)}{\partial a} > 0$.

Proof. See Online Appendix B.2 ■

In the first case, when $\chi = 0$, the results are similar to those in Proposition 1. However, in the second case, as the intermediary gets market power, it must trade off a higher interest rate with less capital, since the incentive to default involves the product of the interest rate and loan amount. When $\chi = 1$ and the enforcement constraint binds, it is not straightforward to characterize how assets and productivity change the interest rate and the optimal loan size.

After bargaining and optimization, instantaneous entrepreneurial profits are then:

$$\pi^*(a, z) = \begin{cases} x(a, z) - ra - \kappa, & \text{if } l = 0 \\ x(k^b, z) - \tilde{r}(k^b - a) - ra - \kappa, & \text{if } l > 0, \end{cases}$$

where the greater of the two is chosen. Financial distortions will impact the income of a borrowing entrepreneur in two ways. First, a below-optimal level of capital, i.e., $k^b < k^u$, will

reduce profits through its impact on $x(k^b, z)$, since the marginal product of capital will exceed r . Second, for a given level of capital, a higher cost of borrowing, $\tilde{r} > r$, will reduce profits through its impact on capital costs, $\tilde{r}(k^b - a)$. Profitability will naturally impact decisions on entry and dynamic accumulation, which we turn to now.

3.2.2 Occupational Choice

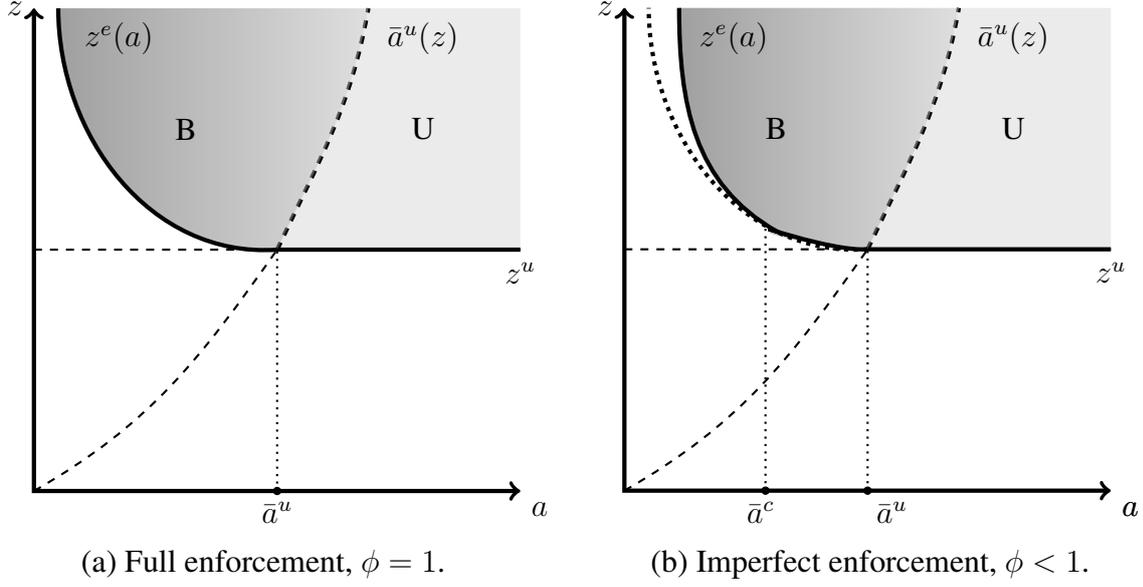
At every moment, agents have the option to either work at a wage w or as an entrepreneur and earn profits. The decision is static, and so they make this decision to maximize their current income flow, $I(a, z) = \max\{w, \pi^*(a, z)\}$. Define the policy function $o(a, z)$ such that $o(a, z) = 1$ if the individual becomes an entrepreneur and zero otherwise.

Financial frictions will influence not only the allocation of capital, but the occupational choice as well. We illustrate this in Figure 3, which shows the occupational choice in (a, z) space for agents facing $\chi = 0$ (perfect competition in the banking sector), a given wage rate, w , and interest rate, r , and two levels of the enforcement variable, ϕ . Online Appendix B.3 contains the formal derivation of the two graphs presented in Figure 3.

Figure 3(a) shows the case of perfect enforcement, i.e., $\phi = 1$, and costly financial intermediation, $\tilde{r}(a, z) - r > 0$. The horizontal line z^u illustrates the cutoff for entrepreneurship in a world with no financial frictions. In this world, all agents with productivity $z > z^u$ would become an entrepreneur producing at their optimal scale regardless of their wealth, and they would share the same marginal productivity of inputs. With financial frictions, however, this is not the case. The threshold is now wealth dependent as illustrated by the solid line $z^e(a)$. The white region below $z^e(a)$ indicates that the extensive margin is distorted as some low-wealth agents become workers despite productivities above the perfect credit z^u threshold. The dark gray shaded area (region B) represents the entrepreneurs who are borrowers. They pay different loan interest rates, depending on their asset a and productivity z , and so the marginal product of capital varies over this region. Agents in the darker region close to the $z^e(a)$ line pay higher interest rates, borrow more, and produce with a higher marginal product of capital. Agents close to the dotted line $\bar{a}^u(z)$ pay a lower interest rate, borrow less, and have a marginal productivity of capital closer to the internal cost of capital, r . The light gray shaded area (region U) displays the agents who are unconstrained entrepreneurs: their wealth exceeds unconstrained capital, $a \geq k^u(z)$. Agents in this region produce at their optimal scale without borrowing and share the same marginal productivity of inputs. Among these entrepreneurs, there is no misallocation of capital.

Figure 3(b) displays the case in which the enforcement of financial contracts is imperfect, such that $\phi < 1$. We still assume that $\chi = 0$. There are two differences compared with perfect enforcement. First, the line $z^e(a)$ becomes steeper when this constraint starts to bind at \bar{a}^u . That

Figure 3: Occupational Choice, $\chi = 0$.



Notes: The light gray shaded area, **U**, contains the measure of unconstrained entrepreneurs. The dark gray shaded area, **B**, displays the measure of constrained borrowers. The white area below the curve $z^e(a)$ represents the measure of workers.

is, imperfect enforcement of financial contracts affects the extensive margin, further constraining poor yet talented agents from becoming entrepreneurs. Second, imperfect enforcement also impacts the intensive margin of the allocation of capital. Focusing on Region B, the entrepreneurs who borrow, as we get closer to the solid line $z^e(a)$, the enforcement constraint binds and entrepreneurs will be producing with a marginal productivity of capital that is above the loan rate they face. However, agents in region B close to the dotted line $\bar{a}^u(z)$ are not limited by the enforcement constraint. Such agents produce with a marginal productivity of capital similar to the loan rate, which varies with their asset and productivity. Region U still represents the unconstrained entrepreneurs.

3.3 Dynamic Optimization

We turn now to the dynamic optimization, which simply involves a savings decision and the stochastic death of entrepreneurial productivity, z , and replacement with a new one, some \tilde{z} .

Given the static optimization that yields instantaneous income, $I(a, z)$, the budget constraint governing the assets of an entrepreneur (a, z) is:

$$\dot{a}(a, z) = I(a, z) + (r - \delta)a - c(a, z). \quad (9)$$

Note that distortions to k^b and \tilde{r} simply influence the dynamics of asset accumulation (and likewise firm growth) through their impacts on income, $I(a, z)$.

They will also influence the dynamics of asset accumulation through the choice of $c(a, z)$ because they impact the incentives to accumulate through the value function and continuation value. Let $V(a, z)$ be the stationary value for individual with the current state (a, z) . The value function satisfies the following stationary Hamilton-Jacobi-Bellman (HJB) equation:

$$\begin{aligned} \rho V(a, z) = & \max_c u(c) + \partial_a V(a, z)(I(a, z) + (r - \delta)a - c) + \\ & \gamma \left[\int_{\mathcal{Z}} V(a, \tilde{z}) \mu(\tilde{z}) d\tilde{z} - V(a, z) \right] \end{aligned} \quad (10)$$

3.4 Equilibrium

We solve for a stationary competitive equilibrium.¹² Individuals differ from one another with respect to their asset and entrepreneurial abilities, (a, z) . Given the invariant distribution of abilities $\mu(z)$, the stationary competitive equilibrium of this economy consists of a stationary distribution of states (a, z) , $H(a, z)$, induced by the decision of the agents and the distribution $\mu(z)$. Prices are given by the wage rate w , the rental price of capital r , and loan interest rates $\tilde{r}(a, z)$. Individuals' optimal behavior was described in detail above, and the policy functions associated with their optimal decisions are $k(a, z)$, $n(a, z)$, $o(a, z)$ and $c(a, z)$. These decisions are consistent with the recursive problem of all agents and with the financial contracts. It remains, therefore, to characterize the market equilibrium conditions and the aggregate law of motion:

1. Equilibrium in the capital market:

$$K := \int_{\{o(a,z)=1\}} k(a, z) H(da, dz) = \int a H(da, dz). \quad (11)$$

2. Equilibrium in the labor market:

$$N := \int_{\{o(a,z)=1\}} n(a, z) H(da, dz) = \int_{\{o(a,z)=0\}} H(da, dz). \quad (12)$$

3. Final goods:

$$\begin{aligned} \int c(a, z) H(da, dz) + \int_{\{o(a,z)=1, k(a,z)>a\}} \tau(a, z) H(da, dz) = \\ \int_{\{o(a,z)=1\}} y(a, z) H(da, dz) - \delta K. \end{aligned} \quad (13)$$

4. The joint distribution $h(a, z)$ evolves according to the following Kolmogorov Forward

¹²Given the continuous-time setup, we can use an efficient numerical algorithm based on Achdou et al. (2022).

equation:

$$0 = -\frac{d}{da} [\dot{a}(a, z)h(a, z)] - \gamma h(a, z) + \gamma \mu(z) \int h(a, \tilde{z}) d\tilde{z}. \quad (14)$$

We have assumed that financial intermediaries' profits are spent outside the economy. Finally, a variant of this fully general equilibrium is that of a small open economy which faces a fixed interest rate, r . In this case, the interest rate is assigned from outside the model, and the capital market clearing equation is dropped as an equilibrium condition.

3.5 Summary of Financial Frictions

Financial frictions have a significant impact when they prevent capital from adequately flowing from the wealthy-but-unproductive to the poor-but-highly-productive. Financial frictions can therefore matter in two ways: (i) statically, by determining which firms receive capital and how much they receive, and (ii) dynamically, by influencing the accumulation of wealth and therefore the endogenous distribution of such firms. The five sources of financial frictions that we have modeled differ from one another in the ways they impact these two channels.

Statically, τ_z is the most benign of the spread-causing frictions, as it leads to high productivity individuals paying the lowest spreads and therefore getting the most capital among borrowers. The quantity constraint, ϕ , misallocates capital directly, allowing high productivity entrepreneurs and wealthy entrepreneurs to access more capital but limiting capital to poor entrepreneurs and low productivity entrepreneurs. Next is τ_0 , which is indiscriminate and hits potential entrepreneurs uniformly. τ_a charges high spreads to poor potential entrepreneurs, and the poor who are most productive have the greatest need for finance. Finally, χ disproportionately hits the poor-and-most-productive, since they have the greatest surplus to borrowing. On its own, χ does not distort the capital allocation, but it can be particularly harmful through the second channel, when combined with a quantity constraint that hits the poor.

Dynamically, all frictions change savings accumulation (and firm growth) in counteracting ways: by decreasing constrained agents' ability to accumulate assets by lowering available income for saving (via the static channel of less capital) but increasing their incentive to save (to self-finance). In addition, however, spreads reduce the ability to accumulate more drastically than an equivalently-binding quantity constraint because they lower income further by increasing costs. On the other hand, the two spread frictions that hit the poor, τ_a and χ (in combination with ϕ), give additional incentives to save, since interest rates fall as assets are accumulated. Similarly, a binding quantity constraint, driven by $\phi < 1$, gives the additional incentive of access to more credit as collateral increases.

4 Calibration

To discipline our quantitative exercises, we calibrate the model to be consistent with macro and micro moments of the Brazilian economy. Our approach is to assign standard values for two parameters common in the literature, and then to jointly calibrate the remaining parameters of the model to the private sector of Brazil during the period 2006-16.

The two assigned values are the inverse intertemporal elasticity of substitution parameter, $\sigma = 1.5$, and the depreciation rate, $\delta = 0.03$. The intertemporal elasticity is in line with most of the literature on consumption surveyed by Attanasio and Weber (2010) and also with the Brazil-specific literature that estimates σ in the range from 1 to 3 (e.g., Gandelman and Hernández-Murillo, 2014; Fajardo, Ornelas, and Farias, 2012). The depreciation rate is in the common range of values used in the macro literature.

The remaining 11 parameters are jointly calibrated to match a set of 12 relevant moments characterizing firm dynamics, concentration, and credit markets.¹³ Firm dynamics and concentration are important determinants of the equilibrium distribution of productivity and wealth. The credit market characteristics help discipline the distortions themselves. The parameters are jointly determined, but we give a rationalization for the choice of moments parameter-by-parameter below.

Our benchmark is a closed economy.¹⁴ We calibrate the subjective discount rate, $\rho = 0.24$, to match an interest rate of 2%, the average real risk-free interest rate over the period from 2005 to 2016. We calculate the risk-free real interest rate as the difference between the real interest rate on Brazilian treasury bills of roughly 6% and the sovereign default risk premium of about 4%.

The production function exponents on capital, $\alpha = 0.33$, and labor, $\theta = 0.39$ determine both capital accumulation and the share of income to entrepreneurs ($1 - \alpha - \theta$). We therefore discipline these by the capital-output ratio, the growth rate of firms (driven by capital accumulation under financial frictions), and the earnings share of the right tail (which is dominated by entrepreneurial income). For Brazil, capital-output ratio of the private sector is 2.55 in 2016 using Feenstra et al. (2015) data, and average annual firm growth in our data is 3.4%. For the income tail, we target the fact that the top 10% of earners receive 56% of total income, according to Morgan (2017). A final production function parameter, the fixed cost, $\kappa = 0.60$, helps determine the minimum efficient scale for an entrepreneur, and hence the average firm size, which

¹³The definition, value, and source of the moments are detailed in Online Appendix A.2.

¹⁴In Online Appendix C, we present a calibration and counterfactual experiments in a small open economy in which we perform the same counterfactuals but fix the interest rate at 2% and do not impose capital market clearing. There arises a distinction between assets and capital, and in the perfect-credit world capital is much higher. Hence, the impacts of spreads and frictions more generally are even stronger relative to this perfect-credit world, but the key channels and messages of the paper are robust to an open economy.

in Brazil is 13 employees per firm.

Next, we have two parameters that determine the distribution and dynamics of productivity. The Pareto parameter, $\eta = 3.67$, determines the thickness of the productivity tail, and is disciplined by the relative importance of large firms in the tail. We target the share of the top 10% of firms in total employment of 0.77, which we calculate in our data. The Poisson arrival rate of a new productivity draw, $\gamma = 0.39$, is disciplined by the exit rate of establishments, which is 11% in Brazil. The large gap between the death rate of productivity and the death rate of firms is already an indicator of potential severe misallocation on the extensive margin.

Finally, we have five parameters related to the financial sector: the limited enforcement quantity parameter, ϕ , the intermediation technology parameters, τ_0 , τ_a and τ_z , and the intermediaries' bargaining power in financial contracting, χ . We discipline these parameters to match the overall level of financial development, measured as the ratio of external finance to GDP, which averages 0.49, and moments targeting the distribution of credit and spreads. In our data, only 26% of entrepreneurs have credit. We generate spreads moments using firm-level spreads (credit-weighted at the firm-level) as their primitives. Taking simple averages across firms, we find that both the average spread is high, 64 percentage points, and varied, with a standard deviation of 40 percentage points.¹⁵ When credit-weighted across firms, the average spread remains quite high at 45%. Still, the sizable difference between the two indicates how negatively correlated the use of credit is with spreads; large firms pay lower spreads and are also the larger borrowers.

The resulting intermediation technology parameters are $\tau_0 = 0.08$, $\tau_a = 0.73$, $\tau_z = 0.94$, and $\chi = 0.10$. These spread-causing parameters impact spread moments in different ways. Heuristically ranking the four spread-causing parameters, χ leads to smallest standard deviation of spread and disproportionately impacts the weighted spread relative to the unweighted spread, followed by τ_0 and χ , while τ_a leads to the largest standard deviation and most impacts the weighted spread. The relatively low level of bank market (i.e., bargaining) power, $\chi = 0.10$, reflects the sizable gap between the simple average spread and the credit-weighted spread and negative correlation between spreads and credit usage that drives that gap.

These frictions explain sizable variation in spreads. Comparing the credit-weighted spread of 0.28 with the value of τ_0 of 0.08, almost 1/3 of the weighted spread comes from intermediation costs, which do not vary with firm characteristics. To put the spread-inducing parameters (τ_a , τ_z , and χ) into perspective, consider two extremes. The intermediation costs imply that an entrepreneur with no assets and a minuscule productivity would face a spread of 175 percentage points, coming purely from intermediation costs. Similarly, a very productive entrepreneur with

¹⁵This is different from the simple average spread and simple standard deviation of spreads presented in Table A1. We washed out factors from the data which are not present in our model, such as differences in maturity and non-performing loans. We accomplish this by using regression (4) in Table 1. See Online Appendix A.2 for more details.

no assets—with profits about three times the equilibrium wage—would pay a spread of about 45 percentage points, a mix of intermediation costs and market power.

The enforcement parameter, $\phi = 0.59$, never binds in the benchmark calibration. We therefore choose the lowest enforcement level consistent with a non-binding constraint. However, this effectively gives us one fewer parameter with which to match the data moments.

Table 2: Calibration and Model Fit

Parameter Values		
Parameter	Description	Value
<i>2 assigned parameters</i>		
σ	Coefficient of relative risk aversion	1.50
δ	Depreciation rate	0.03
<i>11 calibrated parameters</i>		
ρ	Subjective discount rate	0.24
α	Elast. of y with respect to k	0.33
θ	Elast. of y with respect to n	0.39
κ	Fixed cost of production	0.60
η	Curvature of the Pareto distr.	3.67
γ	New productivity arrival rate	0.39
ϕ	Enforcement parameter	0.59
τ_0	Interm. costs - independent factor	0.08
τ_a	Interm. costs - elast. of assets	0.73
τ_z	Interm. costs - elast. of productivity	0.94
χ	Bank barg. power in a loan	0.10
Model Fit		
12 Targeted Moments	Data	Model
Risk-free bond rate	0.020	0.020
Capital-output ratio	2.55	1.89
Average firm growth rate	0.034	0.034
Top 10% earners' income share	0.56	0.52
Average firm size	13	14
Top 10% firms' employment share	0.77	0.71
Firm exit rate	0.11	0.10
External finance to GDP ratio	0.49	0.49
Fraction firms with credit	0.26	0.23
Average spread (unweighted)	0.64	0.69
Average spread (credit-weighted)	0.45	0.28
Standard deviation of spread	0.40	0.34

The above discussion about parameter values and calibration targets are summarized in the top panel of Table 2, while the model fit is shown in the bottom panel. The calibrated values of common parameters generally compare well with previous calibrations in the literature. The micro returns to scale ($\alpha + \theta$) and Pareto parameter (η) both fall in the range (and indeed between) those in similar models calibrated for the United States (e.g., Buera, Kaboski, and

Shin, 2011) and India (e.g., Buera, Kaboski, and Shin, 2020). The value of κ is between those calibrated for services and manufacturing in the two-sector calibration in Buera, Kaboski, and Shin (2011). The calibrated value of $\phi = 0.59$ indicates stronger enforcement than the value of 0.15 for India in Buera et al. (2020), but this is consistent with higher levels of external finance-to-GDP in Brazil (0.49 vs. 0.35), and also a result of the spreads themselves reducing levels of credit. The model is therefore a fairly close calibration to the existing literature, except that (i) the enforcement problem/quantity constraint is weaker, (ii) we require a high discount rate to assure that firms borrow despite the high spreads, and (iii) the arrival rate of new productivity is much higher (compare 0.39 with 0.11 in Buera et al. 2011).

With more moments than parameters, the overall fit of the model is good though not perfect. We capture a high level of spreads and high dispersion in spreads, but the fit is not exact, especially the credit-weighted spread, which is lower in the model. Moreover, hitting these spreads requires the capital-output ratio to be smaller than in the data, though still within the range of commonly accepted values.

In Section 5.2, we will also consider an alternative calibration that bases spread targets on the spread data for working capital rather than calculating moments using loan data across all types of credit. These produce considerably smaller levels and variations of spreads, and so, alternatively, it is also an indicator of the relative importance of spreads in economies with more moderate spreads.

5 Quantitative Analysis

We now use the calibrated model to evaluate the quantitative importance of financial frictions. We do so by comparing the results for output, wages, TFP, capital, and the credit market under the stationary distributions of different alternative parameter values. Finally, we assess the role of spreads in a calibration with more moderate spreads.

5.1 Benchmark Results

We start by looking at the impacts of spreads vs. quantity constraints on aggregates. The top panel of Table 3 summarizes these impacts by presenting results relative to their values in a counterfactual world of perfect credit; i.e., the same parameter values (i.e., no enforcement problem, $\phi = 1$) but also no intermediation costs, $\tau_i = 0$, or intermediary market power, $\chi = 0$, that lead to spreads. This perfect credit benchmark is illustrated in Column (1).

Column (2) shows the full impact of the calibrated spread frictions in our benchmark economy. The τ_i and χ frictions lead to a considerably lower ratio of credit to GDP, just 32% of the level

Table 3: Impacts of Credit Frictions on Development

Value Relative to Perfect Credit:	Perfect Credit (1)	Benchmark (2)	No Spread Disp. (3)	Only Quant. Constr. (4)
<i>Aggregate values relative to perfect credit world:</i>				
GDP	1.00	0.61	0.64	0.76
TFP	1.00	0.72	0.77	0.84
Wage	1.00	0.68	0.76	0.80
Capital	1.00	0.59	0.55	0.75
Credit/GDP	1.00	0.32	0.32	0.32
Interest rate	0.14	0.02	0.02	0.02
Firm growth	0.06	0.03	0.00	0.03
Exit rate	0.30	0.10	0.18	0.15
Avg. firm size	11	14	8	8

Notes: Column (2) uses the values calibrated in Table 2, in particular $\tau_a = 0.73, \tau_z = 0.94, \chi = 0.10, \tau_0 = 0.08$, and $\phi = 0.59$. The other columns keep the credit-to-GDP ratio constant while changing parameters. Relative to benchmark values, Column (1): $\tau_i = \chi = 0$, Column (3): $\tau_a = \tau_z = \chi = 0$ and $\tau_0 = 0.30$, and Column (4): $\tau_i = \chi = 0$ and $\phi = 0.04$, calibrated to match credit/GDP in the benchmark (and data).

in the perfect credit economy. The lack of credit leads to substantially lower capital in the economy, 59% of the level in the perfect credit counterfactual. Moreover, the credit frictions misallocate capital, which leads to TFP being lower, just 72% of the perfect credit economy. In aggregate, the lower capital and less well allocated capital leads to GDP being just 61% of the perfect credit-world GDP. The level of wages is just 68% of their level under perfect credit. The slightly larger impact on GDP relative to wages is a result of the the spreads being an important direct reduction to GDP.

Column (2) is a result of both the level and dispersion of spreads (in addition to some quantity restrictions). We contrast this benchmark with counterfactual explanations for the observed financial underdevelopment. That is, in Columns (3) and (4), we calibrate different parameters to match the credit/GDP ratio in the benchmark model (and data). In Column (3), we eliminate all dispersion in spreads by setting the spread dispersion-causing parameters to zero, $\tau_a = \tau_z = \chi = 0$. We then calibrate $\tau_0 = 0.30$ to match the credit to GDP in the benchmark economy. Relative to the benchmark, an economy with the same level of credit to GDP and the same average spreads but no dispersion exhibits somewhat higher TFP (0.77 vs. 0.72) and somewhat lower capital (0.55 vs. 0.59). Altogether, the higher TFP and lower capital lead to only slightly higher levels of output (0.64 vs. 0.61). Overall, however, although spread dispersion lowers output, the *dispersion* of spreads is not the primary cause of output losses in the benchmark calibration but rather the high *level* of spreads is.

In Column (4), we eliminate all spreads (levels and dispersion) by setting $\tau_a = \tau_z = \tau_0 = \chi = 0$, and instead increase the enforcement friction (by decreasing ϕ to 0.04) in order to again match benchmark credit to GDP. This counterfactual is closest in spirit to the existing literature (e.g., Buera, Kaboski, and Shin, 2011), with only quantity constraints but no spreads. The impacts on aggregates are much milder than in the benchmark model with spreads. The levels of GDP (0.76 vs. 0.61), wages (0.80 vs. 0.68), TFP (0.84 vs. 0.72), and capital (0.75 vs. 0.59) are all markedly higher. Hence, if only quantity restrictions were modeled, the predicted impacts would be 61% (i.e., $(1-0.75)/(1-0.59)$) as large on capital, 62% as large on GDP, and 57% as large on TFP. Thus, the impacts are over 50% (e.g., $(100\%-61\%)/61\%=64\%$) larger in a model incorporating spreads.

The bottom panel examines the model implications for the interest rate on savings and firm dynamics. All simulations with frictions yield a lower value for the interest rate than the perfect credit world. Although capital falls by slightly more than TFP, the interest rate on savings is substantially lower in the benchmark, just 2% relative to 14% in the perfect credit world. With spreads and quantity frictions, there is a decline in demand for credit at any interest rate. These frictions also increase the supply of savings by providing strong motives for saving for precautionary and self-financing reasons, and to secure better credit spreads. All of this drives down the interest rate, and these forces are largely independent of the source of financial frictions.

In the perfect credit world, firms enter at their optimal size, and only change abruptly in response to a productivity change that leaves them as entrepreneurs. Average firm size is constant with age, but firm growth is positive (0.06) on average because increases are larger in percentage terms than the mirror decreases.¹⁶ Comparing the benchmark with the perfect credit world, firm growth is slower (0.03 vs. 0.06), exit (which equals entry) is smaller (0.10 vs. 0.30), and the average firm has slightly more workers (14 vs. 11) in the benchmark model. Comparing Columns (2) to Columns (3) and (4), low exit rates are driven by financial frictions across the board. However, the benchmark has faster growth than the model without dispersion, larger firms, and especially lower exit rates.¹⁷

In sum, spread frictions are an (even *the most*) important source of the losses from credit market imperfections. Modeling financing spreads matters since it is quantitatively important for the aggregate impacts of financial frictions on development because it leads to larger aggregate losses.

Given this finding, Table 4 delves into the driving forces behind spread frictions in more depth

¹⁶For example, growing from 5 to 10 workers is a 100 percent increase, while shrinking from 10 to 5 is only a -50 percent increase.

¹⁷Indeed, the smaller firms in the economy without dispersion, a result of no growth on average and higher exit rates, is the main reason that wages are higher in the no dispersion economy: more entrepreneurs means workers are scarcer.

by evaluating the roles of the various types of spread-causing frictions: market power (governed by χ), the uniform intermediation cost (τ_0), the asset-dependent intermediation cost (governed by τ_a), and the productivity-dependent intermediation cost (governed by τ_z). The top panel gives the impacts on aggregates (expressed relative to the perfect credit values), the middle panel presents the remaining spread and credit moments that we target, and the bottom panel shows some measures related to firm dynamics.

Columns (2)-(5) shut off one of these individual frictions one at a time, and the results demonstrate that the z -dependent cost is the most powerful force in the calibration, while market power has essentially no impact. Eliminating the former leads to the largest gains in output (0.61 to 0.73), for example, because it reduces spreads the most (e.g., average weighted spreads fall by 16 percentage points). It also increases firm growth, entry/exit, and firm size the most. In contrast, eliminating market power has no effect because market power is so weak in the benchmark (and does not even interact with the quantity constraint which is nonbinding).

In these exercises, the total independent impacts of turning off each of the frictions on output is smaller than the overall impacts of spread frictions. In Online Appendix D, we instead turn on each single friction starting from the perfect credit economy, and the total of these individual exercises greatly exceeds the overall impact. Both underscore that the multiple individual frictions have smaller marginal impacts, either because they substitute for one another or there is a weakening marginal effect of increasingly higher spreads.

Columns (6)-(9) instead load up all the spreads onto a single individual friction, while eliminating the others, in order to demonstrate how problematic the different frictions *could* be. Here we attempt to keep these comparable by targeting the credit-weighted average spread of 0.28 in each of the alternative calibrations. However, market power, χ , (Column 6) and asset-dependent intermediation costs, τ_a (Column 8) are not capable of producing such high levels of spreads on their own. Market power is bounded, and in the case of asset-dependent spreads, spreads are avoidable since assets are endogenous. In both cases, we simply choose values that approach their maxima.¹⁸

¹⁸For market power, we choose χ of 0.99. For τ_a , the calibrated value is enormous, 457, indicating a person with no assets would be offered an interest rate of 45,700%! (Such a person of course does not borrow.)

Table 4: Isolated Impacts of Spread-Causing Frictions

	Eliminating Frictions					Single Friction Calibrations			
	Benchmk (1)	No Market Power (2)	No Uniform Cost (3)	No a - depend. Cost (4)	No z - depend. Cost (5)	All Market Power [†] (6)	All Uniform Cost (7)	All a - depend. Cost (8)	All z - depend. Cost (9)
<i>Aggregate values relative to perfect credit world:</i>									
GDP	0.61	0.61	0.65	0.64	0.73	0.48	0.64	0.43	0.63
TFP	0.72	0.72	0.75	0.77	0.81	0.57	0.78	0.58	0.75
Wage	0.68	0.68	0.72	0.74	0.78	0.46	0.77	0.48	0.72
Capital	0.59	0.60	0.64	0.59	0.73	0.56	0.55	0.42	0.60
Credit/GDP	0.32	0.32	0.55	0.36	0.81	0.58	0.36	0.01	0.30
<i>Firm credit spread moments:</i>									
Interest rate	0.02	0.02	0.04	0.03	0.07	0.06	0.03	-0.03	0.02
Avg. (unweighted)	0.69	0.67	0.55	0.40	0.33	0.02	0.28	0.21	0.51
Avg. (weighted)	0.28	0.27	0.18	0.28	0.12	0.02*	0.28	0.19*	0.28
Std. deviation	0.34	0.33	0.33	0.06	0.28	0.01	0.00	0.03	0.11
Frac. with credit	0.23	0.23	0.28	0.39	0.63	0.32	0.58	0.00	0.30
Firm growth	0.03	0.03	0.08	0.00	0.18	0.04	0.00	0.03	0.00
Exit rate	0.10	0.10	0.12	0.15	0.27	0.02	0.20	0.02	0.13
Avg. firm size	14	14	17	11	26	7	9	7	11

Notes: Column (1) parameter values are those calibrated in Table 2. The other columns keep the interest rate constant while changing parameters. Relative to these values, Column (2): $\chi = 0$, Column (3): $\tau_0 = 0$, Column (4): $\tau_a = 0$, Column (5): $\tau_z = 0$, Column (6): $\tau_i = 0$ and $\chi = 0.01$, Column (7): $\tau_a = \tau_z = \chi = 0$ and $\tau_0 = 0.28$, Column (8): $\tau_0 = \tau_z = \chi = 0$ and $\tau_a = 457$, and Column (9): $\tau_0 = \tau_a = \chi = 0$ and $\tau_z = 1.65$. For Columns (6) and (8), the calibrated value is chosen to match the weighted spread in the benchmark.

[†]: The single parameter alone cannot match the weighted spread of 0.28. Hence, Columns (6) and (8) yield spreads substantially below target but approaching their peaks.

We verify that market power and asset-dependent intermediation costs can have strong impacts. The results in Column (6) illustrate that the negligible results for market power in Column (2) are due to the low calibrated levels and not because intermediary market power is inherently benign. Indeed, although maximal market power can only produce an average credit-weighted spread of 0.02, it can lower output by more than half (0.48) relative to the perfect credit. Similarly, asset-dependent spreads in Column (8) can only produce an average credit-weighted spread of 0.19, but it lowers output to 0.43. Both do so by greatly lowering entry/exit, indicating that productivity shocks do not lead to exit. In both cases, poor-but-productive agents would keep little of their entrepreneurial income and so would have little incentive to save in anticipation or ultimately enter. As their entry is choked off, wealth and productivity become increasingly decoupled. The higher incentive to escape the endogenous spreads leads to firm growth after entry, nevertheless. In the case of market power, total credit is not disproportionately limited, nor is the fraction of entrepreneurs who borrow. In the case of asset-dependent intermediation costs, however, credit overall is choked off, as only a negligible fraction borrow. Both scenarios are more extreme than in the benchmark calibration. In both cases, firms are also substantially smaller, so there are more entrepreneurs but not the most productive entrepreneurs.

In contrast, Columns (7) and (9) show that while the uniform intermediation cost, τ_0 , and productivity-dependent intermediation cost, τ_z , are able to generate large average spreads on their own, their impact on output is less than in the benchmark economy—0.64 and 0.63, respectively, compared to 0.61 in the benchmark. Entry/exit rates remain relatively high, 0.20 and 0.13, respectively, vs. 0.10 in the benchmark. Again, τ_0 and τ_z lower firm growth, as firm growth is essentially eliminated when spreads are loaded up on them. Finally, we again see that the interaction of frictions is important in driving sharper losses. Focusing on spreads, the z -dependent cost leads to the highest unweighted spreads and standard deviation among the four frictions, and demonstrates the feature that drives its relatively large role in the calibrated benchmark.

To summarize, first, the productivity-dependent intermediation cost (τ_z) is the dominant independent driver of the aggregate impacts in the benchmark economy, but interactions with other frictions also matter substantially. Second, market power and asset-dependent intermediation costs can have steep aggregate impacts by choking off entry and making entry hinge on wealth rather than productivity. Third, uniform and productivity-dependent intermediation causes lower firm growth, whereas asset-dependent intermediation costs increase it.

5.2 Moderate Spread Results

We now evaluate the impact of the level and dispersion of spreads in a model with more moderate levels of spreads. To discipline this analysis, we calibrate to the level and dispersion of spreads based on only working capital loans. While we believe the observed variation of spreads across loan types is important variation in the real cost of capital across firms, it is also true that our model has only within-period borrowing, for which working capital is perhaps more relevant.¹⁹ In addition, a more moderate calibration may be more indicative of the relevance of spreads in other economies, where the level of spreads and dispersion is lower.

Concretely, we replace the three spread moments in Table 2 to the empirical analogs for working capital, lowering the targeted average unweighted spread from 0.64 to 0.38, the average weighted spread from 0.45 to 0.14, and the standard deviation of spreads from 0.40 to 0.23. The major change in the calibration is a change in the relative importance of the various financial frictions. Namely, the relatively benign uniform cost and the productivity-specific intermediation cost both fall ($\tau_0 = 0.08$ to 0.04 and, especially, $\tau_z = 0.94$ to 0.29). At the same time, τ_a , the asset-specific intermediation cost parameter, rises mildly from 0.73 to 0.91, while χ , the bank bargaining power, rises markedly from 0.10 to 0.52. Moreover, the market power now interacts with a lower enforcement parameter (ϕ falls from 0.59 to 0.20) that yields a quantity constraint that now binds for some.²⁰ The calibrated fit (included in Online Appendix E) is quite similar, with slightly better fit of the spread moments, but with a larger average firm size than in the data (17 vs. 13).

Table 5 shows the analogous aggregate impacts in Table 3 but for this calibration. Our conclusions with respect to aggregate impacts are robust to this alternative calibration of the benchmark model, but dispersion plays a larger role. Specifically, the benchmark drops in output, TFP, and capital are very similar, with a somewhat stronger drop in wages (40% vs. 32% in Table 3) and weaker drop in credit/GDP (62% vs. 68%). However, in this moderate spread economy, the alternative models imply much weaker impacts on aggregates. While frictions in the benchmark model yield a 43% drop in GDP, the model with no spread dispersion yields only a 27% drop, indicating that both the level and dispersion of spreads play an important role in this world with more moderate spreads. This happens because of the relative importance of the different frictions: market power and asset-dependent spreads are more damaging than the z -dependent spread in the earlier calibration because the former limits the entry of productive firms. In the no dispersion model, firm growth is not completely eliminated under moderate spreads. Again, this analysis shows that in a world with more moderate spreads modeling both

¹⁹As suggested by Figure 2, using investment loans would yield similar moments. The key is excluding earmarked credit (which exhibit much lower interest rates) and especially revolving credit and other credit (with much higher levels and variation in rates).

²⁰In addition, γ , the arrival rate of productivity, falls somewhat from 0.39 to 0.30 and κ , the fixed cost of production, rises from 0.60 to 0.66. All other parameters are unchanged.

Table 5: Impacts of Moderate Credit Frictions on Development

Value	Perfect		No	Only
Relative to	Credit	Benchmark	Spread	Quant.
Perfect	Credit	Benchmark	Disp.	Constr.
Credit:	(1)	(2)	(3)	(4)
<i>Aggregate values relative to perfect credit world:</i>				
GDP	1.00	0.57	0.73	0.81
TFP	1.00	0.69	0.82	0.87
Wage	1.00	0.60	0.80	0.83
Capital	1.00	0.57	0.68	0.81
Credit/GDP	1.00	0.38	0.38	0.38
Interest rate	0.14	0.02	0.01	0.01
Firm growth	0.05	0.04	0.04	0.05
Exit rate	0.30	0.07	0.16	0.15
Avg. firm size	12	17	11	10

Notes: Column (2) uses the values $\tau_a = 0.91$, $\tau_z = 0.29$, $\chi = 0.52$ and $\tau_0 = 0.05$ and the other parameter values are those calibrated in Online Appendix Table E5. The other columns keep the credit-to-GDP ratio constant while changing parameters. Relative to benchmark values, Column (1): $\tau_i = \chi = 0$, Column (3): $\tau_a = \tau_z = \chi = 0$ and $\tau_0 = 0.15$, and Column (4): $\tau_i = \chi = 0$ and $\phi = 0.04$, calibrated to match credit/GDP in the benchmark (and data).

spreads and their dispersion is critical to evaluating the losses from financial frictions.

6 Life Cycle Dynamics in Data and Model

We now examine the firm dynamics in the model and data in more depth as an additional testable implication of the model's mechanisms. To compare with the data, we use the model to generate a Monte Carlo simulation of a population of agents. From this sample, we generate a comparably sized sample of firms.²¹ Recall that our data for Brazil contain annual observations of firm size (number of employees), spreads, and age for firms who borrow, but we lack data on capital and, consequently, productivity. We therefore focus our analysis on the life cycle dynamics of firms. We first evaluate the life cycle pattern of spreads to evaluate our mechanism, then turn to its implications for firm growth patterns, before evaluating misallocation over the lifecycle.

²¹The samples are not identical, since entry is endogenous in the Monte Carlo simulation, but both samples are so large that the difference is irrelevant.

6.1 Spread Patterns

By construction, our model reproduces some moments of the level and variation of spreads, but it also predicts that spreads have components that vary both across firms and over their life cycles. Much of the former variation is exogenous, whereas the latter is endogenous. An important question is whether these patterns are quantitatively consistent with those in the data. Table 6 compares the results of regressions of $spread_{i,t}$ on firm i 's characteristics. Columns (1) and (2) simply reproduce the relevant coefficients from the regression results in Columns (4) and (5) of Table 1. (We omit the constants which are less meaningfully comparable since the data regressions also include risk-related controls, loan controls, and time fixed effects.) We compare these to the benchmark model analogs in Columns (3) and (4).

The size and life cycle profiles of spreads in the benchmark model match the data reasonably well though not perfectly. Qualitatively, we match that large firms pay lower spreads and that spreads are decreasing and convex with age. Specifically, both model and data show substantial and significant decrease in spreads with firm size, and a significant, declining but convex life cycle of spreads. Quantitatively, the relationships are stronger in the model but not egregiously. The strength of these relationships is roughly twice as strong in the model, and the standard errors are smaller as well. (Analogous of the two panels in Figure 1 showing the model patterns relative to the data patterns are included in Online Appendix F.) Size and life cycle together explain a substantial share though not all of the variation: 27.2% in the data and 54.4% in the model. Individually, the semi-elasticity coefficient on size of -14.98 in Column (4) is about twice the -7.3 in the data in Column (3), while the estimated quadratic age the model estimates in Column (4) imply a drop of 29 percentage points in the spread between entry and age 10. The corresponding drop in the data is roughly 11 percentage points (recall Figure 1 and see Online Appendix F for comparable graphs of the simulated data).

Moreover, both magnitudes are reasonable in relation to other work. For example, Midrigan and Xu (2014) report drops in average product of capital (which equals marginal product of capital in their model and our model) of 15 percentage points in China and 21 percentage points in Korea (but an increase of 25 percentage point in Colombia over the life cycle). The spread dynamics are important, since they impact the ability and incentives to accumulate assets endogenously. We turn now to the implications of spreads for firm size dynamics.

6.2 Firm Growth

To capture patterns of firm size with respect to credit and life cycle in the model and data, we regress the growth of firm i , $\frac{n_{i,t+1}}{n_{i,t}}$ on log size, a quadratic in age, $h_{i,t}$, an indicator of whether

Table 6: Spreads and Firm Characteristics: Data vs. Model

	Data [†]		Model	
	(1)	(2)	(3)	(4)
Size (ln)	-7.472*** (0.151)	-7.309*** (0.152)	-17.282*** (0.008)	-14.979*** (0.009)
Age		-0.129*** (0.0339)		-3.603*** (0.007)
Age ²		0.001*** (0.0003)		0.075*** (0.0001)
Loan controls	Yes	Yes	-	-
Time FE	Yes	Yes	Yes	Yes
Firm FE	No	No	No	No
Observations	6,926,846	6,926,846	3,209,321	3,209,321
R-squared	0.272	0.272	0.469	0.544

Notes: The dependent variable is $Spread_{i,t}$. † The “Data” results reproduce regression coefficients in Columns (4) and (5) of Table 2. Both model and data samples include only firms with credit. Standard errors are in parentheses; * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$

or not a firm borrowed, $\mathbb{I}_{loan,i,t}$, the spread, $spread_{i,t}$, and a firm fixed effect, β_i :

$$\frac{n_{i,t+1}}{n_{i,t}} = \beta_i + \beta_n \log n_{i,t} + \beta_a h_{i,t} + \beta_{a^2} h_{i,t}^2 + \beta_{loan} \mathbb{I}_{loan,i,t} + \beta_{\bar{r}} spread_{i,t} + \beta_i + \varepsilon_{i,t}. \quad (15)$$

The results are presented in Table 7. We again compare the results for the data to those of the simulated model, but here we add two alternative models without spread variation as well: (i) the model with a single common spread but no dispersion, and (ii) the status quo model with only a quantity constraint. Comparing the first two columns with the data and our benchmark model, we see similar patterns. (We again omit the constants since their comparison is less meaningful, given the set of controls in the empirical regression.) First, the age coefficients in both reflect the fact that most firms are relatively young in both the data and benchmark model. They capture a declining, convex relationship of growth over the first 15 years of life cycle and the corresponding concave increase in size over the first 15 years. Second, large firms grow slower than smaller firms. Both the life cycle and firm size relationships are somewhat stronger in the benchmark model than in the data, but again not egregiously so. Third, getting a loan is associated with subsequent growth: 4 percentage points faster growth in the data and about 5 percentage points in the model. The only difference between model and data is that high interest rates are associated with faster growth in the model, a combination of high willingness to borrow among the most productive with strong constraints, whereas high spreads are associated with (very) weakly slower growth in the data. Regressions in both model and data explain a small share of the variation in growth.

Table 7: Firm Growth Patterns - Firm Growth(t+1,t): Data vs. Model

	Data	Model		
	(1)	Bench. (2)	No Disp. (3)	Quant. Constr. (4)
Size (ln)	-0.568*** (0.00089)	-0.77919*** (0.00084)	-0.77281*** (0.00084)	-0.55569*** (0.00041)
Age	-0.0196*** (0.00016)	-0.08905*** (0.00032)	-0.08912*** (0.00028)	-0.09876*** (0.00014)
Age ²	0.00014*** (1.31e-06)	0.00019*** (4.45e-06)	-0.00021*** (4.63e-06)	0.00023*** (2.48e-06)
Got loan	0.0401*** (0.0007)	0.04788*** (0.0038)	0.63176*** (0.00202)	0.52167*** (0.00123)
Spread	-0.00028*** (4.98e-06)	0.01182*** (0.00006)		
Loan controls	Yes	-	-	-
Time FE	Yes	Yes	Yes	Yes
Firm FE	Yes	Yes	Yes	Yes
Observations	6,582,972	31,698,562	22,414,620	23,355,214
R-squared	0.274	0.11	0.1444	0.2006

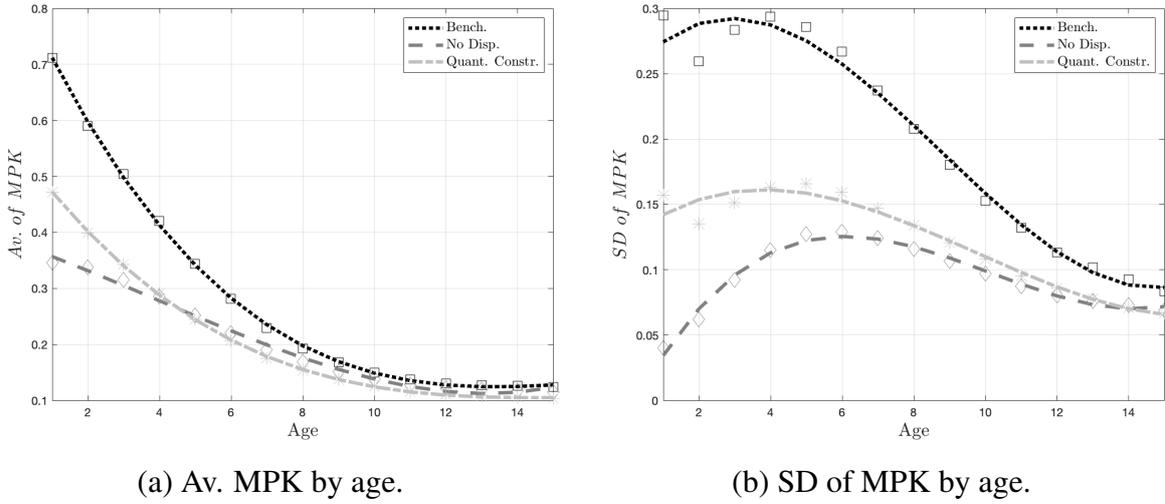
Notes: t statistics in parentheses; * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$. “Bench.” uses the benchmark parameter values in Table 2. “No Disp.” uses the no dispersion calibration, where $\tau_a = \tau_z = \chi = 0$ and $\tau_0 = 0.30$. “Quant. Constr.” uses the only quantity constraint calibration, where $\tau_i = \chi = 0$ and $\phi = 0.04$.

Columns (3) and (4) are similar to the benchmark model, with a few exceptions. First, naturally, neither model has variation in spreads, and so that variable is omitted. Second, the no dispersion model shows a declining growth rate with age, but the relationship is concave rather than convex, and therefore different than the data. (Recall from Table 3 that average firm growth is much lower in the no dispersion model, but this cannot be illustrated since the constants cannot be compared.) Second, the quantity-constraint model shows a somewhat weaker relationship with firm size, more in line with the data. Third, however, both the no dispersion in spreads model and the quantity-constraint model show much faster growth for firms that received loans compared to this relationship in the data. Fourth, most of the firms in simulations receive credit in both the model with no dispersion and the model with only the quantity constraint, whereas in the data, only 26% of firms receive credit.

6.3 Misallocation over the Life-Cycle and Model Identification

Finally, in Figure 4 we examine the extent of capital misallocation over the life cycle, which vary considerably across the three models (benchmark, no dispersion and quantity constraint only). Recall from Table 3 that, in the aggregate, productivity is substantially lower in the benchmark model than in the no dispersion and quantity constraint models. (Output in the

Figure 4: Misallocation over the Life Cycle



Notes: “Bench.” uses the benchmark parameter values in Table 2. “No Disp.” uses the no dispersion calibration, where $\tau_a = \tau_z = \chi = 0$ and $\tau_0 = 0.30$. “Quant. Constr.” uses the only quantity constraint calibration, where $\tau_i = \chi = 0$ and $\phi = 0.04$. The dotted black line is the best polynomial fit of the benchmark model. The squares are the associated values. The dark gray dashed line is the best polynomial fit of the no dispersion calibration model. The diamonds are the associated values. The light gray dashed line is the best polynomial fit of the quantity constraint calibration model. The stars are the associated values.

benchmark and no dispersion models is nevertheless similar because of lower capital accumulation in the no dispersion case). We find that average productivity patterns with age are quite similar, so we assess the allocation of capital across entrepreneurs.

Specifically, the left panel of Figure 4 shows the average marginal product of capital (MPK) over the life cycle for the three models. The average MPK starts out much higher in the benchmark model, reflective of the high average spreads that asset-poor entrants face and lower levels of capital they produce with. In the case of no dispersion, spreads for entrants are no larger than for other firms and so they utilize more capital upon entry and have lower MPK. The quantity constraint lies intermediate between the two. As assets are accumulated over time, the MPK for all three models converges to a level just above 0.1, so on average it is young firms who are short capital and bear the brunt of financial frictions, but this is especially the case for the benchmark model.

A more direct measure of capital misallocation across heterogeneous firms is the standard deviation of the marginal product of capital across firms. The right panel of Figure 4 shows that the life cycle patterns of this measure of misallocation vary considerably across the three models. In particular, with firms paying different interest rates, the dispersion in MPK is much higher in the benchmark model especially for younger firms. As assets are accumulated, spreads fall and converge, lowering the standard deviation of MPK. In the case of no dispersion in interest rates,

variation in the MPK simply stems from some firms financing fully internally while others face the uniform spread on external finance, and the dynamics reflects a change in that fraction. Again, the quantity constraint falls intermediate between the two. While these differences in misallocation are more persistent over the lifecycle than the differences in average MPK, they again are concentrated in the early years.

7 Conclusion

In this paper, we explored the effects of financing costs on aggregate development and firm dynamics. Using rich administrative loan-level data sets, we presented evidence of the high level and variation of interest rate spreads on firms' credit in Brazil.

Moreover, we augmented a standard model of credit-constrained entrepreneurs with interest rate spreads that arise from intermediation costs and financial intermediaries' market power. We calibrated the model to match key characteristics of the Brazilian economy. The quantitative results show that credit spreads have larger impacts on development aggregates than the collateral constraints typically considered in the literature. The implied dynamics of spreads and growth over the firm life cycle are broadly consistent with empirical patterns, and the importance of spread dispersion implies that spreads are most distorting for young entrants. Together, our findings therefore indicate that financial frictions are more important than previously believed and that interest rate spreads are an important friction to consider.

Our study also motivates future work on the causes of credit spreads to improve financial development. Spreads arising from market power or falling disproportionately on small firms are particularly harmful, so they should get more focus in policy discussions and research. In particular, empirically identifying the sources of these frictions is important.

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Online Appendix for

“Financing Costs and Development”

by Tiago Cavalcanti, Joseph P. Kaboski, Bruno Martins, and Cezar Santos

A Data Appendix

This appendix provides a more detailed description of the data (together with summary statistics) and the definition, sources, and values of the moments targeted in our calibration.

A.1 Data Description

Data on bank loans to all formal firms in Brazil are from the Brazilian Public Credit Registry (SCR - *Sistema de Informações de Crédito*). This is a confidential loan-level database, managed by the Central Bank of Brazil. It contains information of all formal loans granted from January 2006 until December 2016. We observe the lender, borrower, loan type (investment, working capital, revolving, etc.), size of the loan, the interest rate on the loan, the loan maturity, default rates, and whether or not it was at an earmarked interest rate. This dataset allows us to construct information on the borrower-lender relationship, such as the length of a firm-bank relationship.

The other main dataset that we use in our empirical analysis is RAIS (*Relação Anual de Informações Sociais*), a matched employer-employee administrative dataset covering all formal firms in Brazil. This is a mandatory annual survey maintained by the Ministry of Labor. RAIS provides information on firms, such as sector of activity and location, and information on employees, which we use to construct firm-level measures of employment and labor compensation. It is also possible to identify the date of entry and exit of firms. With this dataset, we can capture important firm dynamics for all formal firms in Brazil.

Using the unique firm tax identifier, we merge the SCR and RAIS datasets. Financial firms, public administration, non-governmental organizations and multilateral agencies were excluded in our sample. Our database therefore comprises more than 25 million observations between 2006 and 2016.

To be precise about how the dataset is used, the Brazilian Central Bank has a loan-level dataset in each year. The underlying loan-level data are aggregated to construct credit flows and stocks at the level of firm-bank-loan type-year combination. This is the dataset we were provided. Similarly, we use loan-weighted averages to construct average spreads, maturity, non-performing loans, and other measures at these levels. We have the firm identifier, so we can

control for firm fixed effects or aggregate loans at the firm level weighting by the loan amount. In the latter, the loan interest rate corresponds to the weighted average of all loans a firm had in a particular year. Similar procedures can be used to construct loan characteristics at the firm level, but our main regressions are based on firm-bank-loan type observations.

Table A1 provides descriptive statistics for the main variables used in our empirical analysis. As we can see, there are almost 25 million observations and statistics for the following variables are provided: firm size (number of employees and wage bill), firm age in years, spread (difference between the loan interest rate and the deposit interest rate) rate, average maturity, non-performing loan (NPL), loan type (working capital, investment, foreign trade), earmarked loan, duration (in months) of the firm-bank relationship and number of banks a firm got loans in a particular year. We also have indicator variables for the location (state) of the firm, firm legal nature and economic sector.

Table A1: Summary Statistics: Credit Data (SCR)

	N	mean	sd	p10	p50	p90	min	max
Firm wagebill	25,441,646	66547.98	1827883.21	675.86	4989.79	49174.29	0.00	1.10e+09
Firm age	25,441,646	15.67	20.63	2.56	9.93	29.59	0.00	118.62
Spread	25,441,646	77.10	93.26	3.08	31.51	218.27	-14.32	607.78
NPL	25,441,646	0.05	0.43	0.00	0.00	0.15	0.00	1996.88
Default	25,441,646	0.04	0.20	0.00	0.00	0.00	0.00	1.00
Maturity	25,441,646	13.79	32.62	1.27	3.87	35.29	0.00	651.83
Earmarked loan	25,441,646	0.48	0.50	0.00	0.00	1.00	0.00	1.00
BNDES loan	25,441,646	0.00	0.04	0.00	0.00	0.00	0.00	1.00
Firm-Bank relat.	25,441,646	81.84	141.92	2.35	36.99	185.55	0.00	1407.20
Number of banks	25,441,646	2.89	1.99	1.00	2.00	5.00	1.00	36.00

A.2 Moments

Here we provide the definition, source and value of the moments used in Section 4 of the paper to internally calibrate the model parameters.

1. Exit rate: 11%. Source: RAIS. We calculate the exit rate using the RAIS dataset from 2006 to 2016.
2. Average firm size (number of employees): 13. Source: RAIS.
3. Average firm growth rate (growth in number of employees): 3.4%. Source: RAIS.
4. Top 10-percentile employment share: 77%. Source: RAIS.
5. Top 10-percentile earnings share: 55.6%. Source: Morgan (2017).
6. Capital-output ratio: The capital-output ratio is calculated from Feenstra et al. (2015), the Penn World Tables (PWT) 10.01. We use the share of gross capital formation at current PPPs and the perpetual inventory method to construct the capital to output ratio, assuming that the Brazilian economy is on the balanced growth path from 1995 to 2005. The capital to output ratio is then 2.55 in 2016.
7. Real risk free interest rate: 2%. For the real interest rate in Brazil, we took the average rate from 2005.1 to 2016.12 of the monthly Over/Selic interest rate (Brazilian Central Bank rate) minus the inflation rate measured by the IGP-DI (General price index from Vargas Foundation). We then annualized the monthly average real interest rate, and we get 5.87%. We also deducted country default risks, measured by the sovereign default spreads. Damodaran (2020) shows that default spreads varied from 2 to 4% in the period (see Figure 13 of this paper) and the value in April. Therefore, we set the interest rate in the small open economy to 2%.
8. Credit to output ratio: we target a value of 48.7%. Following Buera, Kaboski, and Shin (2011), we define this by the sum of the credit to non-financial corporations, private bond market capitalization, and stock market capitalization.
9. The average spreads once some factors are washed out: 64%. We calculate this by using the predicted value of the Regression (8) in Table 1. We set maturity at 1 year (as in our model) and non-performing loan (lag, contemporaneous and lead) at 0 (there is no default in our model). We then calculate the average predicted spread and the resulting value is 64%, which is about 13 percentage points lower than the unconditional average spread. Source: Our dataset based on the SCR and RAIS.
10. Weighted average spreads: From the predicted spread above, we calculate the credit-weighted spread of 45%. Source: Our dataset based on the SCR and RAIS.

11. Standard deviation of spreads: 40%. Following the same procedure as above, we calculate the standard deviation of spreads once some loan effects are washed out. The value we target is: 40%, which is about 53 percentage points lower than the unconditional standard deviation of spreads. Source: Our dataset based on the SCR and RAIS.
12. Entrepreneurs with credit: 26%. We use an extensive margin measure of credit. Roughly 26% of entrepreneurs have credit. Source: Our dataset based on the SCR and RAIS.

B Theory Appendix

We provide proofs of Propositions 1 and 2 together with an analytical derivation of Figure 1.

B.1 Proof of Proposition 1

Notice that $x_k = r + \tau(a, z)$ and we have that

$$\tilde{r}(a, z) = r + \tau(a, z) + \frac{\chi}{k^b(a, z) - a} (y^b(a, z) + \tau(a, z)a - \kappa - \tilde{w}(a, z)).$$

The proof when $\chi = 0$ is trivial.

When $\chi = 1$ and $\tau(a, z) = \tau_0$, then by the implicit function theorem, we have that

$$\frac{\partial \tilde{r}(a, z)}{\partial z} = \frac{\frac{\partial y^b(a, z)}{\partial z} - (\tilde{r}(a, z) - (r + \tau_0)) \frac{\partial k^b(a, z)}{\partial z} - \frac{\partial \tilde{w}(a, z)}{\partial z}}{k^b(a, z) - a}.$$

This can be rewritten as:

$$\frac{\partial \tilde{r}(a, z)}{\partial z} = \frac{y^b(a, z) - (\tilde{r}(a, z) - (r + \tau_0))k^b(a, z) - (1 - \alpha - \theta) \left(z a^\alpha \left(\frac{\theta}{w} \right)^\theta \right)^{\frac{1}{1-\theta}} I_{w < \tilde{\pi}}}{z(1 - \alpha - \theta)(k^b(a, z) - a)},$$

where $I_{w < \tilde{\pi}}$ is an indicator function which takes value equal to one when $w < \tilde{\pi}(a, z)$ and zero otherwise. The denominator of this expression is clearly positive. The numerator is positive for any $S^e \geq 0$.

It can also be shown that

$$\frac{\partial \tilde{r}(a, z)}{\partial a} = \frac{\tilde{r}(a, z) - r - \frac{\partial \tilde{w}(a, z)}{\partial a}}{k^b(a, z) - a}.$$

The denominator for this expression is always positive for entrepreneurs (a, z) with $a <$

$k^u(a, z)$ and $S^e \geq 0$. If $w > \tilde{\pi}(a, z)$, then $\frac{\partial \tilde{w}(a, z)}{\partial a} = 0$ and

$$\frac{\partial \tilde{r}(a, z)}{\partial a} = \frac{\tilde{r}(a, z) - r}{k^b(a, z) - a} > 0,$$

which is positive, otherwise financial intermediaries surplus would be negative. If $w < \tilde{\pi}(a, z)$, then

$$\frac{\partial \tilde{r}(a, z)}{\partial a} = \frac{\tilde{r}(a, z) - \alpha \left(z a^\alpha \left(\frac{\theta}{w} \right)^\theta \right)^{\frac{1}{1-\theta}} \frac{1}{a}}{k^b(a, z) - a} < 0.$$

The numerator for this case has to be negative for an agent (a, z) with $a < k^u(a, z)$ and $S^e \geq 0$. In order to see this, notice that $\alpha \left(z a^\alpha \left(\frac{\theta}{w} \right)^\theta \right)^{\frac{1}{1-\theta}} \frac{1}{a}$ corresponds to the marginal productivity net of labor cost of an additional capital at the collateral of an entrepreneur who is borrowing. This object has to be larger than her cost of capital, $\tilde{r}(a, z)$, otherwise she will not borrow.

If $\tau(a, z) = \tau_0$, $S^e \geq 0$ and $a < k^u(a, z)$ but the incentive compatible constraint is not binding, then clearly $\frac{\partial k^b(a, z)}{\partial a} = 0$. It can also be shown that

$$\frac{\partial k^b(a, z)}{\partial z} = \frac{y^b(a, z) - (\tilde{r}(a, z) - (r + \tau_0))k^b(a, z) - (1 - \alpha - \theta) \left(z a^\alpha \left(\frac{\theta}{w} \right)^\theta \right)^{\frac{1}{1-\theta}} I_{w < \tilde{\pi}}}{(1 - \alpha - \theta)z(\tilde{r}(a, z) - (r + \tau_0))}.$$

This is clearly positive for any $S^e \geq 0$, $a < k^u(a, z)$ and when the incentive compatible constraint does not bind.

When $\chi \in (0, 1)$, we have that in equilibrium $x_k = r + \tau(a, z)$, which is independent of χ . Therefore $\frac{\partial k^b(a, z)}{\partial \chi} = 0$. In addition,

$$\frac{\partial \tilde{r}(a, z)}{\partial \chi} = \frac{y^b(a, z) + \tau(a, z)a - \kappa - \tilde{w}(a, z)}{k^b(a, z) - a} > 0.$$

B.2 Proof of Proposition 2

When $\chi = 0$ and entrepreneurs' bargaining power is equal to one, then it is optimal to set $S^b = 0$. Then equation (7) implies

$$\phi x(k^b(a, z)) = (r + \tau)k^b(a, z),$$

and equation (8) requires:

$$\tilde{r}(a, z) = r + \tau(a, z).$$

Then the interest rate on loan will be negatively related with the asset value of each entrepreneur, a , and with her managerial productivity, z . Using the incentive compatible constraint we can show that $\frac{\partial k^b(a, z)}{\partial a} > 0$ and $\frac{\partial k^b(a, z)}{\partial z} > 0$, as required.

When $\chi = 1$ and $\tau(a, z) = \tau_0$, then it can be shown that

$$\frac{\partial \tilde{r}(a, z)}{\partial z} = \frac{(\phi x_k - \tilde{r})\tilde{w}_z(a, z) + \tilde{r}(1 - \phi)x_z}{(1 - \phi)x_k(k - a)}.$$

If $w > \tilde{\pi}(a, z)$, then $\tilde{w}_z(a, z) = 0$ and this expression is clearly positive. In addition, for this case $\frac{\partial k^b(a, z)}{\partial z} = -\frac{x_z}{x_k} < 0$. When $w < \tilde{\pi}(a, z)$, then $\tilde{w}_z(a, z) = x_z(a) > 0$ and we cannot determine the sign of $\frac{\partial \tilde{r}(a, z)}{\partial z}$. The first term of the numerator of the above equation is negative ($\phi x_k - \tilde{r} = \frac{\tilde{r}(\alpha + \theta - 1)k - \alpha \tilde{r}a}{(1 - \theta)k} < 0$.) while the second is positive. Moreover, $\frac{\partial k^b(a, z)}{\partial z} = \frac{x_z(a) - (1 - \phi)x_z}{(1 - \phi)x_k} < 0$, which can be positive or negative.

Notice also that:

$$\frac{\partial \tilde{r}(a, z)}{\partial a} = \frac{(\phi x_k - \tilde{r})(r + \tilde{w}_a(a, z)) + \tilde{r}(1 - \phi)x_k}{(1 - \phi)x_k(k - a)},$$

and

$$\frac{\partial k^b(a, z)}{\partial a} = \frac{r + \tilde{w}_a(a, z)}{(1 - \phi)x_k}.$$

If $w > \tilde{\pi}(a, z)$, then $\tilde{w}_a(a, z) = 0$ and $\frac{\partial k^b(a, z)}{\partial a} = \frac{r}{(1 - \phi)x_k} > 0$ and $\frac{\partial \tilde{r}(a, z)}{\partial a} = \frac{(\phi x_k - \tilde{r})r + \tilde{r}(1 - \phi)x_k}{(1 - \phi)x_k(k - a)}$.

It can be shown that the first term of the numerator is negative while the second is positive. Therefore, it is not clear how the loan interest rate varies with assets a .

If $w < \tilde{\pi}(a, z)$, then $\tilde{w}_a(a, z) = x_k(a) - r$ and $\frac{\partial k^b(a, z)}{\partial a} = \frac{x_k(a)}{(1 - \phi)x_k(k)} = \frac{1}{1 - \phi} \left(\frac{k}{a}\right)^{\frac{1 - \alpha - \theta}{1 - \theta}} > 1$. In addition, $\frac{\partial \tilde{r}(a, z)}{\partial a} = \frac{(\phi x_k - \tilde{r})x_k(a) + \tilde{r}(1 - \phi)x_k}{(1 - \phi)x_k(k - a)}$. Once more it is not possible to sign the numerator of the above expression and therefore it is not clear how the loan interest rate varies with assets a .

For the case in which $\chi \in (0, 1)$ and $\tau(a, z) = \tau_0$, we can easily show that

$$\frac{\partial k^b(a, z)}{\partial \chi} < 0,$$

while

$$\frac{\partial \tilde{r}(a, z)}{\partial \chi} = -\frac{((1 - \theta - \alpha)\tilde{r}k + \alpha \tilde{r}a)\frac{\partial k^b(a, z)}{\partial \chi}}{k^b(a, z) - a} > 0.$$

B.3 Analytical Derivation of Figure 3

Consider the case in which $\chi = 0$. Let $d \leq a$ be the amount of assets entrepreneurs use in their business, and let l be loans, such that $k = d + l$. Clearly, since $\tilde{r} = r + \tau(a, z) \geq r$ for all finite a , then if $l > 0$, then $d = a$. The problem of the entrepreneur can be rewritten as

$$\pi(a, z) = \max_{n, d, l \geq 0} z(d + l)^\alpha n^\theta - wn - \tilde{r}l - rd - \kappa, \quad (16)$$

subject to

$$l \leq \frac{\phi(z(d+l)^\alpha n^\theta - wn)}{\tilde{r}}, \quad \text{with } \tilde{r} = r + \tau_0 + \frac{\tau_a}{1+a} + \frac{\tau_z}{1+z}, \quad (17)$$

$$d \leq a. \quad (18)$$

The Lagrangean associated with this problem is:

$$L = z(d+l)^\alpha n^\theta - wn - \tilde{r}l - rd - \chi + \lambda \left[\frac{\phi(z(d+l)^\alpha n^\theta - wn)}{\tilde{r}} - l \right] + \mu[a-d]$$

The Kuhn-Tucker conditions are:

$$\frac{\partial L}{\partial n} = \left(\theta \frac{y}{n} - w \right) \left(1 + \lambda \frac{\phi}{\tilde{r}} \right) \leq 0, \quad n \geq 0, \quad \frac{\partial L}{\partial n} n = 0, \quad (19)$$

$$\frac{\partial L}{\partial d} = \alpha \frac{y}{k} \left(1 + \lambda \frac{\phi}{\tilde{r}} \right) - r - \mu \leq 0, \quad d \geq 0, \quad \frac{\partial L}{\partial d} d = 0, \quad (20)$$

$$\frac{\partial L}{\partial l} = \alpha \frac{y}{k} \left(1 + \lambda \frac{\phi}{\tilde{r}} \right) - \tilde{r} - \lambda \leq 0, \quad l \geq 0, \quad \frac{\partial L}{\partial l} l = 0, \quad (21)$$

$$\mu[a-d] = 0, \quad (22)$$

$$\lambda \left[\frac{\phi(z(d+l)^\alpha n^\theta - wn)}{\tilde{r}} - l \right] = 0. \quad (23)$$

Case 1: If $0 < d < a$, then $\mu = 0$ and $\lambda = 0$. Therefore:

$$\theta \frac{y}{n} = w, \quad \alpha \frac{y}{k} = r, \quad \text{and} \quad \alpha \frac{y}{k} < \tilde{r}.$$

It can be shown that

$$k^u(z) = \left(z \left(\frac{\alpha}{r} \right)^{(1-\theta)} \left(\frac{\theta}{w} \right)^\theta \right)^{\frac{1}{1-\alpha-\theta}}, \quad n^u(z) = \left(z \left(\frac{\alpha}{r} \right)^\alpha \left(\frac{\theta}{w} \right)^{1-\alpha} \right)^{\frac{1}{1-\alpha-\theta}}.$$

And

$$y^u(z) = \left(z \left(\frac{\alpha}{r} \right)^\alpha \left(\frac{\theta}{w} \right)^\theta \right)^{\frac{1}{1-\alpha-\theta}},$$

with

$$\pi^u(z) = (1 - \alpha - \theta)y^u(z) - \kappa.$$

Therefore, $\pi^u(z) \geq w$ defines a threshold ability level z^u given by

$$z^u = \left(\frac{w + \kappa}{1 - \alpha - \theta} \right)^{1-\alpha-\theta} \left(\frac{r}{\alpha} \right)^\alpha \left(\frac{w}{\theta} \right)^\theta,$$

such that for all (a, z) with $a > k^u(z)$, and $z \geq z^u$ agents are entrepreneurs. Notice that z^u is independent of a . Since $k^u(z)$ is increasing with z , we can define a threshold of assets $\bar{a}^u(z)$,

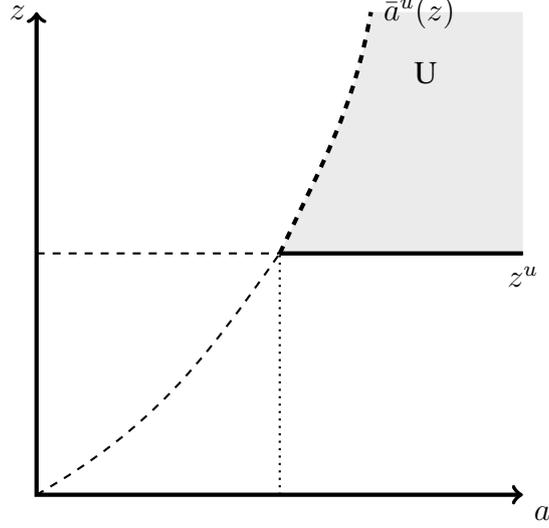


Figure B1: Case 1: $k^u(z) \leq a$. Light gray shaded area shows the measure of agents who are unconstrained entrepreneurs.

such that all agents with $z > z^u$ and $a > \bar{a}^u(z)$ are unconstrained entrepreneurs. Figure B1 shows the region in which entrepreneurs are unconstrained and do not borrow.

Case 2: If $d = a > 0$ and $l = 0$, then $\mu > 0$ and $\lambda = 0$. Consequently:

$$\theta \frac{y}{n} = w, \quad \alpha \frac{y}{k} = r + \mu, \quad \text{and} \quad \alpha \frac{y}{k} < \tilde{r}.$$

It can be shown that

$$\begin{aligned} k^{nb}(a, z) &= a, \\ n^{nb}(a, z) &= \left(z a^\alpha \left(\frac{\theta}{w} \right) \right)^{\frac{1}{1-\theta}}, \\ y^{nb}(a, z) &= \left(z a^\alpha \left(\frac{\theta}{w} \right)^\theta \right)^{\frac{1}{1-\theta}}, \end{aligned}$$

and

$$\pi^{nb}(a, z) = (1 - \theta)y^{nb}(a, z) - ra - \kappa.$$

Condition $\pi^{nb}(a, z) \geq w$ defines a threshold ability level $z^{nb}(a, z)$ given by

$$z^{nb}(a, z) = \left(\frac{w + \kappa + ra}{1 - \theta} \right)^{1-\theta} \left(\frac{w}{\theta} \right)^\theta \frac{1}{a^\alpha},$$

such that for all agents with $k^{nb}(a, z) = a$, and $z \geq z^{nb}(a)$ agents are entrepreneurs. Observe

that since $1 - \alpha - \theta > 0$, then $\lim_{a \rightarrow 0} z^{nb}(a) = \infty$. It can be shown that

$$\text{sign} \left(\frac{\partial z^{nb}(a)}{\partial a} \right) = \text{sign} \left((1 - \alpha - \theta)ra - \alpha(w + \kappa) \right).$$

Notice that since $\alpha \frac{y}{a} = r + \mu$ at $z^{nb}(a)$, such that $(1 - \theta)y^{nb}(a, z) - ra - \kappa = w$, we have

$$\text{sign} \left(\frac{\partial z^{nb}(a)}{\partial a} \right) = \text{sign}(-\mu a).$$

This is clearly negative, as long as $\mu > 0$. Therefore as $a \rightarrow \bar{a}^u$, then $z^{nb}(a) \rightarrow z^u$.

Case 3: If $d = a > 0$ and $0 < l < \frac{\phi(z(d+l)^{\alpha} n^{\theta} - wn)}{\bar{r}}$, then $\mu > 0$ and $\lambda = 0$. Consequently:

$$\theta \frac{y}{n} = w, \quad \alpha \frac{y}{k} = r + \mu, \quad \text{and} \quad \alpha \frac{y}{k} = \tilde{r}.$$

It can be shown that

$$\begin{aligned} k^b(a, z) &= \left(z \left(\frac{\alpha}{\tilde{r}} \right)^{(1-\theta)} \left(\frac{\theta}{w} \right)^{\theta} \right)^{\frac{1}{1-\alpha-\theta}}, \\ n^b(a, z) &= \left(z \left(\frac{\alpha}{\tilde{r}} \right)^{\alpha} \left(\frac{\theta}{w} \right)^{1-\alpha} \right)^{\frac{1}{1-\alpha-\theta}}, \\ y^b(a, z) &= \left(z \left(\frac{\alpha}{\tilde{r}} \right)^{\alpha} \left(\frac{\theta}{w} \right)^{\theta} \right)^{\frac{1}{1-\alpha-\theta}}, \end{aligned}$$

and

$$\pi^b(a, z) = (1 - \alpha - \theta)y^b(a, z) + (\tilde{r} - r)a - \kappa.$$

Therefore, given that $\tilde{r} = r + \tau_0 + \frac{\tau_a}{1+a} + \frac{\tau_z}{1+z}$, the inequality $\pi^b(a, z) \geq w$ defines an ability level $z^b(a, z)$ given by

$$z^b(a, z) = \left(\frac{w + \kappa - (\tilde{r} - r)a}{1 - \alpha - \theta} \right)^{1-\alpha-\theta} \left(\frac{r}{\alpha} \right)^{\alpha} \left(\frac{w}{\theta} \right)^{\theta},$$

such that for all agents with $a < k^b(a, z) < \frac{\phi(z(d+l)^{\alpha} n^{\theta} - wn)}{\bar{r}}$, and $z \geq z^b(a, z)$, then agents are entrepreneurs. Observe that $\frac{\partial z^b(a, z)}{\partial a} < 0$.

Cases 2 and 3 imply that for all $a \in [0, \bar{a}^u]$, there will be a productivity level $z^e(a, z) = \max\{z^u, \min(z^{nb}(a, z), z^b(a, z))\}$ such that $z^e(\bar{r}, w, \bar{a}^u(r, w)) = z^u(r, w)$, $\frac{\partial z^e(\bar{r}, w, a)}{\partial a} < 0$, and $\lim_{a \rightarrow 0} z^{nb}(a, z) = \infty$. In addition, whenever $z \geq z^e(\bar{a}^u, z)$, then the agent is an entrepreneur. See Figure B3

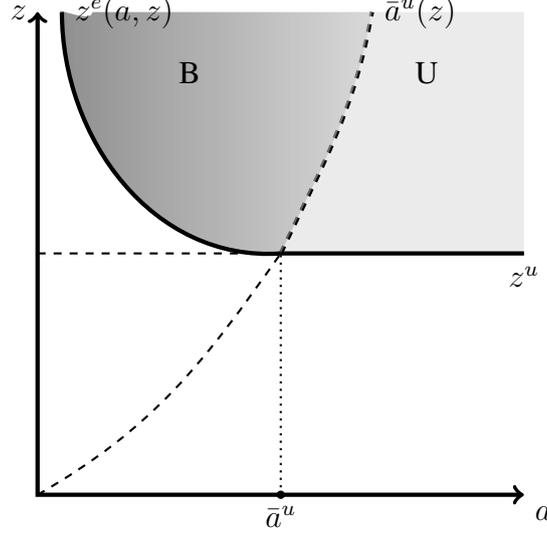


Figure B2: Cases 2 and 3: $k^u(z) < a$ and $0 \leq k - a < \frac{\phi(z(a+l)^{\alpha}n^{\theta}-wn)}{\tilde{r}}$. Light gray shaded area shows the measure of agents who are unconstrained entrepreneurs. Dark gray shaded area shows the measure of agents who are not constrained borrowers.

Case 4: If $d = a > 0$ and $l = \frac{\phi(z(a+l)^{\alpha}n^{\theta}-wn)}{\tilde{r}}$, then $\mu > 0$ and $\lambda > 0$. Consequently:

$$\theta \frac{y}{n} = w, \quad \alpha \frac{y}{k} \left(1 + \lambda \frac{\phi}{\tilde{r}}\right) = r + \mu, \quad \text{and} \quad \alpha \frac{y}{k} \left(1 + \lambda \frac{\phi}{\tilde{r}}\right) = \tilde{r} + \lambda.$$

Given that the amount of capital is constrained, it must be the case that $\alpha \frac{y}{k} > \tilde{r}$. The labor first-order condition yields:

$$n(w; k^c, z) = \left(z (k^c)^{\alpha} \left(\frac{\theta}{w} \right) \right)^{\frac{1}{1-\theta}},$$

where k^c solves

$$k^c = a + \frac{\phi(z(k^c)^{\alpha}n(w; k^c, z)^{\theta} - wn(w; k^c, z))}{\tilde{r}}.$$

This equation defines

$$k^c = k^c(\tilde{r}, w; z, a), \quad \text{with} \quad \frac{\partial k^c}{\partial a} > 0, \quad \frac{\partial k^c}{\partial z} > 0.$$

The derivatives can be checked using the Implicit Function Theorem.

We have that

$$y^c(\tilde{r}, w; z, a) = \left(z k^c(\tilde{r}, w; z, a)^{\alpha} \left(\frac{\theta}{w} \right)^{\theta} \right)^{\frac{1}{1-\theta}}$$

and

$$\pi^c(\tilde{r}, w; z, a) = (1 - \theta)(1 - \phi)y^c(\tilde{r}, w; z, a) - ra - \kappa.$$

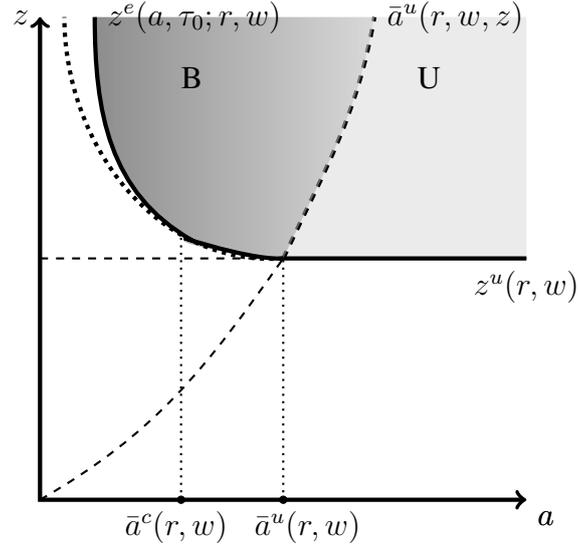


Figure B3: Cases 4: $k^u(r, w) < a$ and $k - a = \frac{\phi(z(a+l)^{\alpha}n^{\theta} - wn)}{\tilde{r}}$. Light gray shaded area shows the measure of agents who are unconstrained entrepreneurs. Dark gray shaded area shows the measure of agents who are not constrained borrowers.

Condition $\pi^c(\tilde{r}, w; z, a) \geq w$ defines a threshold ability level $\bar{z}^c(\tilde{r}, w; a)$, which is decreasing in a as long as $\lambda > 0$. We can show that $\lim_{a \rightarrow 0} \bar{z}^c(\tilde{r}, w; a) = \infty$. Observe that when $\lambda = 0$ and $l = \frac{\phi(z(a+l)^{\alpha}n^{\theta} - wn)}{\tilde{r}}$, then for agents who are indifferent to be entrepreneurs or workers, we have that $\bar{z}^c(\tilde{r}, w; a) = \bar{z}^b(\tilde{r}, w; a)$. This defines a value $\bar{a}^c(w, \tilde{r})$, such that whenever $a < \bar{a}^c(w, \tilde{r})$ and $\bar{z}^b(\tilde{r}, w; a) \leq z \leq \bar{z}^c(\tilde{r}, w; a)$, the leverage constraint is binding. For such agents, then $l = \frac{\phi(z(a+l)^{\alpha}n^{\theta} - wn)}{\tilde{r}}$ and $\lambda > 0$, and $\bar{z}^c(\tilde{r}, w; a) > \bar{z}^b(\tilde{r}, w; a)$, in order to compensate for the low capital used. Therefore for any $\bar{z}^b(\tilde{r}, w; a) \leq z \leq \bar{z}^c(\tilde{r}, w; a)$ and $a < \bar{a}^c(w, \tilde{r})$, the occupational choice is restricted by the leverage ratio. This is shown in the figure below.

C Quantitative Results for the Small Open Economy

Table C2: Small Open Economy: Impacts of Credit Frictions on Development

Value	Perfect Credit		No Spread	Only Quant. Constr.
Relative to Perfect Credit:	(1)	Benchmark (2)	Disp. (3)	(4)
<i>Aggregate values relative to perfect credit world:</i>				
GDP	1.00	0.33	0.34	0.41
TFP	1.00	0.72	0.76	0.84
Wage	1.00	0.36	0.41	0.43
Capital	1.00	0.09	0.09	0.12
Credit/GDP	1.00	0.08	0.08	0.08
Interest rate	0.02	0.02	0.02	0.02
Firm growth	0.08	0.03	-0.02	0.04
Exit rate	0.29	0.10	0.17	0.15
Avg. firm size	9	14	9	8

Notes: Column (2) parameter values are those calibrated in Table 2. The other columns keep the interest rate constant while changing parameters. Relative to benchmark values, Column (1): $\tau_i = \chi = 0$, Column (3): $\tau_a = \tau_z = \chi = 0$ and $\tau_0 = 0.30$, and Column (4): $\tau_i = \chi = 0$ and $\phi = 0.04$, calibrated to match credit/GDP in the benchmark (and data).

Table C3: Small Open Economy: Isolated Impacts of Spread-Causing Frictions

	Eliminating Frictions					Single Friction Calibrations			
	Benchmk (1)	No Market Power (2)	No Uniform Cost (3)	No a - depend. Cost (4)	No z - depend. Cost (5)	All Market Power [†] (6)	All Uniform Cost (7)	All a - depend. Cost (8)	All z - depend. Cost (9)
<i>Aggregate values relative to perfect credit world:</i>									
GDP	0.33	0.33	0.36	0.35	0.43	0.25	0.35	0.24	0.35
TFP	0.72	0.72	0.72	0.75	0.74	0.58	0.77	0.59	0.75
Wage	0.36	0.36	0.40	0.40	0.47	0.26	0.42	0.25	0.38
Capital	0.09	0.10	0.12	0.10	0.19	0.08	0.09	0.06	0.10
Credit/GDP	0.08	0.08	0.17	0.09	0.31	0.13	0.09	0.00	0.00
<i>Firm credit spread moments:</i>									
Interest rate	0.02	0.02	0.02	0.02	0.02	0.07	0.02	0.02	0.02
Avg. (weighted)	0.28	0.17	0.27	0.11	0.12	0.03*	0.28	0.17*	0.28
Avg. (unweighted)	0.69	0.67	0.55	0.40	0.33	0.02	0.28	0.17	0.51
Std. deviation	0.34	0.33	0.33	0.06	0.28	0.01	0.00	0.01	0.11
Frac. with credit	0.23	0.23	0.28	0.38	0.46	0.52	0.58	0.00	0.29
Firm growth	0.03	0.03	0.05	-0.00	0.12	0.04	-0.01	0.03	0.00
Exit rate	0.10	0.10	0.11	0.14	0.14	0.03	0.19	0.02	0.12
Avg. firm size	14	14	17	11	21	9	9	7	11

Notes: Column (1) parameter values are those calibrated in Table 2. The other columns keep the interest rate constant while changing parameters. Relative to these values, Column (2): $\chi = 0$, Column (3): $\tau_0 = 0$, Column (4): $\tau_a = 0$, Column (5): $\tau_z = 0$, Column (6): $\tau_i = 0$ and $\chi = 0.01$, Column (7): $\tau_a = \tau_z = \chi = 0$ and $\tau_0 = 0.28$, Column (8): $\tau_0 = \tau_z = \chi = 0$ and $\tau_a = 457$, and Column (9): $\tau_0 = \tau_a = \chi = 0$ and $\tau_z = 1.65$. For Columns (6) and (8), the calibrated value is chosen to match the weighted spread in the benchmark.

[†]: The single parameter alone cannot match the weighted spread of 0.28. Hence, Columns (6) and (8) yield spreads substantially below target but approaching their peaks.

D Additional Results for One Friction at a Time

Table D4: Benchmark: One Friction at a Time

	Benchmark (1)	Only Market Power (2)	Only Uniform Cost (3)	Only a - depend. Cost (4)	Only z - depend. Cost (5)
GDP	0.61	0.99	0.82	0.88	0.68
TFP	0.72	1.00	0.90	0.89	0.79
Wage	0.68	0.99	0.91	0.86	0.81
Capital	0.59	0.98	0.74	0.97	0.64
Credit/GDP	0.32	0.99	0.81	1.04	0.65
Interest rate	0.019	0.144	0.100	0.112	0.036
Avg. Unweighted	0.692	0.005	0.084	0.195	0.302
Avg. Weighted	0.282	0.007	0.084	0.020	0.169
Std. Deviation	0.343	0.003	0.000	0.246	0.063
Frac. with credit	0.232	0.962	0.909	0.761	0.602
Firm growth	0.034	0.060	0.055	0.228	0.033
Exit rate	0.098	0.297	0.292	0.287	0.262
Avg. firm size	14	11	11	30	16

Notes: Column (1) provides results for the benchmark, with the parameters from Table 2. The other columns shut off all spread-causing parameters but one: Column (2) has $\tau_i = 0$ and $\chi = 0.10$; Column (3) has $\chi = \tau_a = \tau_z = 0$ and $\tau_0 = 0.08$; Column (4) has $\chi = \tau_0 = \tau_z = 0$ and $\tau_a = 0.73$; Column (5) has $\chi = \tau_a = \tau_0 = 0$ and $\tau_z = 0.93$.

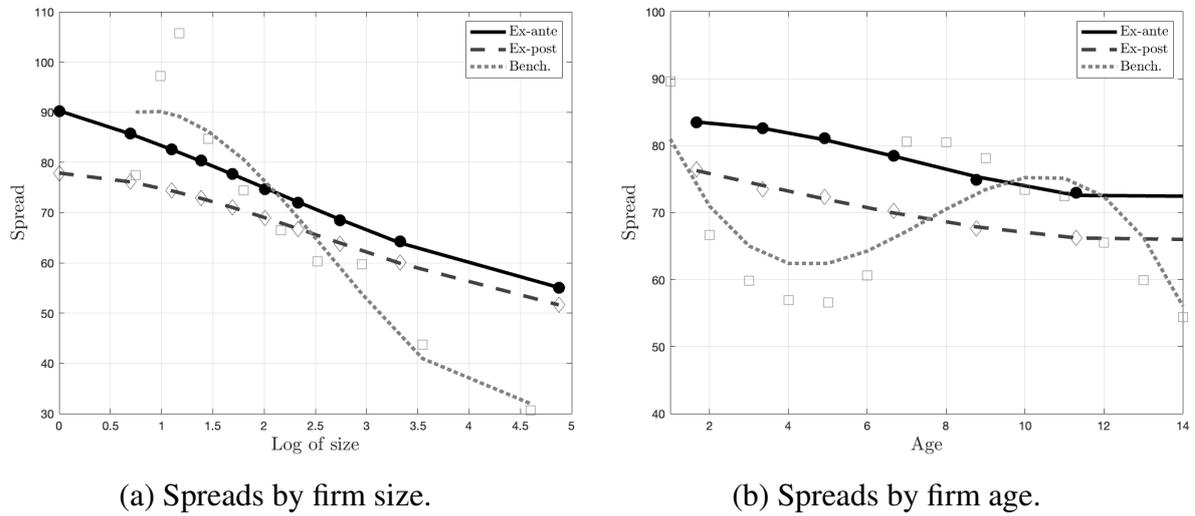
E Calibration for the Scenario with Moderate Spreads

Table E5: Calibration and Model Fit, Moderate Spreads

Parameter Values		
Parameter	Description	Value
<i>2 assigned parameters</i>		
σ	Coefficient of relative risk aversion	1.50
δ	Depreciation rate	0.03
<i>11 calibrated parameters</i>		
ρ	Subjective discount rate	0.24
α	Elast. of y with respect to k	0.33
θ	Elast. of y with respect to n	0.39
κ	Fixed cost of production	0.66
η	Curvature of the Pareto distr.	3.41
γ	New productivity arrival rate	0.30
ϕ	Enforcement parameter	0.20
τ_0	Interm. costs - independent factor	0.05
τ_a	Interm. costs - elast. of assets	0.91
τ_z	Interm. costs - elast. of productivity	0.29
χ	Bank barg. power in a loan	0.52
Model Fit		
12 Targeted Moments	Data	Model
Risk-free bond rate	0.020	0.019
Capital-output ratio	2.55	1.92
Average firm growth rate	0.034	0.035
Top 10% earners' income share	0.56	0.55
Average firm size	13	17
Top 10% firms' employment share	0.77	0.75
Firm exit rate	0.11	0.07
External finance to GDP ratio	0.49	0.57
Fraction firms with credit	0.26	0.30
Average spread (unweighted)	0.28	0.34
Average spread (credit-weighted)	0.14	0.14
Standard deviation of spread	0.23	0.19

F The Relationship between Spreads and Firm Characteristics: Model and Data

Figure F4: Ex Ante and Ex Post Spreads



Notes: Ex ante spreads are based on contracted interest rates. Ex post spreads are calculated by setting the interest rate to -100% for loans in default.